SIMULATION OF X-RAY SIGNALS

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ABSTRACT

A parameterized form of the local emissivity is used for the simulation of soft X-ray signals obtained on the WENDELSTEIN W VII-A stellarator with a 30 diode array. Numerical calculation of the line integrals for the different viewing angles and for a set of rotation angles covering one full signal period provides simulated periodic signals. In addition radial profiles of the line integrated emission averaged over some time interval or at specific times, the relative amplitude modulation and the relative phase of the oscillations are calculated. These have to be fitted to the corresponding measured signals and profiles in order to get a reliable picture of the local emissivity. The model can take into account two poloidally asymmetric contributions of the type $m = 1,2,3$ or 4 ($m$=poloidal mode number). Each asymmetry can be generated in two ways (modulation of intensity and of geometry parameters). Besides an uniform rotation of the asymmetric terms some specific simple time evolution of the signals can be included (non-uniform rotation, growth of oscillations, sawtooth oscillations). The various input parameters are illustrated and the result of a simulation procedure is presented for a particular discharge in W VII-A.
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6. Summary and Discussion
1. INTRODUCTION

For the analysis of instabilities like MHD- and Tearing Modes a knowledge of the spatial X-ray emissivity distribution has been proved to be a helpful tool. This is because the electron temperature is equalized on magnetic surfaces in the case of a sufficient high parallel electron heat conductivity even in the case of a fast time evolution and rotation of the modes. The spatial distribution of the other two essential plasma parameters entering the expression for the X-ray intensity, the plasma density and $Z_{\text{eff}}$, may not be closely similar to the time dependent magnetic field topology because of the inertia of the ions. The X-ray intensity distribution should therefore be some representation of the magnetic field structure.

Experimentally the main difficulty arises from the fact that direct measurements of the local X-ray intensity are not available but only line integrated signals from which the local intensities have to be reconstructed. With the detector arrangement used on the Wendelstein W VII-A stellarator consisting of 30 surface barrier diodes /1/ (see sketch in fig. 1) tomographic methods for the evaluation of the local X-ray emissivity will fail in the case of more than one mode present with possibly different rotation frequencies. Therefore an attempt was made to simulate the X-ray diode signals with a model by which the local X-ray emissivity in the plane where the diodes are situated is parameterized in a proper way. Since a high flexibility of the model was desirable, and since finer details of the experimental signals should be reproduced e.g. the number and exact radial position of phase reversals, double frequencies, non-sinusoidal oscillations etc. a large number of parameters are used in the model. Although many of these parameters can be deduced directly from the X-ray signals with a rather good accuracy e.g. poloidal mode numbers, radial positions of asymmetries, profile half widths etc., some iterative work is required to find the appropriate values of the parameters.
fig. 1 Sketch of the diode array geometry
In the special case of the \( m = 1 \) mode attempts have already been made to simulate the time evolution of the \( m = 1 \) island structure before and after the \( m = 1 \) internal disruptions /2,3/. In these papers some new aspects of the theoretical interpretations of the disruption process were discussed.

The parameters defined in the present approach do not relate to a specific theoretical model, although it is possible to shape the surface in a way similar to predictions of theory. The primary aim of this work is to give a satisfactory explanation for the whole pattern of X-ray signals for specific times and to identify the type of the instabilities. It should be noted that the computer programme was developed from a routine originally used to simulate static asymmetric radial flux profiles of impurity radiation and \( \alpha \) emission in the horizontal and vertical directions /4/.

2. Structure of the Local X-Ray Emissivity Distribution

2.1. Independent Variables, Coordinate System

The cross sections of the unperturbed magnetic surfaces of the \( \omega \) VII-A Stellarator are almost elliptical due to the superposition of the external \( 1 = 2 \) helical field. So the surface function \( \psi = e^2 x^2 + y^2 = \text{const.} \) is used to describe the static elliptical shape of the plasma cross section \((x', y') = \text{intrinsic coordinate system of the ellipse, } e = \frac{b}{a} = \text{eccentricity of the ellipse}\). The ellipses describing the cross sections of magnetic surfaces are inclined in a different way at different toroidal positions against a fixed coordinate system \( x, y \) originating at the magnetic axis (fig. 2). Using the polar coordinates \( r, \zeta \) \((x = r \cos \zeta, y = r \sin \zeta) \) \( \psi \) is expressed by

\[
\psi = r^2 (e^2 \cos^2 (\zeta - \zeta_0) \cdot \sin^2 (\zeta - \zeta_0)) = b^2
\]

with \( \zeta_0 = \text{angle between } x \text{- and } x' \text{ axes} \)

\( b = \text{minor radius of ellipse} \)
fig. 2 Sketch of coordinate system

Since on all $\Psi = \text{const.}$ surfaces the same physical properties of the plasma are expected the independent variables describing points on the same ellipse are chosen to be $\xi$ and $\zeta$ rather than $r,\tau$ (fig. 2).

$\xi$ is defined by the radius $r$ at an angle of $45^0$ with respect to the $x',y'$-system (fig. 2). This quantity is considered to be an appropriate representation of a mean ellipse radius. Then from eq. (1) the following relationship is derived:

$$\xi = r\sqrt{\frac{2}{1 + e^2}} \cdot \sqrt{e^2 \cos^2 (\zeta - \zeta_0) + \sin^2 (\zeta - \zeta_0)} \quad (1a)$$

$= \text{const. for } \Psi = \text{const.}$
In order to define and compare radial distances for emissivity distributions which are produced by plasmas with cross sections of different eccentricities (i.e. different external helical fields) in the following sections use will be made of the radial variable $\xi$.

### 2.2. Basic Parameterization of $I(\xi, \zeta)$

The spatial structure of the local X-ray emissivity $I(\xi, \zeta)$ across the plasma cross section is essentially described by three terms. The first term, containing no poloidal asymmetry besides the static elliptic plasma shape, is the dominant contribution to the mean X-ray intensity radial profile. The two other terms include additional poloidal asymmetries by which two different modes may be taken into account.

$$I(\xi, \zeta) = I_0(\xi) + I_1(\xi) H_1(\xi) + I_2(\xi) H_2(\xi)$$  \hspace{1cm} (2)

with

$$I_j(\xi) = \frac{I_j}{1 + \left| \frac{\xi - \xi_{dj}}{\xi_{hj}} \right|^{\alpha_j}} \hspace{1cm} (j = 0,1,2)$$  \hspace{1cm} (3)

$$H_i(\xi) = \left| \cos\left( \frac{m_i (\xi - \zeta_i)}{2} \right) \right|^{\chi_i - C_i} \hspace{1cm} (i = 1,2)$$  \hspace{1cm} (4)

where

- $I_j$ : maxima amplitudes of $I_j(\xi)$
- $\xi_{dj}$ : location of the maxima
- $\xi_{hj}$ : half-width radius of $I_j(\xi)$
- $\alpha_j$ : slope parameters of radial profiles $I_j(\xi)$
- $m_i$ : define type of the poloidal asymmetry contained in $H_i(\xi)$ (m = 1,2,3,4)
\[ \xi_i : \text{poloidal angles of the maxima of} \quad I_i(\xi) \cdot H_i(\xi) \]

\[ X_i : \text{slope parameter of poloidal asymmetries} \]

\[ C_i : \text{constant with} \quad \int_0^\pi |\cos \zeta|^X d\zeta / \pi \geq C_i \geq 0 \]

Normally \( C_i = \int_0^\pi |\cos \zeta|^X d\zeta / \pi \) is taken. This results in a total intensity profile (averaged over a complete poloidal turn) which is smooth and is given by \( I_O(\xi) \) alone. However, if a relative minimum of \( I_0(\xi) + I_i(\xi) \cdot H_i(\xi) \) appears for \( \xi > \xi_{di} \) in the direction where \( H_i(\xi) \) has its absolute minimum (\( \xi = \xi_i + \frac{\pi}{m_i} \)), \( C_i \) is lowered until the minimum vanishes. This is because in the presence of magnetic islands a continual decrease or a flattening of the temperature around the X-points of the separatrix can be expected rather than the occurrence of a local temperature minimum. Alternately an arbitrary value of \( C_i \) can be chosen. So it is possible to produce some flattening or enhancement of the average intensity profile at the radial position of the asymmetry. The displacement \( \xi_{do} \) usually is zero. By means of setting \( \xi_{do} \neq 0 \) hollow intensity profiles can be generated.

2.3. Additional Parameters

Based on experiences with simulations of X-ray signals using the definition (2) some additional parameters for further tailoring the \( I(\xi, \zeta) \) distribution are introduced. The effect of these parameters will be illustrated in chapter 5.2.

The parameters \( f_i \) and \( g_i \) (i = 1, 2) allow for a different radial shape of \( I_i(\xi) \) inside the radial position \( \xi_{di} \) where \( I_i(\xi) \) has its maximum:

\[ \begin{align*}
\xi < \xi_{di} & : \quad \xi_{hi} \rightarrow \xi_{hi} \cdot f_i \\
\alpha_i & : \quad \alpha_i \rightarrow \alpha_i \cdot g_i
\end{align*} \]
Then using the parameters \( h_i, k_i \) \((i = 1, 2)\) the poloidal slope parameters \( \chi_i \) of the asymmetric contributions \( I_i(\xi) \cdot H_i(\xi) \) can be varied linearly depending on \( \xi \) and that in a different way inside and outside \( \xi_{di} \):

\[
\xi < \xi_{di} : \quad \chi_i \rightarrow \chi_i - \frac{h_i}{\xi_{hi}} (\xi - \xi_{di})
\]

for

\[
\xi > \xi_{di} : \quad \chi_i \rightarrow \chi_i + \frac{k_i}{\xi_{hi}} (\xi - \xi_{di})
\]

This makes \( H_i \) also dependent on \( \xi : H_i(\xi) \rightarrow H_i(\xi, \xi) \) \((i = 1, 2)\).

The asymmetric modulation on the intensity (see eqs. (2) - (4)) is in particular suitable for the description of possible local maxima of the intensity in the presence of island structures. Other effects can be considered which may be included by a modulation of a geometrical quantity rather than of the intensity: the cross sections of magnetic flux surfaces may be deformed or shifted in the presence of kink modes or during the formation of magnetic islands by tearing modes /5, 6, 7/. Especially inside the inner separatrix of the island structure a radial compression of the flux surfaces between the magnetic axis and the 0-point(s) and a radial expansion towards the X-point(s) is expected. Also an inward shift of the 0-points and an outward shift of the X-points respectively is predicted. Similar effects may be produced by an internal \( m = 1 \) kink mode. In order to include these additional non-axisymmetric effects in \( I(\xi, \zeta) \) the term \( I_0(\xi) \) in eq. (2) was replaced by additional terms dependent also on \( \zeta \):

\[
I_0(\xi) \rightarrow I_0(\xi, \zeta) = \sum_{i=1}^{2} a_i I_0(x_i) + \sum_{i=1}^{2} \frac{a_i I_0(x_i)}{1 + \frac{|\xi - \xi_{do}|}{\xi_{hoi}(\zeta)}} + \sum_{i=1}^{2} \frac{a_i I_0(x_i)}{1 + \frac{|\xi - \xi_{do}|}{\xi_{ho}}}
\]

\[
\equiv \sum_{i=1}^{2} I_{0i}(\xi, \zeta) + \sum_{i=1}^{2} I_{0i}(\xi, \zeta)
\]  (5)
The non-axisymmetric contributions included in $\xi_{\text{hoi}}(\zeta)$ are expressed by parameters $\epsilon_i$ with $\epsilon_i \neq 1$. Depending on the values of $\epsilon_i$, the coefficient $a_i$ take the values:

$$
a_1 = 1, \quad a_2 = 0 \quad \text{for} \quad \epsilon_1 \neq 1, \quad \epsilon_2 = 1
$$

$$
a_1 = 0, \quad a_2 = 1 \quad \text{for} \quad \epsilon_1 = 1, \quad \epsilon_2 \neq 1
$$

$$
a_1 = 0.5, \quad a_2 = 0.5 \quad \text{for} \quad \epsilon_1 \neq 1, \quad \epsilon_2 \neq 1
$$

Since the case $m_i = 1$ is treated in a different way (only for this case $b_i$ and $c_i$ are important), let us first discuss eq. (5) for $m_i \neq 1$ ($m_i = 2, 3, 4$):

For $m_i \neq 1$ the second term in (5) vanishes; $\epsilon_i = 0$ and $b_i = 1$.

The contours of $\xi_{\text{hoi}}(\zeta)$ consist of elliptic segments retaining the original value $\xi_{\text{ho}}$ in the direction of the angle bisector between the major and minor radii with

$$
\xi_{\text{hoi}}(\zeta) = \xi_{\text{ho}} \sqrt{\frac{\epsilon_r^2 + 1}{\epsilon_r^2 (1 - \cos m_i (\zeta - \zeta_i)) + 1 + \cos m_i (\zeta - \zeta_i)}}
$$

(m_i = 2, 3, 4)

This formulation may be also considered as a special Fourier expansion of the perturbation on $\xi_{\text{ho}}$. Especially due to the higher order terms the desired shape of the surfaces (elliptical, triangular or quadratic) is achieved.

The angles $\zeta_i$ are the same as defined in eq. (4) ($\zeta = \zeta_i$ corresponds to the direction of the minor axis of the elliptic segments). The parameters $\epsilon_{ri}$ are the radial dependent eccentricities of the elliptic segments with

$$
\epsilon_{ri} = 1 - (1 - \epsilon_i) \quad \frac{1}{1 + \left| \frac{\zeta - \zeta_i}{\zeta_{wi}} \right|^{B_i}}
$$

(i = 1, 2)
where

\[ \varepsilon_i : \text{maximum eccentricity (at } \xi = \varepsilon_{vi}) \]

\[ \varepsilon_{vi} : \text{radius where } \varepsilon_{ri} \text{ has its maximum value} \]

\[ \varepsilon_{wi} : \text{half width radius for } \varepsilon_{ri} \]

\[ \beta_i : \text{slope parameter for } \varepsilon_{ri} \]

**Case \( m = 1 \):**

Since for the \( m = 1 \) case a shift of the surfaces larger than the respective radii should be principally possible with almost no asymmetric deformation eq. (6) was not applied for \( m = 1 \).

First it is assumed that outside a certain maximum radius \( \xi_q \) there is no additional poloidal asymmetric contribution:

\[
\text{for } \xi > \xi_q : \quad \xi_{hoi}(\xi) = \xi_{ho} = \text{const} \]

\[
b_i = 1 \]

\[
c_i = 0 \]

Inside \( \xi_q \) the radial profile is considered to consist of the sum of a constant term which is equal to the intensity at \( \xi = \xi_q \) (right term of eq. (5)) and a modified intensity distribution which is displaced from the center.

\[
\text{for } \xi < \xi_q : \quad c_i = 1 \]

\[
b_i = \varepsilon_i \quad (\text{maximum value of the displaced intensity contribution inside } \xi_q) \]

\[
\xi_{hoi}(\xi) = \varepsilon_{wi} = \text{const.} \quad (\text{corresponding half width radius}) \]

\[
\xi_{do} = 0 \]

\[
\alpha_0 = \beta_i \quad (\text{modified slope parameter inside } \xi_q) \]
The displacement of the intensity distribution inside $\xi_q$ is introduced by replacing

$$\xi \rightarrow \sqrt{\xi^2 + \xi_{v_1}^2 + 2\xi_{v_1} \cdot \xi \cos (\zeta - \zeta_i)}$$

(8)

with

$\xi_{v_1}$ : radial displacement of the intensity distribution inside $\xi_q$

$\zeta_i$ : poloidal angle of the displaced center

Actually eq. (8) describes a rotation of displaced $\psi = \text{const.}$ contours around the axis.

Some examples of emissivity distributions containing the different types of non-axisymmetric terms will be presented in 5.2.

3. Geometry and Line Integration

For the simulation of the X-ray signals, integration must be performed along the line of sight of each diode (see fig. 1). For this purpose first an ideal spatial resolution of the detectors is assumed. In reality the radial resolution is better than -1cm in the equatorial plane through the plasma center. (The mean plasma radius is about 11 cm).

Along each line of sight S determined by the angle $\theta$ and the distance L between plasma center and the common viewing slit only one independent variable is varied which is defined by the distance s (see fig. 3). The point where $s = 0$ is given by the lowest distance between the torus axis and S. In order to include a horizontal shift D of the plasma center with respect to the torus axis, the integration is performed along a line S' which is parallel displaced by the distance D. The value $I(\xi, \zeta)$ for each given s on the line S' is found by using the relations
\[ r = \sqrt{(L \cdot \sin \theta - D \cdot \cos \theta)^2 + s^2} \]  \hspace{1cm} (9)

\[ \sin \xi = \frac{((L - D \cdot \cot \theta) \cdot \cos \theta - s) \cdot \sin \theta}{r} \]  \hspace{1cm} (10)

and eq. (1)

for \( \theta = 0 \):

\[ r = \sqrt{D^2 + s^2} \]

and \( \sin \xi = -\frac{D}{r} \)

**fig. 3** Sketch of integration line

For the simulation of the 30 X-ray signals

\[ \phi (\theta_k, \zeta_1, \zeta_2) = \int_{S_k} I(\xi, \zeta, \zeta_1, \zeta_2) \, ds \]  \hspace{1cm} \text{has to be known for} \ k = 1, \ldots, 30

and as a function of the angles \( \zeta_1, \zeta_2 \) which describe the rotation of the modes.
In order to reduce the number of the numerical integrations the terms containing only \( \zeta_1 \) or \( \zeta_2 \) respectively are integrated separately for an equal number of angles \( \zeta_1, \zeta_2 \):

\[
\phi_i(\theta_k, \zeta_i^n) = \frac{1}{S_k} \int_{0}^{a} I_{0i}^{0}(\xi, \zeta, \zeta_i^n) + I_{0i}^{b}(\xi, \zeta, \zeta_i^n) + I_{i}(\xi, \zeta) \cdot H_{i}(\xi, \zeta, \zeta_i^n) \text{d}s
\]

\( (i = 1, 2; \quad k = 1, \ldots, 30; \quad n = 1, \ldots, n_f) \) \hfill (11)

The angles \( \zeta_i^n \) are equally spread over an angle of \( \frac{2 \pi}{m_i} \) degrees corresponding to one signal period of \( \phi_i(\zeta_i) \) \( (i = 1, 2) \):

\[
\frac{n_f}{m_i} = \frac{2 \pi}{m_i} \left( \frac{\zeta_{0i}}{m_i} + \frac{\zeta_i^n}{m_i} \right)
\]

where the \( \zeta_{0i} \) are used to define the azimuthal direction for which the maxima of the two asymmetric contributions \( I_1, I_2 \) are coincident.

Then for each required combination of angles \( \zeta_1, \zeta_2 \) \( \phi(\theta_k, \zeta_1, \zeta_2) \) is derived from a quadratic interpolation procedure of the \( \phi_i(\theta_k, \zeta_i^n) \)-values.

Also the effect of a non-ideal spatial resolution of the detectors can be considered without the need of additional integrations at extra \( \theta \)-angles. Since the angle between adjacent chords \( S \) is roughly equivalent to the aperture angle of the detectors it is reasonable to perform quadratic interpolations between the \( \phi(\theta_k) \)\( /8/ \). Then the original values for \( \phi(\theta_k) \) can be replaced by an average value of \( \phi(\theta) \) in an interval \( \theta_k - \Delta/2 < \theta < \theta_k + \Delta/2 \) where \( \Delta \) determines the spatial resolution (see also ref. /9/).
4. Time Evolution of the Signals

4.1. Rotation of the Poloidal Asymmetries

In the presence of kink- and tearing modes the oscillating X-ray signals can be assumed to originate from the rotation of non-axi-symmetric contributions in the local emissivity distribution. The rotation may be caused by the diamagnetic drift or by the toroidal electrical field. /1/.

In order to describe the time evolution of the X-ray signals the functions \( \zeta_1(t) \) and \( \zeta_2(t) \) have to be defined. In practice this is done by specifying the number of signal periods arising from each mode structure during an arbitrary time interval \( t = 0 \) to \( T \). The parameters \( \zeta_{01} \), \( \zeta_{02} \) defined in eq. (12) determine the initial values of the rotation angles for \( t = 0 \).

Then we can write

\[
\zeta_i(t) = \frac{N_i}{m_i} \cdot \frac{2\pi}{T} \cdot t + \frac{\zeta_{0i}}{m_i} \quad (i = 1, 2) \quad (13)
\]

where \( N_i : \) number of signal periods of \( \phi (\theta_k, \zeta_i(t)) \) in the time interval \( 0 \leq \frac{t}{T} \leq 1 \)

Thus a uniform rotation of two modes with possibly different rotation frequencies and some fixed phase relation among one another (mode coupling /1/) can be described.
4.2. Non-uniform Rotation

Occasionally the oscillating X-ray signals exhibit waveforms clearly differing from sinusoidal oscillations. Experimentally there is evidence for varying rotation frequencies; the rotation may even stop completely or cease irregularly for a short time interval. This "mode locking" \( /1/ \) is believed to originate from a localized interaction of a large amplitude mode with the wall or some obstacle limiting the plasma aperture. In order to facilitate the decision whether a partial mode locking or the poloidal emissivity distribution itself is responsible for the non-sinusoidal oscillations, eq. (12) was modified in the following way:

\[
\xi_i^n = n \frac{2\pi}{n_f m_i} + \frac{\xi_{oi}}{m_i} - \xi_{si} \cdot \sin \left( n \frac{2\pi}{n_f} - \xi_{ti} \right) \\
(14)
\]

\( (i = 1, 2; \ n = 1, \ldots, n_f) \)

Again the \( \xi_i^n \) are equally distributed over one signal period. Then for the time dependencies of the rotation angles (eq. (13)) follows:

\[
\xi_i(t) = \frac{N_i}{m_i} \cdot \frac{2\pi}{T} t + \frac{\xi_{oi}}{m_i} - \xi_{si} \cdot \sin\left( \frac{2\pi}{T} t - \xi_{ti} \right) \\
(15)
\]

where \( \xi_{si} \): maximum amplitude of periodic modulation on rotation angles

\( \xi_{ti} \): defines poloidal position of maximum / minimum velocity of rotation
4.3. Growing Oscillations

The easiest way to simulate growing X-ray signal oscillations is to make the maxima \( I_1, I_2 \) of the asymmetric terms in eq. (3) time dependent:

\[
I_i(t) = I_i \cdot e^{-\gamma(T-t)} \quad (i = 1, 2)
\]

with \( \gamma = \) growth rate of \( I_i \)

As \( I_i(t) \) does not depend on the coordinates, (only on \( \xi_i(t) \)) the integrals

\[
\int_{S_k} I_i(\xi, \zeta, \xi_i) \cdot H_i(\xi, \zeta, \xi_i^n) \, ds \quad (i = 1, 2)
\]

(see eq. (11)) are calculated and the interpolation is made as before with \( I_i(t) = I_i = \) const. Then for each time \( t \) the integrals have to be multiplied by the time dependent factor given in (16).

The growth of modes or islands can also be described by time dependent parameters \( \xi_{hi} \) (see eq. (3) or \( \xi_{hoi}(\zeta) \) (see eq. (5)). However this would require many more integrations because the line integral for each asymmetric term has to be calculated not only for one signal period but for all the \( N_i \) \((i = 1, 2)\) signal periods in the time interval 0 to \( T \) (with \( N_i \) defined in eq. (13)).
4.4. Sawtooth Signals

Sawtooth signals are generated by a progressive peaking of the symmetric emissivity distribution \( I_0(\xi) \) (see eqs. (2), (3)) during the time interval 0 to \( T \) defined in 4.1.. At \( t = T \) a sudden re-establishment of the original profile \( (t = 0) \) is assumed. If it is required that the profile does not change in time for a given radial position \( \xi_{in} \) (sawtooth inversion radius) it would be more difficult to retain the parameterization for the profile \( I_0(\xi) \) of eq. (3). This is because then the parameters \( I_0, \xi_{ho}, \alpha_0 \) would depend on each other in a complicated way. Therefore a sinusoidal radial modulation of \( I_0 \) (see eq. (3) and also eq. (5)) changing linearly in time was introduced.

\[
I_0 + I_0(t) = \begin{cases} 
I_0 + D_a \cdot \sin \left( \frac{\xi_{in} - \xi}{\xi_{in}} \cdot \frac{\pi}{2} \right) \cdot \frac{t}{T} & \text{for } \xi < \xi_{in} \\
I_0 - D_b \cdot \sin \left( \frac{\xi_{in} - \xi}{\xi_{in} - \xi_c} \cdot \pi \right) \cdot \frac{t}{T} & \text{for } \xi > \xi_{in}
\end{cases}
\]

(17)

with

\[
\begin{align*}
I_0(t) &= 0 & \text{for } I_0(t) < 0 \\
I_0(t) &= I_0 & \text{for } \xi > \xi_c
\end{align*}
\]

where: \( D_a, D_b \) : define amplitudes of sawteeth inside/outside \( \xi_{in} \)

\( \xi_{in} \) : inversion radius of sawteeth

\( \xi_c \) : outer limit for occurrence of sawtooth signals \( (\xi_c > \xi_{in}) \)
5. Examples, Results

5.1. Output of the Computer Programme

With a set of input parameters described in the previous chapters, the output available from the FORTRAN - simulation routine consists of:

- plots of the assumed parameterized local emissivity for different rotation angles (contour plots or 3-dimensional pictures), linear or logarithmic
- radial profiles of the input local emissivity along different poloidal angles (for instance along lines through maxima and minima of asymmetric contributions)
- simulated diode signals for 30 channels distributed over the plasma cross section (relative amplitudes $\Delta A/A$ as a function of time or rotation angle)
- radial profiles of the line integrated flux (time averaged and at specific times (or angles))
- radial profile of the relative amplitudes $\frac{\Delta A}{A} (R)$
- radial profile of the relative phase shift of the different simulated diode signals

The radial position $R = R_k$ ($k = 1$ to $30$), which are attributed to each line of sight for the different $\Theta_k$-angles, are again defined by the variable $\xi$. The value of $\xi$ is derived from the respective elliptical surface $\psi$ for which the line of sight is a tangent (eq. (1a)). The relationship between $R_k$ and $\Theta_k$ is given by:

$$R_k = \frac{2}{(1+e^2) \left[ \tan^2(\xi_0 - \Theta_k) + e^2 \right]} \cdot e \cdot L \left[ \cos \xi_0 - \tan(\xi_0 - \Theta_k) \sin \xi_0 \right]$$

with $e, \xi_0, \Theta_k, L$ previously defined in eqs. (1) and (9)
All the plots compiled in the last four items are used for comparison with the corresponding experimental data in order to find a realistic form of the local emissivity.

5.2. Dependence of \( I(\xi, \zeta) \) on Input Parameters

In order to illustrate the various possibilities of shaping the local emissivity distribution some contour plots are shown in the following figures:

In the upper row of fig. 4 the basic eqs. (2) - (4) are used to generate asymmetric distributions with single poloidal mode numbers \( m = 1/2/3 \) for a circular plasma \((a,b,c)\) \((I_0 = 1, I_1 = 0.3/0.4/0.4, I_2 = 0, \xi_{do} = 0, \xi_{ho} = 5.0/4.0/4.0, \xi_{d1} = 3.6/7.5/7.5, \xi_{h1} = 1.8/2.0/2.0, \alpha_0 = 4.0, \alpha_1 = 3.5/6.0/6.0, X_1 = 3.0/2.0/2.0)\).

In the second row the \( m = 1,2,3 \) asymmetries arise only from the parameter \( \xi_{1\neq 1} \) defined in section 2.3. where the amount of the asymmetric deformation is given by the parameter \( \xi_{1\neq 1} \) \((4d,e,f)\). Note that the definitions given in 2.3. are different to some extent for \( m = 1 \) and \( m = 2,3,4 \) respectively.

\[(I_0 = 1, I_1 = 0, I_2 = 0, \xi_{do} = 0, \xi_{ho} = 5.0/4.0/4.0, \alpha_0 = 4.0; m = 1 : \xi_{1\neq 1} = 0.4, \xi_{v1} = 1.3, \xi_{w1} = 2.7, \beta_1 = 3.5, \xi_{q} = 6.5; m = 2,3 : \xi_{1\neq 1} = 0.65, \xi_{v1} = 0, \xi_{w1} = 5.0, \beta_1 = 6.0)\]

Finally in the third row combination of the effects illustrated in the previous rows are presented \((4g,h,i)\). The short straight lines on the side of each contour plot mark the lines of sight for the X-ray diodes. The rotation angle \( \xi_{1\neq 1} \) (eq. (4)) is zero.
fig. 4  Contour plots for $m = 1, 2, 3$ (circular)

a-c) : use of $I_1(\xi, \zeta)$

d-f) : use of $I_0(\xi, \zeta)$

g-i) : combination of $I_1(\xi, \zeta)$ and $I_0(\xi, \zeta)$
fig. 5  Contour plots for a $m = 2$ asymmetric distribution in the case of a static elliptic plasma deformation (for two rotation angles) (same parameters as in fig. 4b, but $e = 0.6$, $\epsilon_0 = -45^0$)

fig. 6  Local distribution containing two different asymmetric terms.
(a: $m = 1,2$ ; b: $m = 2,3$)
The eccentricity of the static elliptical plasma deformation due to the \( l = 2 \), \( m = 5 \) helical windings depends on the applied helical field of the W VII-A Stellarator. In fig. 5 the distribution shown in fig. 4b is modified to account for an elliptical cross section by setting the parameters \( e = 0.6 \), \( \xi_0 = -45^\circ \) (\( e, \xi_0 \) defined in (1)).

Fig. 6 shows the possibility of including two asymmetric contributions of different types. This may be used to simulate the coupling between two modes /1/. (Fig. 6a: \( m = 1, 2 \); fig. 6b: \( m = 3, 2 \)). The two asymmetries can rotate each with an individual speed or in such a way to provide phase locked oscillations.

\[
(I_{0,1,2} = 1.0, 6, 0.2; \quad \xi_{do} = 0.5, 0, 8.5; \quad \xi_{h0,1,2} = 4.0, 1.6, 1.6; \quad \alpha_{0,1,2} = 4.0, 6.0, 0.6; \quad \epsilon = 1, 0; \quad \xi_1, 2 = 3.0)
\]

The central plot of fig. 7 (7c) presents again the identical distribution of fig. 4b which is then modified by several parameters. In fig. 7a the parameter \( X_1 \) (defined in eq. (4)) was changed from \( X_1 = 2.0 \rightarrow 4.0 \), resulting in a poloidal compression of the island structure. The effect of making \( X_1 \) dependent on \( \xi \) is shown in 7b (the parameters \( h_1, k_1 \) defined in 2.3. are altered from \( h_1 = 0 \rightarrow -2 \), \( k_1 = 0 \rightarrow +2 \)). This changes the direction of the contour lines across the island region in such a way that a more circular or elliptical shape is achieved. In fig. 7d the halfwidth of the \( m = 2 \) term is reduced from \( \xi_{h1} = 2.0 \rightarrow 1.2 \). Finally in 7e the radial shape of the asymmetric term is changed inside \( \xi_{d1} \) by setting \( f_1 = 1 \rightarrow 1.3 \); \( g_1 = 1 \rightarrow 1.4 \) with \( f_1, g_1 \) defined in 2.3.
fig. 7  Modification of the distribution containing a m = 2 asymmetric term (7c):
    a) $x_1 = 2.0 \rightarrow 4.0$
    b) $h_1 = 0 \rightarrow -2$
    d) $\xi_{h1} = 2.0 \rightarrow 1.2$
    e) $f_1 = 1 \rightarrow 1.3$
    k_1 = 0 \rightarrow +2
    g_1 = 1 \rightarrow 1.4
    c) same as shown in 4b
For the case given in fig. 4i the set of output plots from the simulation programme is completed in fig. 8. First fig. 8a shows a 3-dimensional view of the assumed emissivity distribution (logarithmic scale). Then in 8b profiles of the local distribution along diameters through one of the maxima (full line) and one of the minima (dashed line) of the asymmetric contribution are given. The corresponding intersections are marked in 8a by the two thick lines. The thirty fluctuating signals plotted in 8c result from the rotation of the emissivity distribution. They can be compared with the measured X-ray diode signals (relative amplitude modulation). The vertical solid lines mark the times (or the rotation angles) where line integrated radial intensity profiles are given in fig. 8d. (The two dashed lines around the full line in the lower half of fig. 8d). The radial dependence of the line integrated intensities averaged over one rotation period is indicated by the solid line in fig. 8d (I(R) scale). The lowest dashed curve (DI(R) scale) gives the radial profile of the relative profile of the relative signal amplitudes. Finally from the upper dashed curve (Phase scale) the relative phase shift between the thirty signals can be derived. Due to the different directions of the line integrations (see fig. 1) the equal phase line of the oscillations is tilted. The sign and the amount of the inclination is dependent on the direction of rotation and the m-number of the asymmetry respectively.
fig. 8  Additional plot-output from the simulation programme
a) 3-dimensional view of the distribution given in
fig. 4i
b) Local radial profiles through maxima (full line) and
minima (dashed line) of the asymmetric terms
fig. 8  c) Fluctuations of the line integrated emissivities due to rotation of the distribution (simulated diode signals)

d) Solid line: Line integrated radial profile averaged over poloidal structure
   2 dashed curves around solid line: Profiles for specific rotation angles (marked by vertical bars in 8b)
   Lower dashed curve: Radial profile of signal modulation
   Upper dashed curve: Radial dependence of phase angle of modulation
fig. 9  Modification of the signals in the case of a periodic non-uniform rotation (Again the local emissivity given in 4c is used):

\[ \xi_{sl} = 15^0, \quad \xi_{tl} = 30^0 \]
The time behaviour for the rotation of the spatial emissivity distribution as given in fig. 4c and 8 is modified in fig. 9 by means of the parameters \( \zeta_{s1} \) and \( \zeta_{t1} \) (see eq. (15)). The periodic changing rotation frequency now results essentially in non-sinusoidal waveforms of the fluctuations. This effect modifies in particular the slope of the equal phase line and generates signal components which have an opposite phase for signals around 5 to 13 cm with respect to the signals from the opposite side (-5 to -13 cm). This may be used to differentiate whether non-sinusoidal waveforms are generated by a partly locked island structure or by the poloidal structure itself. In the latter case all the signals at the same radii but from opposite sides would have the same shape and only a phase shift would appear. 
\[
(\zeta_{s1} = 15^0, \quad \zeta_{t1} = 30^0)
\]

The next figure (fig. 10) shows an example for sawtooth signals with a superimposed growing \( m = 1 \) oscillation. The signal generation is based on the distribution of fig. 4a but additionally the parameters \( \gamma \) (growth rate of \( m = 1 \) oscillation, see eq. (16)), \( D_a \), \( D_b \), \( \xi_{in} \), \( \xi_{c} \) (see eq. (17)) are used to achieve the characteristic time behaviour of the signals. 
\[
(\gamma_1 = 3.0, \quad D_a = 0.8, \quad D_b = 0.5, \quad \xi_{in} = 5.5, \quad \xi_{c} = 11.0)
\]
fig. 10  Simulation of sawtooth signals and growing oscillations.
(Derived from distribution given in 4a, but $\gamma = 3.0$, $D_a = 0.8$, $D_b = 0.5$, $\xi_{in} = 5.5$, $\xi_c = 11.0$)
fig. 11a  Example: Simulation of experimentally measured X-ray signals (strong $m = 2$ asymmetries with a $m = 1$ contribution, shot 18886)

11a left side: Measured diode signals
11b right side: Simulated diode signals
fig 11b 11b left side: Comparison of measured and simulated radial profiles (line integrated intensity, relative amplitude and phase)
11b right side: Contour plots of fitted intensity distribution for two rotation angles
Table I 1)

Parameters used for the simulation shown in fig. 11 (Shot 18886)

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1) unit of lengths: cm
5.3. Comparison with Experimental Data

Several attempts to simulate experimental X-ray signals and profiles have already been presented in ref. /1/. One of these examples is again shown in fig. 11. The experimentally measured X-ray signals (given on the left side of fig. 11a) exhibit growing oscillations which originate from a large $m = 2$ and a weaker $m = 1$ distortion of the local emissivity distribution which is shown on the right of fig. 11b for two rotation angles. The simulated signals are given on the right side of 11 a. The left side of fig. 11b contains measured and simulated radial profiles of averaged line integrated intensities, the relative modulation amplitudes and relative phase shifts of the signals. The measured signals and profiles are remarkably similar to those actually measured.

Without repeating a detailed discussion about the physical nature of the instability in this particular discharge the following details should be mentioned. In order to reproduce the characteristic radial dependence of the waveforms (for instance from diode 21 to diode 25 the maxima become progressively more peaked) the poloidal extent of the $m = 2$ asymmetry had to be reduced by using the parameter $k_2 
eq 0$ (see 2.3.). Additionally as was already discussed in 5.2., a periodic retardation of the rotation was introduced by use of the parameters $\xi_{s1,2}$ and $\xi_{t1,2}$ (see eq. (15)). By this mean the slower rise of the periodic signals at -5.4 to -6.5 cm compared with the decrease and the reverse effect at +7.7 to 8.3 cm respectively could be achieved. It also contributes to the broadening of the signal maxima at $\pm$ 4 to 5 cm and the corresponding minima inside $\pm$ 2.5 cm. Table I lists the set of parameters defined in the previous chapters for the simulation presented in fig. 11.
6. Summary and Discussion

The behaviour of MHD-modes can be influenced in a wide range by means of the adjustable external helical field in the W VII-A Stellarator /1,10/. At different experimental conditions various modes and combinations of modes occur which are detected by Mirnov-coils /10/ and soft X-ray diodes. In many cases inversion of the line integrated X-ray emission at different angles into a local emissivity distribution is impossible due to the plasma observation from only one side. Therefore the inverse approach was applied to get some information about the non-axisymmetric structure of the emissivity. Assuming some parameterized form of the local emissivity distribution, simulated X-ray signals are generated by integration along the lines of sight. Even though the distribution found after properly adjusting the parameters may not be unequivocable this method allows statements about the m-numbers, radial positions and extensions of the instabilities and superpositions of modes. In particular when the X-ray signals exhibit several phase reversals asymmetrically spread over the plasma cross section a direct determination of the modes causing the oscillations is very difficult. In this case a reasonable simulation of the signals would help to identify the type of modes.

Simulations were performed for almost all modes detected in the W VII-A plasma: \((m,n) = (1,1), (2,2), (2,1), (3,2), (4,3)\) where \((1,1)-\) and \((2,1)-\) modes are coupled in most cases showing the same frequency in the signals. Also the \((2,2)-\)mode only appeared in combination with a dominant \((3,2)-\)mode.

In some cases the mode also provides a possible explanation of the fine structure superimposed on the periodic signals like double frequencies and non-harmonic distortions. Comparing simulations with out and with inclusion of the realistic spatial resolution of the detectors (\(\approx 1 \text{cm}\)) indicated a small effect, mostly in the vicinity of phase reversals. At these particular radial positions a phase shift as well as a change of the amplitude of the oscillations may be produced. This rather weak influence of a 1 cm spatial resolution was also found for symmetrical Abel inverted emission profiles /9/.
The X-ray measurements are performed together with Dr. P. SmULDERS.

Special thanks to him also for many discussions about the interpretation of the signals.
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