CALCULATION OF ELECTROSTATIC FIELDS IN VAPOUR SHIELDS EVOLVING AT ABLATING SURFACES

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Abstract

Three phenomena are considered in connection with the problem of electrostatic shielding of eroding materials bombarded by magnetically confined plasma particles: (a) the potential jump at the hot plasma – cold vapour interface, (b) the electric field distribution in the interior of the vapour layer, and (c) the boundary conditions which the electric field must satisfy at the eroding wall.

Introduction

When a solid surface is directly exposed to the intense energy flux carried by hot incident plasma particles (divertor plates in tokamaks, pellets injected into plasmas), the surface begins to evaporate (ablade). In a relatively short time (μs time scale), a high-density and relatively cold partially ionized cloud forms at the surface, which intercepts at least a fraction of the energy carriers originating from the plasma and penetrating the cloud.

The incident particle fluxes are depleted by collisional interaction with the cloud particles. The particle and energy fluxes are reduced by elastic and elastic/inelastic scattering processes, respectively. While the incident ions are stopped at the cloud periphery, the energetic electrons penetrate further inward. As a result of the different stopping lengths, the cloud initially receives a negative excess charge, as a result of which the energy flux carried by the electrons is impaired. As time goes on, the ionized cloud particles streaming away from the surface form a return current, which tends to balance the current carried by the incident particles. The magnitude of the return current is a function of the cloud conductivity, which may vary from zero at the eroding surface to the Spitzer value at the fully ionized cloud periphery, which is in contact with the hot background plasma.

Under steady-state conditions, no further charge accumulation may take place and the total current given by the sum of the currents carried by the various hot and cold charge carriers must vanish in any cloud cross-section. The resultant electric field is defined by the condition of vanishing total current (ambipolarity).
This electric field affects the magnitude of the incident particle fluxes and thus the energy flux at the eroding surface. The self-consistent calculation of this field is thus a condition for accurate determination of the erosion rate itself.

In early pellet ablation analyses [1], [2] it was assumed that the incident electron flux is balanced by an outward-directed flux of cold thermal electrons at any cloud cross-section. The effects of ions and of a possible mismatch between the regions where the electrons are stopped and the cold electrons produced (by collisional ionization) were not taken into account. In later analyses it was proposed by Rozhansky [3, 4] that electrostatic shielding may have an important effect on the magnitude of the energy flux reaching the surface. It has also been shown that an additional potential drop may develop at the cold-cloud - hot-plasma interface ("double layer"), causing additional shielding [3], [5]. The stopping length calculations performed in connection with an ablation analysis, as reported in [6], used an improved model, based on the Braginskii equations, but still neglecting ion effects and the alterations needed to describe the partially ionized cloud zone.

In this report, a model is proposed for calculating the electrostatic shielding field by taking into account also the effect of the cold ions and the formation of a double layer at the cloud - plasma interface. Continuum approach is used. The results obtained are directly applicable to models in which the energy distribution functions are approximated by discrete energy groups.
Double Layer at the Hot Plasma - Cold Cloud Interface

The scenario considered is shown in Fig. 1. An eroding wall is covered by a low-temperature, high-density, partially ionized collisional plasma (vapour) that is expanding in the direction normal to the wall. The space outside the vapour blanket is occupied by a high-temperature, low-density plasma. The particle fluxes are denoted by $\Gamma$. Subscripts $e$, $i$, $h$, and $c$ denote electron, ion, hot, and cold, respectively.

An expression is sought for the value of the potential jump $\Delta \Phi$ across the interface ("double layer") separating the two regions under the following assumptions:

- the secondary electron emission is negligible,
- the background (hot) plasma is collisionless,
- the cold ion current $\Gamma_{ic}$ can be neglected (strictly true only in a coordinate system attached to and moving with the interface).

Also the effect of the hot ions penetrating into the double layer and stopped shortly beyond this sheath is neglected in this approximation.

Let us now consider the conditions of ambipolarity and charge neutrality.

A. The condition of zero current (see Fig. 1):

\[
\Gamma_{eh} - \Gamma_{ih} - \Gamma_{ec} = 0,
\]

Figure 1
where

$$\Gamma_{eh} = \Gamma_{eh}^+ - \Gamma_{eh}^-.$$  

Superscripts $^+$ and $^-$ denote particles moving towards the eroding surface (to the left) and away from it (to the right), respectively. Defining now a critical velocity

$$\frac{1}{2} m_e v_{\text{crit}}^2 = e\Delta \Phi, \quad \nu_{\text{crit}} \equiv \left(2 \frac{e}{m_e} \Delta \phi \right)^{1/2},$$

where $v_{\text{crit}}$ represents a threshold velocity. All hot electrons coming from the right with energies less than $e\Delta \phi$ are reflected.

Denoting the Maxwellian energy distribution function by $f_M$,

$$f_M = \frac{n_e(\infty)}{(2\pi k T_{eh})^{1/2}} \exp\left(-\frac{1}{2} \frac{m_e v_x^2}{k T_{eh}} \right),$$

we have

$$\Gamma_{eh}^+ = \int_0^\infty f_M v_x dv_x.$$  

The reflected fraction is represented by the integral

$$\Gamma_{eh}^- = \int_0^{v_{\text{crit}}} f_M v_x dv_x,$$

whereas the passing fraction by

$$\Gamma_{eh} = \Gamma_{eh}^+ - \Gamma_{eh}^- = \int_{v_{\text{crit}}}^{\infty} f_M v_x dv_x.$$  

With regard to the cold electron current $\Gamma_{ec}$:

$$\Gamma_{ec} = v_{ec} n_{ec},$$

where the velocity $v_{ec} = v_{eco} + v_{\text{crit}} \approx v_{\text{crit}}$.

The velocity $v_{eco}$, representing the initial velocity of the eroded particles, is usually negligible compared with the velocity acquired across the potential sheath. The magnitude
of $n_{ec}$ is determined from the condition of charge neutrality in any cross-section (see below).

Obviously, the magnitude of the hot ion current is given by

$$\Gamma_{ih} = \left(\frac{kT_i}{m_i}\right)^{1/2} n_i.$$

B. The condition of charge neutrality

Obviously, $n_{eh}^+ = \int_0^\infty f_M du_z = \frac{1}{2} n_i(\infty)$,

and

$$n_{eh}^- = \int_0^{v_{erit}} f_M du_z.$$

The condition of charge neutrality can thus be written for the high-temperature collisionless domain as

$$n_{eh}^+ + n_{eh}^- + n_{ec} = n_i(\infty).$$

$$\frac{1}{2} n_i(\infty) \quad \frac{1}{2} n_i(\infty)$$

Since $n_{eh}^+ = \frac{1}{2} n_i(\infty)$, in order to satisfy the above equality, the sum of the last two terms on the l.h.s. of the equation must also be equal to $\frac{1}{2} n_i(\infty)$.

Hence the density of the cold electrons streaming to the right is given as

$$n_{ec} = \int_0^\infty f_M du_z - \int_0^{v_{erit}} f_M du_z = \int_0^{v_{erit}} f_M du_z,$$

where the last term represents the deficiency of the hot electrons lost to the cold region.

Returning now to the condition of zero current, the sum of the particle fluxes can be written as

$$\int_{v_{erit}}^{\infty} f_M u_z du_z - \left(\frac{kT_i}{m_i}\right)^{1/2} n_i(\infty) - \left(\frac{e}{m_e}\Delta \phi\right)^{1/2} \int_{v_{erit}}^{\infty} f_M du_z = 0.$$

Making the substitution $\Delta \phi^* \equiv \frac{e \Delta \phi}{kT_{ih}}$ and integrating the above expressions, we obtain
\[
\exp(-\Delta \phi^*) - \left(2\pi \frac{m_e}{m_i}\right)^{1/2} - \sqrt{\Delta \phi^*} \sqrt{\pi e rf(\sqrt{\Delta \phi^*})} = 0
\]

\[F_1(\Delta \phi^*) \quad F_2(\Delta \phi^*)\]

Figure 2

The variations of the functions \(F_1(\Delta \phi^*)\) and \(F_2(\Delta \phi^*)\) are shown in Fig. 2. The solution (numerical or graphical) of the above equation yields

\[
\Delta \phi^* \approx 1.7 \text{ to } 1.8.
\]

Since \(\exp(1.7) \approx 5.6\), the electrostatic screening effect of the double layer is substantial:

\[
\Gamma_{eh} = \Gamma_{eh}(z \to \infty) \exp(-\Delta \phi^*).
\]

The effect of the hot ions stopped at the hot plasma - cold cloud interface on the magnitude of this shielding factor is still to be assessed.
Determination of the Electrostatic Field Distribution in the Vapour Cloud Interior

The scenario considered is shown in Fig. 3. The region extends from the eroding surface on the l.h.s. up to the cloud - plasma interface at the r.h.s. boundary. The substance may be completely unionized at the eroding surface and fully ionized in the vicinity of the cloud - plasma interface.

Denoting the current density carried by the species $l$ by $j_l$, one can write the zero current condition for an arbitrary cloud cross-section as

$$j = j_{eh} + j_{ec} + j_{ic} = 0,$$

where

$$j_{eh} = \sum_k e \Gamma_{eh}^{(k)}$$

(multi-group approximation is used for simulating the energy distribution function, $k$ being the group index).

Since $j_{ec} = -n_e e u_e$, and $j_{ic} = n_i e u_i$, one needs two equations for determining $u_e$ and $u_i$. The equations sought are the respective momentum conservation equations. For the electron component one may write

$$-\frac{\partial}{\partial z} (n_e k T_e) + e n_e \frac{\partial \phi}{\partial z} - m_e n_e \nu_{eN} C_{eN}^{(u)} (u_e - u_N)$$

$$-m_e n_e \nu_{ei} C_{ei}^{(u)} (u_e - u_i) - \left( \frac{\nu_e N}{\nu_{ei} + \nu_{eN}} C_{eN}^{(T)} + \frac{\nu_{ei}}{\nu_{ei} + \nu_{eN}} C_{ei}^{(T)} \right) n_e \frac{\partial k T_e}{\partial z} = 0.$$

The neutral component velocity $u_N$ is given by the solution of the respective hydrodynamic/gasdynamic problem. The terms appearing in the parentheses in the last
term on the l.h.s. are collision-frequency-weighted or (could also be) ionization-fraction-weighted. These terms are defined in an ad hoc manner to take into account the possible presence of a neutral component in the cold plasma. As before, \( \phi \) denotes the (unknown) electrostatic potential.

Prior to going to the ion momentum equation, let us consider two limiting cases:

(a) **Fully ionized plasma, \( \nu_e N = 0 \):**

In this case

\[
 j = -n_e e (u_e - u_i); \quad (\nu_{ei})_{\text{eff}} \equiv \nu_{ei} C^u_{ei}
\]

\[
 j = \frac{e}{m_e \nu_{ei} \text{eff}} \left\{ \frac{\partial}{\partial z} (n_e kT_e) - e n_e \frac{\partial \phi}{\partial z} + C^T_{ei} n_e \frac{\partial}{\partial z} kT_e \right\}
\]

where \( E \equiv -\frac{\partial \phi}{\partial z} \) and \( (\nu_{ei})_{\text{eff}} \equiv \nu_{ei} C^u_{ei} \).

The coefficients

\[
 C^u_{ei} = 0.51, \quad C^T_{ei} = 0.71,
\]

represent the Braginskii coefficients known for fully ionized plasmas.

Introducing now the electrical conductivity \( \sigma \),

\[
 \sigma \equiv \frac{n_e e^2}{m_e \nu_{ei} \text{eff}},
\]

one obtains the Braginskii equations containing the thermoelectric current constituent

\[
 j = \sigma E + \frac{n_e e}{m_e \nu_{ei} \text{eff}} \left( 1 + C^T_{ei} \right) \frac{\partial}{\partial z} (kT_e) + \frac{e(kT_e)}{m_e \nu_{ei} \text{eff}} \frac{\partial n_e}{\partial z} =
\]

\[
 = \sigma E + \frac{\sigma}{e} (kT_e) \left[ 1.71 \frac{\partial}{\partial z} (\ln kT_e) + \frac{\partial}{\partial z} (\ln n_e) \right].
\]

(b) **Weakly ionized gas, \( \nu_{ei} = 0, u_i = 0 \) (negligible ion effects):**

Defining now

\[
 \nu_e \text{eff} \equiv \nu_e N C^u_e N,
\]

one can write the electron momentum conservation equation for this in the following form:

\[
 -(1 + C^T_{eN}) n_e \frac{\partial}{\partial z} (kT_e) - kT_e \frac{\partial n_e}{\partial z} + e n_e \frac{\partial \phi}{\partial z} =
\]
\[ + \frac{m_e}{e^2} n_e \nu e_n C_{eN}^u j \left(1 - \frac{u_N}{u_e}\right) = 0; \]

hence

\[ j = \frac{n_e e^2}{m_e \nu e_{\text{eff}}} E + \frac{e k T_e}{m_e \nu e_{\text{eff}}} \frac{\partial n_e}{\partial z} + \frac{n_e e}{m_e \nu e_{\text{eff}}} \left(1 + C_{eN}^T\right) \frac{\partial k T_e}{\partial z}, \]

or

\[ j = \sigma E + \frac{e k T_e}{e} \frac{\partial}{\partial z} (\ln n_e) + \frac{e k T_e}{e} \left[ \frac{\partial}{\partial z} (\ln k T_e) \right] \left(1 + C_{eN}^T\right). \]

Introducing now

\[ \sigma k T_e / n_e e^2 \equiv D, \]

one obtains an equation written in a form common for weakly ionized gases:

\[ j = \sigma E + n_e e \left( D \frac{\partial}{\partial z} (\ln n_e) + D^T \frac{\partial}{\partial z} (\ln k T_e) \right). \]

Turning now to the ion momentum equation:

\[ - \frac{\partial}{\partial z} \left(n_i k T_i\right) - e n_i \frac{\partial T_i}{\partial z} + m_e n_i \nu e_i C_{ei}^u (u_e - u_i) - \mu_i n_i n_i C_{iN}^u (u_i - u_N), \]

\[ - \frac{\mu_i n_i \nu i N C_{iN}^T n_i}{m_e \nu e_i + \mu_i n_i \nu i N} \frac{\partial}{\partial z} (k T_i) + \frac{\nu e_i}{\nu e_i + \nu e_N} C_{ei}^u n_i \frac{\partial}{\partial z} (k T_e) = 0. \]

As before,

\[ C_{ei}^u = 0.51, \quad C_{ei}^T = 0.71, \quad \text{and} \quad C_{eN}^u = \frac{e}{\mu e N m_e \nu e N}; \]

where \( \mu e N \) denotes electron mobility, and
\( C_{eN}^u = \frac{1}{1.13} \) is for hydrogen [7]. Furthermore, \( C_{eN}^T = 0, C_{iN}^u \approx 1, C_{iN}^T \approx 0; \)

and \( \mu_i N = \frac{m_i m_N}{m_i + m_N} \) is the reduced mass.

If carbon is considered, the friction terms corresponding to \( ei \) collisions must be summed up over the six ionization levels:

\[ 0 \equiv - \frac{\partial}{\partial z} (n_e k T_e) + e n_e \frac{\partial \phi}{\partial z} - m_e \nu e N n_e C_{eN}^u (u_e - n_N) - \]

\[ - \sum_{k=1}^{6} C_{ei}^u k m_e n_e \nu e k (u_e - u_{ki}) - \left( \frac{\nu e N}{\nu e_i + \nu e_N} C_{ei}^T + \frac{\nu e_i}{\nu e_i + \nu e_N} \right) n_e \frac{\partial k T_e}{\partial z}, \]
where
\[
\nu_{ek} \equiv \frac{4\sqrt{2\pi}e^4Z^2_{ik}n_{ik}}{3meT_e^{3/2}}.
\]

Introducing now \(Z_{eff} \equiv \sum_{i=1}^{s} \frac{n_{ik}Z^2_{ik}}{ne}\) and \(\bar{u}_i \equiv \sum_{i=1}^{s} \frac{n_{ik}Z^2_{ik}}{\sum n_{ik}Z^2_{ik}}\),

the electron momentum equation can be rewritten in the following form:

\[
-\frac{\partial}{\partial z}(nekT_e) + en_e \frac{\partial \phi}{\partial z} - me\nu_{eN}neC_{eN}^u(u_e - u_N) -
-C_{ei}^{u}me\nu_{ei}(u_e - \bar{u}_i) - \left( \frac{\nu_{eN}}{\nu_{ei} + \nu_{eN}}C_{eN}^T + \frac{\nu_{ei}}{\nu_{ei} + \nu_{eN}}C_{ei}^T \right) ne \frac{\partial}{\partial z}(kT_e) = 0.
\]

The Braginskii coefficients appearing in the above equations are functions of \(Z_{eff}\) and

are displayed in the table below:

<table>
<thead>
<tr>
<th>(Z_{eff})</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{ei}^u)</td>
<td>.51</td>
<td>.44</td>
<td>.40</td>
<td>.38</td>
</tr>
<tr>
<td>(C_{ei}^T)</td>
<td>.71</td>
<td>.91</td>
<td>1.02</td>
<td>1.09</td>
</tr>
</tbody>
</table>

\(C_{eN}^T \approx 0, \quad C_{eN}^u \approx 1.\)

In the case of carbon, a set of six momentum equations is needed, corresponding to the

color number of ionization states, to determine \(u_{ik}, k = 1 \text{ to } 6\). These can be written, in a

color single temperature approximation, in the following form:

\[
-\frac{\partial}{\partial z}(n_{ik}kT_i) - Z_{ke}n_{ik} \frac{\partial \phi}{\partial z} + me\nu_{ek}C_{ei}^u(u_e - u_{ik})
- \mu_{kN}n_{ik}C_{kN}^u(u_{ik} - u_N) - \frac{\mu_{kN}n_{ik}}{me\nu_{ei} + \mu_{kN}n_{ik}} C_{kN}^{T} \frac{\partial}{\partial z}(kT_e)
+ \frac{\nu_{ei}}{\nu_{ei} + \nu_{eN}} Z_{kn}n_{ik}C_{ei}^T \frac{\partial}{\partial z}(kT_e) = 0
\]

In the expression above, \(\mu_{kN}\) denotes, as before, reduced mass.

If some of the ionization levels are empty, or some dominate, the system of equations

presented can be somewhat simplified.
Boundary Condition at the Eroding Wall

If there are neither cold electrons nor cold ions available in the vicinity of the wall, in order to ensure the zero net current condition, the slowed-down, stopped, and reversed initially hot electrons must carry the return current as well (see Fig. 4):

\[ \Gamma_{eb}^+ + \Gamma_{eb}^- = 0 \]

The reversed beam-electrons represent the "cold-electron current"; the E-field strength is such as to ensure \( n_{eb}^- v_{eb}^- = n_{eb}^+ v_{eb}^+ \) at any cross-section. If \( n_{eb}^- = n_{eb}^+ \) (in the absence of scattering), then \( v_{eb}^- = -v_{eb}^+ \) at any cross-section \( z = \text{const} \).

In a general case, local thermal ionization may exist. In this case, considering hydrogen plasma, the condition of zero current can be written in the following form (the effect of the incident hot ions is neglected also in this case, see Fig. 5):

\[ \Gamma_{eb} - \Gamma_{ec} + \Gamma_{ic} \equiv 0 \]

![Figure 5]

The electron and ion momentum equations necessary for determining the velocities of the cold species \( u_e, u_i \), implicitly present in \( \Gamma_e \) and \( \Gamma_i \), can be written as

\[ 0 \equiv -\frac{\partial}{\partial z}(n_e kT_e) + n_e e \frac{\partial \phi}{\partial z} - m_e n_e \nu_e N C_{eN}^u (u_e - n_N) - m_e n_e \nu_e i C_{ei}^u (u_e - u_i) \]
\[ -\left( \frac{\nu_{eN}}{\nu_{\Sigma}} C_{eN} + \frac{\nu_{ei}}{\nu_{\Sigma}} C_{ei} \right) n_e \frac{\partial}{\partial z} (kT_e) = 0, \]

\[ 0 = -\frac{0}{\partial z} (n_i kT_i) - e n_i \frac{\partial \phi}{\partial z} + m_e n_e \nu_{ei} C_{ei}(u_e - u_i) - \mu_{iN} n_e \nu_{iN} C_{iN} W_i(u_i - u_N) - \]

\[ \frac{\mu_{iN} \nu_{iN}}{m_e \nu_{ei} + \mu_{iN} \nu_{iN}} C_{iN} n_e \frac{\partial}{\partial z} (kT_i) + \frac{\nu_{ei}}{\nu_{\Sigma}} C_{ei} n_e \frac{\partial}{\partial z} (kT_e), \]

where

\[ \nu_{\Sigma} \equiv \nu_{ei} + \nu_{eN}. \]

These expressions are identical to those used for the cloud interior. In the case of carbon, the equations become correspondingly more complicated.

Once the distribution of the electrostatic potential \(\phi\) is found, the resulting shielding effect, i.e. the modified penetration depth of the charged energy carriers into the vapour cloud and the resulting heat deposition profile, can be found by stopping length calculations.

Results of scenario calculations performed with an ablation code, implemented with the electrostatic shielding model presented here, shall be reported elsewhere.
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References


