On Pellet Ablation in Hot Plasmas and the Problem of Magnetic Shielding

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Abstract

Theories published on pellet ablation phenomena are reviewed. Some inconsistencies are shown to exist in some of the studies available. Models are developed and estimates are given for pellet ablation rates in hot plasmas with and without magnetic fields present. The possible existence of a self-regulating ablation mechanism is indicated.
The injection of frozen hydrogen-isotope pellets in hot plasmas is considered as a possible means of increasing the plasma density in present tokamak experiments\textsuperscript{1, 2} and for replenishing particle losses in the next generation of tokamak installations. The ultimate purpose of pellet injection experiments is the development of a method for the cold refueling of future reactor-size CTR experiments. The idea of refueling by means of pellet injection originates from Spitzer and co-workers\textsuperscript{3}. The alternative refueling methods (gas bleed-in, neutral injection, cluster injection) as applied to reactor-size experiments either lead to ionization and particle capture at the plasma boundary, thus generating inverted density profiles, or require very high injection energies to ensure penetration times shorter than the ionization time of the particles. Acceleration of a large number of particles to the required velocity (typically $5 \times 10^{12}$ particles/s per watt reactor output
are considered to be the necessary refueling rate \(^4\) would require prohibitively large and expensive injector installations. The injection power requirements are somewhat relaxed if cluster injection is considered \(^5\), but only pellet injection with estimated injection velocities of the order of \(10^4\) m/s (approx. 1 eV of energy per particle) offers a real alternative. The magnitude of the required injection velocity is determined in this case by the estimated ablation rates.

Pellet ablation in a hot plasma is affected by a number of physical phenomena. The surface of a pellet exposed suddenly to a plasma is bombarded directly by the plasma particles until the ablation products form a protective blanket around it. Some of the electrons impinging on the pellet surface are subject to elastic scattering, the rest undergo excitational and ionizational collisions. The higher the energy of the incident electrons the higher is the number of ionizing events \(^6\), while the energy transferred to an ion pair produced approaches an asymptotic value. At the same time, the electrons transfer momentum to the pellet. Also the ions impinging on the pellet surface transfer momentum and energy to it. The ions may be subject to charge exchange collisions: the backscattered hot neutrals are then ionized by the surrounding plasma and the cold ions left behind increase the degree of ionization of the pellet mass. With time the ablation products form a moderately warm plasma blanket around the pellet, which may be partially or fully ionized, depending upon the energy flux to the pellet. The blanket, heated at its outer boundary by the surrounding plasma and cooled at its inner boundary by the pellet, shields
the pellet, partially or totally, from the incident hot plasma particles and acts, at the same time, as a heat reservoir for the pellet itself. There is thus a continuous exchange of mass, momentum and energy at the pellet-blanket and blanket-plasma interfaces, resulting in a continuous phase transition from the condensed phase to ionized matter.

In this paper, we are concerned primarily with the first phase of the ablation process, i.e. the determination of the blanket parameters and the resulting ablation rates as functions of the incident energy flux. If the parameters of the surrounding plasma are given, the momentum and energy fluxes incident on the pellet surface can be estimated in the absence of shielding by

\[
\dot{\rho} \propto \frac{1}{4} n_e v_{\text{eth}} (m_e v_{\text{eth}}) + \frac{1}{4} n_i v_{\text{ith}} (m_i v_{\text{ith}}), \quad (0.1)
\]

\[
\phi \propto \frac{1}{4} n_e v_{\text{eth}} \left( \frac{3}{2} kT_e \right) + \frac{1}{4} n_i v_{\text{ith}} \left( \frac{3}{2} kT_e \right), \quad (0.2)
\]

where \( v_{\text{th}} = (8 kT/m)^{1/2} \); and \( T_i \leq T_e \).

Hence, while electrons and ions play an equal role in the transfer of momentum, energy is transported primarily by the electrons. If the incident electrons have a Maxwellian distribution, it can be shown by integrating over all electron energies 6, 8 that

\[
\phi \propto \frac{1}{2} n_e v_{\text{eth}} (kT_e). \quad (0.3)
\]

Since the electron thermal velocity under experimental conditions of practical interest is likely to be much higher than the pellet injection velocity, the heating of the
pellet by the incident particles can be considered as spherically symmetric and the effect of the pellet motion on the ablation process may be neglected. (The effect of a magnetic field has not yet been considered.) Hence the phase transition time associated with the sublimation or ionization of the pellet can readily be estimated in a "zero-dimensional" approximation by writing the energy balance for a spherical pellet heated uniformly over its surface:

$$4 \pi r_p^2 \phi \Delta t = \frac{4}{3} \pi r_p^3 n_c \epsilon_{ph},$$  \hspace{1cm} (0.4)

where $r_p$ is the pellet radius, $n_c$ the particle density of the condensed phase, and $\epsilon_{ph}$ the phase transition energy per particle. Hence the ablation time $\Delta t$, the average ablation front velocity $u = r_p/\Delta t$, and the mass flux $\dot{m} = u \varphi_c$ can readily be expressed as functions of the incident energy flux ($\varphi_c = m_a n_c$, $m_a$ is the atomic mass):

$$\Delta t = \frac{1}{3} n_c r_p \epsilon_{ph}/\phi, \quad u = 3\phi/n_c \epsilon_{ph}, \text{ and } \dot{m} = 3 m_a \phi/\epsilon_{ph}.$$  \hspace{1cm} (0.5)

Note that the ablation front velocity and the mass flux in this approximation are directly proportional to $\phi$. Taking typical reactor plasma conditions and pellet parameters ($n_e \approx 10^{14} \text{ cm}^{-3}$, $T \approx 10 \text{ keV}$, $\phi \approx 10^{13} \text{ W/m}^2$, $n_c \approx 5 \times 10^{22} \text{ cm}^{-3}$, $r_p \approx 1 \text{ cm}$, $\epsilon_{ph} = 0.005 \text{ eV}$ and $36.2 \text{ eV}$ for sublimation and for producing a hydrogen ion pair [7], respectively) we have $\Delta t_{\text{subl}} \approx 1.4 \times 10^{-7} \text{ s}$, and $\Delta t_{\text{ioniz}} \approx 10^{-4} \text{ s}$, respectively.
Hence if the pellet is to penetrate a plasma column with a minor radius of a \( \geq 1 \) m without being fully ionized, an injection velocity of the order of \( 10^4 \) m/s and higher would be required.

The assumption of instantaneous phase transition at the pellet surface is well justified for the vaporization (sublimation) process, but it may yield incorrect rates for the ionization process, particularly at low plasma temperatures. Indeed, if the energy flux is not sufficient for instantaneous ionization at the pellet surface, the vaporized material may form a cold cloud around the pellet, thus increasing the surface area exposed to the incident energy flux. The radius of this could can be estimated if the expansion velocity of the ablated particles and the time required for their ionization are known:

\[
 r_{vap} \approx v_{vap} \Delta t_i; \text{ where } \Delta t_i = \Delta t_i(n_e, T_e). \tag{0.6}
\]

Assuming free expansion in vacuum

\[
 \left( \frac{8kT_C}{m} \right)^{1/2} \lesssim v_{vap} \lesssim \left( \frac{4\gamma}{(\gamma-1)m} \right)^{1/2} \tag{0.7}
\]
where ε/m is the energy per unit mass imparted to the pellet and is of the order of the sublimation energy (0.005 eV) per particle mass. The ionization time may then be compared with the value that can be obtained from equ. (0.5) by allowing for the increased effective surface:

\[ \Delta t = \frac{1}{3} n_c r_p \Phi^{-1} \left( \frac{r_p}{r_{\text{vap}}} \right)^2 \]  

(0.8)

Experiments performed on the Pulsator Tokamak at Garching\(^2\) yielded ablation times of the order of 300 μs for deuterium pellets approx.0.6 mm in size at plasma parameter values of \(n_e \approx 10^{13} \text{ cm}^{-3}, T_e < 10 \text{ eV} (\phi \approx 2.0 \times 10^7 \text{ W/m}^2)\). The time-resolved electron density measurements showed that the ionization of the pellet mass was completed by the end of the ablation process. The \(H_\alpha\) line emission indicated the presence of a cloud approx. 5 cm in radius around the pellet. For the above plasma parameters equ. (0.5) to (0.8) yield \(\Delta t_{\text{vap}} \approx 280 \mu\text{s}, \Delta t_{\phi} \approx 50 \mu\text{s}, 262 < v_{\text{vap}} < 1500 \text{ m/s}, r_{\text{vap}} \approx 8 \text{ cm}, \text{ and } \Delta t \approx 320 \mu\text{s} (T_c \approx 10^5 \text{K}).\)

Note that in the above approximation the energy expended on raising the kinetic and internal energies of the ablation products has been completely neglected. The flux reduction caused by the presence of the gas blanket around the pellet and the effect of the finite penetration depths of the incident particles have not been taken into account either. A further significant reduction of the ablation rates may be caused by possible electrostatic and magnetic shielding.
Electrostatic shielding may take place if the pellet is negatively charged by the incident electrons. The magnitude of this effect is determined by secondary emission phenomena, i.e. by the number of electrons emitted per incident electron. The secondary emission coefficients is a function of the incident electron energy and of the target material; and it may be greater or less than unity. Condensed hydrogen isotope pellets may be treated as dielectrics. However, the secondary emission process is coupled in our case with the process of removal of molecular layers from the pellet surface owing to vaporization. The net effect of these simultaneous processes has not yet been investigated and is beyond the scope of this analysis.

Magnetic shielding is caused by the reduction of the effective plasma transport properties across the magnetic field. If $\omega_{ci} \tau_{ii} \gg 1$ ($\omega_{ci}$ and $\tau_{ii}$ denote the ion cyclotron frequency and the ion-ion collision time), the thermal conductivity of a plasma along the magnetic field lines is dominated by electron conduction, while that across the field lines is dominated by ion conduction, owing to the larger Larmor radii of the ions. Chang has shown that the thermal flux carried by the ions along and across the magnetic field lines are

$$\phi^- \approx 1.3 n_i v_{ith} (kT_i) \quad \text{and} \quad \phi^+ \approx 0.67 v_{ith} n_i kT_i / (\omega_{ci} \tau_{ii}),$$
respectively. Hence, since \( \phi_{\parallel}^e > \phi_{\parallel}^i \left( \frac{m_e}{m_i} \right)^{1/2} \), it follows that \( \phi_{\parallel}^i/\phi_{\parallel}^e < \phi_{\perp}^i/\phi_{\perp}^e \left( \frac{\omega_{ci} \tau_{ii}}{\gamma} \right)^{-1} \). Since \( (\omega_{ci} \tau_{ii}) \gg 1 \) for most cases of practical interest, one has \( \phi_{\parallel}^e \gg \phi_{\perp}^e + \phi_{\perp}^i \). Hence, if no anomalous transport processes are present, only the energy flux transported by the electrons along the magnetic field lines play an essential role, and the pellet with its surrounding blanket is not exposed to a spherically symmetric energy influx.

The ablation of deuterium pellets in hot plasmas in the absence of magnetic fields was considered by Gralnick. The results he obtained from a steady-state ablation wave model were used as inner boundary conditions in subsequent numerical calculations pertaining to the expansion dynamics of the ablated material. (The outer boundary conditions were given by the fusion plasma parameters.) The magnetic shielding effect was considered by Rose in a steady-state spherically symmetric magneto-static approximation. The gas blanket surrounding the pellet was assumed here to be fully ionized and field-free. Chang has proposed a flexible magnetic nozzle model in which he allowed for field penetration into the plasma and plasma motion along the magnetic field lines.
Some of the assumptions made in the above analyses shall be re-examined and corrected in the present work and thus some aspects of the above models shall still be discussed in detail. Ablation rates based on properly defined mathematical models shall be calculated here both in the absence of magnetic fields and with magnetic fields present.

1. ABLATION KINETICS IN THE ABSENCE OF MAGNETIC FIELD

1.1 Gralnick's Model

Gralnick has analyzed the ablation of deuterium condensate in a hot plasma by means of a one-dimensional plane wave approximation using the ideal gas conservation equations. His model consists of two phases: the undisturbed condensed phase and the vaporized phase. The two phases are separated by the vaporization front, in which the energy flux is transported by the plasma particles is deposited. The solid phase is at rest in a laboratory frame of reference, the ablation front moves into the condensate with a velocity \( u_\phi \). In a reference frame moving with the ablation front the conservation equations can be written in the following form (see Fig. 1):

\[
\dot{m} = \mathcal{Q}_c v_c = \mathcal{Q}_1 v_r, \tag{1.1}
\]

\[
\dot{m} (v_r - v_c) = \mathcal{Q}_1 v_r^2 - \mathcal{Q}_c v_c^2 = p_c - p_1, \tag{1.2}
\]

\[
e_c + \frac{p_c}{\mathcal{J}_c} + \frac{1}{2} v_c^2 + q = e_1 + \frac{p_1}{\mathcal{J}_1} + \frac{1}{2} v_r^2, \tag{1.3}
\]
where
\[ e_1 = \frac{1}{\gamma - 1} \frac{P_1}{\rho_1}, \quad q = \frac{\phi}{m} - \frac{\epsilon_{ph}}{m_a}, \quad v_c = -u_\phi, \quad v_r = v_1 - u_\phi, \]
\[ \text{(1.4)} \]

subscript \( c \) denotes condensate and subscript \( 1 \) quantities downstream from the discontinuity (ablation or vaporization front) in a laboratory frame of reference. \( v_r \) is the velocity relative to the discontinuity surface. The quantity \( q \) represents energy flux per unit mass and is thus a function of the ablation rate itself. At sufficiently large flux values the energy spent on phase transition may be neglected in the energy balance, i.e. \( q \ll \phi/m \). The pressure in the condensed matter \( P_c \) was corrected by Gralnick \(^8\) by the amount of the momentum flux carried by the incident plasma particles.

The system of equations (1.1) to (1.4) is supplemented by the Jéguquet condition
\[ v_r^2 = \gamma P_1/\rho_1. \]
\[ \text{(1.5)} \]

The state parameters of the condensed phase \( (\rho_c, P_c, e_c) \) are assumed to be known.

Equations (1.1) and (1.2) yield the well-known Rayleigh relation:
\[ P_c - P_1 = \rho_c v_c^2 (\rho_1^{-1} - \rho_c^{-1}), \]
\[ \text{(1.6)} \]

whereas equ. (1.3) can be reduced, by means of equ. (1.6), to the Hugoniot equation:
\[ e_c - e_1 + q = \frac{1}{2} (P_1 + P_c) (\rho_1^{-1} - \rho_c^{-1}). \]
\[ \text{(1.7)} \]
With the help of equs. (1.5), (1.6), (1.7), and the equation of state Gralnick arrived at the following quadratic relation:

\[
\frac{\gamma+1}{\gamma-1} \left( \frac{v_r}{v_c} \right)^2 + 2 \frac{\gamma+1}{\gamma} \left( \frac{v_r}{v_c} \right) - \frac{e_c + q - \frac{v_c^2}{2}}{v_c^2/2} = 0,
\]

whose solution he gave for \( \gamma = 5/3 \) as

\[
\epsilon \equiv \frac{v_r}{v_c} = 0.4 \pm \left\{ 0.16 + 0.25 \left[ \left( \frac{e_c + q}{2u_\phi^2} \right) - 1 \right] \right\}^{1/2}.
\]

Note that \( \epsilon \) reduces to unity at zero heat flux only if \( e_c/u_\phi^2 + [\gamma(\gamma-1)]^{-1} \), i.e. if \( u_\phi \) approaches the isentropic signal velocity of an ideal gas. Hence Gralnick implicitly applies the ideal gas state equation to the condensed phase as well. Note, furthermore, that in the above equation not only \( \epsilon \) but also \( u_\phi \) and \( q \) are as yet unknowns. At this point, Gralnick introduced two additional assumptions: (a) he replaced the unknown energy flux per unit mass by a given energy input per particle \( q = (1.5 W_0 - E_{H_2})/2 \ m_D \), \( W_0 \approx 36 \) ev, \( E_{H_2} \approx 31.67 \) ev, and \( m_D \approx 3.35 \times 10^{-27} \) kgm; (b) he substituted for the ablation front velocity \( u_\phi = -v_c \) the value obtained for a spherical pellet in a zeroth order approximation (see equ. (0.5)) by assuming Maxwellian electron energy distribution. With \( q \) and \( v_c \) known, \( v_r \) and the rest of the flow parameters can readily be found from the above set of equations. For an assumed set of reactor plasma conditions \( n = 10^{15} \) cm\(^{-3}\), \( T = 10^8 \) O\( \text{K} \), \( \phi \approx 4 \times 10^9 \) W/cm\(^2\) he obtained the following ablation parameters: \( |u_\phi| = 1.3 \times 10^2 \) m/s, \( v_1 \approx 1.3 \times 10^4 \) m/s, \( T_1 \approx 2.4 \times 10^4 \) O\( \text{K} \), and \( \epsilon_1/e_c \approx 10^{-2} \).
Note, however, while the first of the above assumptions is permissible in a mathematical sense (it only decouples the ablation rate parameters from the actual reactor flux conditions), the second assumption makes the problem overdetermined. Indeed, if the state parameters of the condensed phase are given, equs. (1.1) to (1.3) with equ. (1.5) are necessary and sufficient for unique determination of the four unknowns \( v_C, v_R, p_1, \) and \( \mathcal{S}_1 \) (and of the associated quantities \( u_\phi, v_1, \dot{m}, \) and \( e_1 \)) in terms of the given energy flux \( \phi \). Moreover, as shall be shown in the next section, the solution can be obtained in explicit form.

1.2 The Two-Phase Jouguet-Type Ablation Model

For the sake of simplicity, it shall be assumed that the equation of state of an ideal gas applies also to the condensed phase (the problem also being solvable without this assumption). Eliminating the pressures and internal energies from equ. (1.7) by means of equs. (1.4), (1.1), (1.5), and (1.6), we obtain

\[
x^2 - 2x + 1 - 2\mu q / v_R^2 = 0,
\]

where \( x \equiv \mathcal{S}_1 / \mathcal{S}_C = v_C / v_R \) and \( \mu \equiv (\gamma - 1) / (\gamma + 1) \).

Introducing now the notation

\[
\dot{\phi}^2 \equiv 2\mu q \approx 2\mu (\dot{\phi} / \dot{m} - \epsilon_{ph} / m_a) \approx 2\mu \phi / m,
\]

(1.9)
the solution of (1.8) yields

\[ \frac{s}{s_c} = \frac{v_c}{v_r} = 1 \pm \frac{\theta}{v_r}, \]  

(1.10)

\[ -u_\phi = v_c = v_r \pm \theta, \]  

(1.11)

\[ v_1 = \mp \theta. \]  

(1.12)

A combination of equations (1.2), (1.5), (1.10), and (1.11) yields a quadratic equation relating \( v_r \) and \( \theta \):

\[ v_r^2 \pm (\gamma - 1) \theta v_r - \left( c_c^2 + \gamma \theta^2 \right) = 0, \text{ i.e.} \]  

(1.13a)

\[ v_r = \frac{1}{2} (\gamma + 1) \left[ \pm \mu \pm \sqrt{1 + \eta} \right] \theta \]  

(1.13b)

where

\[ \eta \equiv \left( \frac{2}{\gamma + 1} \right)^2 \left( \frac{c_c}{\theta} \right)^2 \text{, and } c_c^2 \equiv \gamma p_c / s_c. \]  

(1.14)

The first double sign in equ. (1.13b) originates from the solution of the quadratic equation (1.8), whereas the second one is from the solution of equ. (1.13a). As shall be seen, the selection of the proper sign combination is unique.

A second relation between \( v_r \) and \( \theta \) is obtained by substituting equ. (1.11) in equ. (1.9):

\[ \theta^2 = \frac{2 \mu \phi}{s_c (v_r \pm \theta)}. \]

Expressing \( v_r \) from this equation and substituting it in equ. (13a), we arrive at an explicit expression for \( \theta \):
\( y^4 + Ay^3 - 1 = 0, \) or \( y^3(y \pm A) = 1, \) where
\[
y = \frac{\hat{v}}{c_c}, \quad \frac{\hat{c}}{c_c} = \left(\frac{2\mu}{\gamma \phi_c c_c} \right)^{1/2}, \quad \text{and} \quad A = (\gamma+1)\frac{\hat{c}}{c_c}.
\] (1.15)

Note that the solution of equ. (1.15) may be positive or negative. Note, furthermore, that replacing \( y \) in (1.15) by \(-y\) only reverses the signs of the second term. Because of this symmetry property it is sufficient to consider only the positive solutions (\( \hat{v} > 0 \)) of equ. (1.15). Since, by definition, \( v_x > 0 \) and \( \sqrt{1+\eta} > \mu \) (\( \gamma = 5/3 \)), we may rewrite equ. (1.13b) in its final form:
\[
v_x = \frac{1}{2} (\gamma+1) (\sqrt{1+\eta} \pm \mu) \hat{v} ; \quad \hat{v} > 0.
\] (1.13c)

We may now identify the two branches corresponding to the positive and negative signs in equs. (1.13c) or (1.15). As can be seen from equs. (1.10) and (1.12), the positive branch corresponds to compression: the compressed matter follows the propagating ablation front. This is a detonation-wave-like solution. The negative branch corresponds to expansion: the flow in this case is directed away from the discontinuity surface. This is a slow-burning-type subsonic flow solution.

Let us now consider the domain of \( y \) variations corresponding to the positive and negative signs in equ. (1.15). Since only positive \( \hat{v} \) values are considered, as can readily be seen,

(a) positive branch: \( 1 \leq y \leq A^{-1/3}; \quad 0 \leq \frac{\hat{v}}{c_c} \leq \left(\frac{2\mu}{\gamma+1}\phi^*\right)^{1/3}, \)

(b) negative branch: \( 1 \leq y \leq A; \quad 0 \leq \frac{\hat{v}}{c_c} \leq 2(\gamma-1)\phi^*, \)

where \( \phi^* = \left(\frac{\phi}{S_c c_c^3}\right)^3 \) (1.14)
is the dimensionless energy flux. The upper and lower limits correspond to $\Phi^* > 1$ and $\Phi^* < 1$, respectively.

Assuming $\Phi^* > 1$, which is certainly the case under reactor-like plasma conditions, and taking the values of $\tilde{\theta}$ given by the upper limits of equs. (1.14), explicit expressions can be obtained for all plasma parameters both for the detonation-like and the slow burning-type solutions. The corresponding expressions are given in the first two columns of Table 1. All quantities denoted by asterisks are nondimensionalized by means of the condensed state parameters ($S_C \% 167 \text{ kgm/m}^3$, $T_C \% 10^6 \text{ K}$, $c_c \% 262 \text{ m/s}$).

For moderate and small values of $\Phi^*$, $\tilde{\theta}$ is determined by solving equ. (1.15) for $y = \tilde{\theta}/\theta_C$ (an iterative solution is straightforward). Ablation product parameters calculated by this method are shown in Fig. 2 as functions of the energy flux $\Phi$ both for the detonation-like and the slow-burning wave solutions. Note that with $\Phi \to 0$ $u^* + c_c$ (see equ. 1.13a with $\tilde{\theta} \to 0$) and thus the dimensionless mass flow $m^* = |u^*|$ defined in accordance with equ. (1.1) approaches to unity instead of zero. For this reason, at low energy fluxes it is advisable to use the product $\Phi v_1$ for determining the ablation rate. As can be seen from Fig. 2, $\Phi v_1 \approx |u^*| S_C$ at $\Phi \approx 10^{10} \text{ W/m}^2$. The results corresponding to $\Phi = 4 \times 10^{14} \text{ W/m}^2$ are quite different from those calculated by Gralnick.

Note the basically different functional dependences on $\Phi^*$ in the two cases considered (detonation-like, slow burning-type solutions). While the detonation-like solution predicts increasing mass flow and increasing ablation rate at higher energy fluxes, in the case of an expansion-type
solution the ablation front velocity and the mass flow decrease with increasing energy flux (in contrast to Gralnick's model, in which $\dot{m}^h \propto \phi^h$ was assumed a priori, and, as a result of overdetermining the problem, an expansion solution ($\phi_1/\phi_c \ll 1$) was found). This second case in fact corresponds to heat being added to an already choked flow (a consequence of the Jouguet condition), which, as is well known in gas dynamics, causes a reduction of the flow throughput.

Summarizing the results of this two-phase Jouguet flow approximation one sees that two types of solutions are possible: one corresponds to compression and reversed flow behind the ablation front, the other to expansion and outwardly directed flow. Intuitively, one would not expect reversed flows and vapor densities higher than solid phase density in ordinary ablation processes. Besides, the incident radiation in this case would be intercepted by the shocked material; hence the basic assumption of the model (energy deposition at the discontinuity surface) would be violated. This model can thus probably be omitted from further considerations. On the other hand, the $\dot{m} \propto \phi^{-1}$ proportionality characterizing the expansion solution cannot be considered as physically realistic either. Hence the two-phase Jouguet model is not generally applicable even if it is posed and solved correctly. The deficiency of this model is due to neglecting an essential physical phenomenon: the intense energy deposition at the surface discontinuity inevitably leads to the formation of a shock wave which penetrates the condensed phase preceding the ablation wave. Part of the energy flux transmitted to the medium is dissipated in the shock wave and the ablation wave propagates in a shock-heated medium. It is thus necessary to
<table>
<thead>
<tr>
<th></th>
<th>DETONATION WAVE-LIKE</th>
<th>SLOW BURNING TYPE</th>
<th>SIMPLE SHOCK SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1^*$</td>
<td>$- \left( \frac{2 \mu}{\gamma+1} \right)^{1/3} \phi^*^{1/3}$</td>
<td>$2(\gamma-1)\phi^*$</td>
<td>$-\left(\frac{a-1}{a}\right)^{2/3} \phi^*^{1/3}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\left( \frac{4}{\gamma^2-1} \right)^{2/3} \phi^*^{2/3}$</td>
<td>$-\frac{1}{(\gamma^2-1)^2} \phi^*^{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$v_r^*$</td>
<td>$\gamma \left( \frac{2 \mu}{\gamma+1} \right)^{1/3} \phi^*^{1/3}$</td>
<td>$2(\gamma-1)\phi^*$</td>
<td>$\frac{1}{a(\gamma-1)}^{1/3} \phi^*^{1/3}$</td>
</tr>
<tr>
<td>$-u_\phi^*$</td>
<td>$\left[ 2(\gamma^2-1) \right]^{1/3} \phi^*^{1/3}$</td>
<td>$\frac{1}{2(\gamma^2-1)} \phi^*^{-1}$</td>
<td>$\frac{1}{a(\gamma-1)}^{1/3} \phi^*^{1/3}$</td>
</tr>
<tr>
<td>$v_0^*$</td>
<td>$\gamma+1$</td>
<td>$\frac{1}{4} \frac{\gamma+1}{(\gamma^2-1)^2} \phi^*^{-2}$</td>
<td>$a = a_{\text{max}}(\phi^*)$</td>
</tr>
<tr>
<td>$m_2^*$</td>
<td>$\gamma \left[ \frac{4}{\gamma+1} (\gamma-1)^2 \right]^{1/3} \phi^*^{2/3}$</td>
<td>$\frac{1}{\gamma+1}$</td>
<td>$\frac{\gamma+1}{2} \frac{a-\mu}{a^{1/3}(\gamma-1)^{2/3}} \phi^*^{2/3}$</td>
</tr>
<tr>
<td>$e_1^*$</td>
<td>$\gamma^2 \left( \frac{2 \mu}{\gamma+1} \right)^{2/3} \phi^*^{2/3}$</td>
<td>$4(\gamma-1)^2 \phi^*^{2}$</td>
<td>$\frac{\gamma+1}{2} \frac{a-\mu}{a^{4/3}(\gamma-1)^{2/3}} \phi^*^{2/3}$</td>
</tr>
</tbody>
</table>
consider a model consisting of at least three phases: the undisturbed condensed phase, the shocked phase, and the ablated material.

1.3 Three-Phase Jouguet-Type Ablation Model

The system considered is shown in Fig. 3 in a laboratory frame of reference. The ablation wave moving to the left with a velocity \( u_\phi \) is preceded by a shock wave penetrating the undisturbed condensed phase with a velocity \( u_s \). The density is assumed to decrease across the ablation front, i.e. the expansion-type solution is assumed to apply to the second discontinuity surface. The conservation equations applied to the discontinuity surfaces and, written in the respective relative reference frames, are as follows:

\[
\dot{\mathbf{m}}_s = -\mathbf{f}_s \dot{u}_s = \mathbf{f}_s (v_s - u_s),
\]

\[
\mathbf{f}_s (v_s - u_s)^2 - \mathbf{f}_s u_s^2 = p_c - p_s,
\]

\[
e_c + \frac{p_c}{\mathbf{f}_s} + \frac{1}{2} u_s^2 = e_s + \frac{p_s}{\mathbf{f}_s} + \frac{1}{2} (v_s - u_s)^2,
\]

\[
\dot{\mathbf{m}}_\phi = \mathbf{f}_s (v_s - u_\phi) = \mathbf{f}_1 (v_1 - u_\phi),
\]

\[
\mathbf{f}_1 (v_1 - u_\phi)^2 - \mathbf{f}_s (v_s - u_\phi)^2 = p_s - p_1,
\]

\[
e_s + \frac{p_s}{\mathbf{f}_s} + \frac{1}{2} (v_s - u_\phi)^2 + q = e_1 + \frac{p_1}{\mathbf{f}_1} + \frac{1}{2} (v_1 - u_\phi)^2,
\]

where \( e = \frac{1}{\gamma - 1} \frac{p}{\mathbf{f}} \), and \( q \approx \frac{u_\phi}{m_\phi} \).
As before, the equations are supplemented by the Jouguet condition:

\[(v_1 - u_\phi)^2 = \gamma \frac{P_1}{s_1}.\]  

(1.22)

A unique solution of the above system of equations requires, in addition to the Jouguet condition, specification of still another constraint (unknown: \(u_s, u_\phi, v_s, v_1, p_s, p_1, s_s, \) and \(s_1\)). There exist some obvious physical arguments which could lead to an additional condition but, unfortunately, none of these yield a single-valued explicit constraint of the type of equ. (1.22). For example, if steady-state conditions are to prevail, the stagnation pressure downstream from the ablation wave may not be less than the ambient pressure. This condition defines a whole range of admissible total pressure values.

Instead of choosing any particular and, to some extent, artificial constraint at this point, we shall first obtain a series of solutions with one of the flow variables as a free parameter. Analysis of the solutions thus obtained may then define the region of physically possible plasma parameter values.

Note that equus. (1.16) to (1.18) are the conventional shock wave equations and their solution is unique if one of the shock parameters is given. We shall thus proceed as follows: prescribing a compression ratio

\[a = s_s/s_c = s_s^x\]  

(1.23)
we compute the state variables behind the shock wave by means of the Rankine-Hugoniot relations:

\[
\begin{align*}
p_s^* &= \frac{a-u}{1-\alpha u}; \quad u_s^* = m_s^* = - \left( \frac{2}{\gamma+1} \frac{a}{1-\alpha u} \right)^{1/2} \\
v_s^* &= (1 - \frac{1}{a}) u_s^* \quad \text{and} \quad e_s^* = c_s^2 = \frac{1-\alpha u}{1-\alpha u}
\end{align*}
\] (1.24)

The energy dissipated in the shock wave is given by the change of the stagnation temperature of the gas in the laboratory frame of reference:

\[
\Phi_s = m_s \left[ \frac{1}{2} v_s^2 + \frac{\gamma}{\gamma-1} \left( \frac{p_s}{s} - \frac{p_c}{s_c} \right) \right].
\] (1.25)

The energy flux affecting the gas at the ablation front is the difference between the total energy flux deposited in the ablation wave \( \Phi \) and the energy flux expended on shock-heating the condensed matter:

\[
\Phi_{\text{eff}} = \Phi - \Phi_s.
\] (1.26)

Hence defining

\[
y = \frac{\hat{v}}{v_s}, \quad A = (\gamma+1) \frac{\hat{v}_s}{c_s}, \quad \text{and} \quad \hat{v}_s = \left( \frac{2\mu \Phi_{\text{eff}}}{s_c c_s} \right)^{1/2},
\] (1.27)

all flow parameters can readily be computed by finding \( \hat{v} \) from eqn. (15) and following the procedure outlined above.
The value of "a" can be varied between 1 and $a_{\text{max}} = a_{\text{max}}(\phi_s)$, where the lower limit corresponds to vanishing shock wave strength ($\phi_{\text{eff}} \approx 0$, $\phi_s \to 0$), and the upper limit corresponds to the case where all energy is dissipated in the shock wave ($\phi_s \gg \phi$, $\phi_{\text{eff}} \to 0$). For the case $a \to 1$ the slow burning-type solution displayed in Table 1 is applicable, whereas the second case ($a \to a_{\text{max}}$) corresponds to a two-phase shock model, without the Jouguet condition applied. An analytic solution with $\phi_s = \phi$ as given parameter can be obtained for this limiting case as well. Indeed, expressing $\phi_s$, defined by equ. (1.25), in terms of the compression ratio $a$, we obtain, with the help of the Rankine-Hugoniot relations,

$$\phi_s^* = \frac{\phi_s}{\rho C_s c} = \left(\frac{2}{\gamma - 1}\right)^{3/2} \frac{a^{1/2}(a-1)}{(1-\mu a)^{3/2}}.\quad (1.28)$$

This is the equation that defines $a = a_{\text{max}}$ in terms of a given $\phi_s^*$ value. The value of "a" can readily be found from equ. (1.28) by iterative means. Furthermore, assuming $a = a_{\text{max}}$, the shocked flow parameters can be expressed as functions of the dimensionless energy flux $\phi_s^*$ by means of equ. (1.28). The results are given in the last column of Table 1 (see also the corresponding curves in Fig. 2). As can be seen, the functional dependence is the same, as in the case of the detonation-wave-like solution, only the numerical factors being different. This difference is due to the absence of the Jouguet condition in the ordinary shock wave solution. The quantity "a", of course, is itself a function of $\phi_s^*$, but its variation may be neglected if $\phi_s \gtrsim 10^{13}$ W/m$^2$ ($a_{\text{max}} \to 4$).
Varying the value of $\phi$ between 1 and $a_{\text{max}}$, sets of solutions to equs. (1.16) to (1.22) can be obtained for any given $\phi$ value. Typical reduced flow parameter variations are shown in Fig. 4 as functions of $a$ for $\phi = 10^{11}$ w/m$^2$. In the same figure, the ratio of the energy flux dissipated in the shock wave and the total energy flux absorbed is also shown.

The existence of several characteristic flow regions can readily be observed in Fig. 4. For a $\phi > 3.13$ one has $u_{\phi}^* > u_{s}^*$ and thus no steady state exists with two discontinuity surfaces. In the region $\phi < 3.1$ the value of $\mathcal{G}_1^*$ becomes greater than unity. As has been discussed above, this situation is also inadmissible under steady-state conditions. Finally, if $\phi < 3.01$, $v_1 > 0$, i.e. the ablated material streams towards the condensed phase as in the case of detonation and shock waves. This case, too, is very unlikely to occur under normal ablation conditions. The same characteristic parameter regions are found at other incident flux intensities as well. Moreover, at high flux intensities still another constraint may be defined which imposes a lower limit on the value $a$: the temperature $T_1$ should not exceed the ambient plasma temperature.

We may consider the $v_1 \rightarrow 0$ case as the upper limit for ablation rates and the associated plasma parameters. The reduced mass flow rate $\mathcal{G}_{s}^* u_{\phi}^*$ and the temperature $T_{1}^*$ corresponding to this upper limit are plotted in Fig. 5 as functions of the incident energy flux $\phi$. For the sake of comparison, ablation rate and temperature values corresponding to the average compression ratio within the interval $1 \leq a \leq a(v_1 = 0)\mathcal{G}_1^* v_1^*$ and $<T_1^*>$, respectively, are also shown in this figure. As can be seen, at high flux intensities the value of $<T_{1}^*>$ approaches to the limit
represented by the abmient plasma temperature. Realistic ablation parameter values should lie between the respective limit curves shown in Fig. 5.

The ablation rates obtained from the three-phase Jouguet model without shielding effects present are extremely high. The pellet injection velocities associated with these ablation rates under reactor conditions are well over $10^4$ m/s.

2. ABLATION IN THE PRESENCE OF MAGNETIC FIELD

2.1 Rose's Balloon Model

Rose assumed that the ablated material ionizes instantaneously at the pellet surface and expands by blowing a diamagnetic balloon around the pellet. The main feature of this model is the reduced heat flux to the pellet: assuming that the particles are confined to magnetic surfaces, energy is transferred to the ablated material only by those gyrating particles whose orbits dip into the sides of the balloon as they travel along it. Thus the balloon model consists of three regions (spherically symmetric geometry): (a) the cold pellet of radius $r_p$, (b) the fully ionized diamagnetic balloon of radius $r_1$ surrounding the pellet and consisting of a plasma of temperature $T_1$, density $n_1$, $\beta_1 = 1$ ($B_1 = 0$), (c) the unperturbed background plasma with $n_o, T_o, B_o$, where $p_o + B_o^2/2\mu_o = B_{vac}^2/2\mu_o$. The state parameters of the unperturbed plasma are known. It is assumed that $n_e = n_i$ and $T_e = T_i$ in all regions. The ablation products can only escape along the magnetic field lines, and the outflow velocity is assumed to be the local sonic velocity (flow area $\pi r_1^2$). Rose described this
model in a magnetostatic approximation by means of the following equations:

Particle continuity: \[ \pi r_1^2 n_1 v_s = G, \]  
(2.1)

Pressure balance: \[ 2n_1 kT_1 = B_{\text{vac}}^2 / 2 \mu_0, \]  
(2.2)

Energy balance at \( r = r_p \): \[ a_p F_1 = G \epsilon_i \]  
(2.3)

Energy balance at \( r = r_1 \): \[ a_1 F_0 = G(\epsilon_i + 3 k T_1), \]  
(2.4)

where \( G(\text{sec}^{-1}) \) is the particle ablation (ionization) rate, \( F_1 = 0.5 n_1 v \text{e} \text{kT}_1 \) and \( F_0 = 0.5 n_0 v_\text{eoth} \text{kT} \) are energy fluxes (corresponding to Maxwellian distributions) at the respective surfaces, \( a_p = 4\pi r_p^2, a_1 = \pi [ (r_1 + r_c)^2 - r_1^2 ] / 2 \pi r_1 r_c \), \( r_c = mv/eB \ll r_1 \) is the gyro-radius, and \( v_s = (2T_1/m_i)^{1/2} \). Since \( r_\text{ci} v_\text{ith} T_i = r_c e \text{v_\text{eoth} T_e} \), only one species of particles is considered as energy carriers and the energy spent on heating the vaporized gas is neglected. The ablation kinetics (ablation or shock wave propagation, etc.) is not considered in this approximation. The above set of equations is sufficient for unique determination of the four unknowns \( G, r_1, n_1, \) and \( T_1 \):

\[
G = \frac{\pi r_p^2}{\epsilon_i} \frac{B_{\text{vac}}}{2 \mu_0} \left( \frac{8kT_1}{\pi m_e} \right)^{1/2},
\]  
(2.5)

\[
\frac{r_1}{r_p} = \left( \frac{32}{\pi} \frac{m_a}{m_e} \right)^{1/4} \left( \frac{kT_1}{\epsilon_i} \right)^{1/2},
\]  
(2.6)

\[
n_1 = \frac{B_{\text{vac}}}{4 n_0 kT_1},
\]  
(2.7)

\[
\frac{kT_1}{\epsilon_i} = \frac{1}{3} \left[ \left( \frac{n_0 kT_0}{B_{\text{vac}}^2 / 2 \mu_0} \right) \left( \frac{32}{\pi} \frac{m_e}{m_a} \right)^{1/4} \left( \frac{r_\text{ci}}{r_p} \right) \left( \frac{kT_0}{\epsilon_i} \right)^{1/2} - 1 \right].
\]  
(2.8)
A thorough analysis of Rose's solution was given by Chang\textsuperscript{7}. He showed that the plasma temperatures obtained from this solution are much too low ($T_1 \approx 1$ eV under reactor plasma conditions) to justify the assumption of a fully ionized diamagnetic balloon around the pellet. Chang also showed that, in the framework of this model, physically possible solutions ($T_1 > 0$) are only possible for certain pellet size and magnetic field strength combinations.

### 2.2 Chang's Magnetic Nozzle Model

Since the blanket surrounding the ablating pellet is not likely to be fully ionized, Chang\textsuperscript{7} allowed for the presence of magnetic field in the blanket by prescribing the ratio

$$
\epsilon \equiv \frac{B_1}{2 \mu_0 p_1}
$$

(2.9)

where subscript 1 denotes parameter values around the pellet and at the entrance section of the "magnetic nozzle". His model is described by means of the following equations (see also Fig.6):

$$
\mathcal{S}_{av} = \mathcal{S}_1 v_1
$$

(2.10)

$$
p/\mathcal{S} = p_1/\mathcal{S}_1
$$

(2.11)

$$
\mathcal{S} v_1^2 - \mathcal{S}_1 v_1^2 = p_1 - p
$$

(2.12)

$$
p + B_2^2/2\mu_0 = \text{const} = p_0 + B_0^2/2\mu_0 = B_{vac}^2/2\mu_0
$$

(2.13)

$$
aB = a_1 B_1
$$

(2.14)
where \( a \) is the stream tube cross-section: \( a = \pi r^2 \).

Equation (2.14) implies that the stream tubes are identical with flux tubes, i.e. the plasma is only allowed to move along the magnetic field lines. Hence the ablation products stream away from the pellet as if they were enclosed by a (flexible) nozzle. The pressure variation is coupled to the magnetic field strength variation along the nozzle by means of equ. (2.13). Chang has assumed that the flow becomes sonic at the nozzle throat:

\[
v_\infty = \left( \frac{\mathcal{E}_\infty}{\mathcal{E}_p} \right)^{1/2}, \tag{2.15}\]

and that the throat area is equal to the pellet cross-section:

\[
a_\infty = a_p; \quad r_\infty = r_p. \tag{2.16}\]

He further assumed that the throat pressure is given by the surrounding (known) plasma pressure:

\[
p_\infty = p_o, \quad B_\infty = B_o, \tag{2.17}\]

i.e. undisturbed background plasma conditions prevail at the throat of the nozzle. Similarly to Rose's approach, the ablation kinetics is neglected also in this approximation; the ablation rate \( G \) is assumed to be given by

\[
G = \pi r^2 p \rho n_1 v_{1th} kT_i / \mathcal{E}_i. \tag{2.18}\]
As can be seen, the first three of the above equations correspond to isentropic nozzle flow, and hence, for stations between \( a_1 \) and \( a_2 \), the relation known from ideal gas dynamics apply (equ. 4.8 of Chang\(^7\)):

\[
\left( \frac{a_2}{a_1} \right)^2 = \left( \frac{r_p}{r_1} \right)^4 = \frac{\frac{P_1}{P_0}}{\frac{P_1}{P_0} - 1} \left[ 1 - \frac{2}{r_1} \right] \left( \frac{P_1}{P_0} \right)^\frac{2}{r_1} = \frac{\frac{P_1}{P_0}}{\frac{P_1}{P_0} - 1} \left[ 1 - \frac{2}{r_1} \right] \left( \frac{P_1}{P_0} \right)^\frac{2}{r_1}.
\] (2.19)

Considering equs. (2.14) and (2.16) together, one notices that only those magnetic flux surfaces that were initially embedded in the cold pellet remain trapped in the ablation products. The flux tube, which was originally attached to the cold pellet surface, forms the boundary of the magnetic nozzle at later times, i.e. no field or particle diffusion takes place across this boundary. Hence the external magnetic field does not really penetrate the ablation products and the model is in this respect not different from Rose's diamagnetic balloon model.

Note, furthermore, that it follows from equs. (2.9) and (2.13) that

\[
P_1 = \frac{B_{\text{vac}}^2/2\mu_0}{1 + \frac{\varepsilon}{\varepsilon + 1}}, \quad \text{and} \quad \left( \frac{B_1}{B_{\text{vac}}} \right)^2 = \frac{\varepsilon}{\varepsilon + 1}.
\] (2.20)

Furthermore, equs. (2.14), (2.16), and (2.17) yield

\[
\left( \frac{r_1}{r_p} \right)^4 = (1 - \beta) \left( \frac{\varepsilon + 1}{\varepsilon} \right), \quad \text{where} \quad \beta = \frac{P_0}{B_{\text{vac}}^2/2\mu_0}
\]

is a given parameter. Since

\[
\frac{P_1}{P_0} = \frac{B_{\text{vac}}^2/2\mu_0}{1 + \frac{\varepsilon}{\varepsilon + 1}} = \frac{2\mu_0}{\beta B_{\text{vac}}^2} = \frac{1}{\beta(1 + \varepsilon)}
\]
we obtain
\[
\frac{r_1}{r_p}^4 = \frac{1-\beta}{1-(p_1/p_o)^\beta} \tag{2.21}
\]

which is a second relation between \((r_1/r_p)\) and \((p_1/p_o)\) incompatible with that obtained from ideal gas dynamic considerations (equ. 2.19). Hence, the system of equations treated by Chang is also overdetermined.

There is no simple way to modify this model. To remove one of the constraints used, one would necessarily have to change a number of other assumptions as well. For example, if we remove the \(r_0 = r_p\) constraint, the pressure could be computed from equ.(2.20). However, \(r_0 \neq r_p\) is equivalent to allowing for field penetration and thus equs. (2.14) and (2.17) would have to be modified as well.

Since field penetration is a diffusion process, any steady-state approximation allowing for non-vanishing magnetic fields in the ablation plasma should include some assumption regarding the inward diffusion of the magnetic field and the outward motion of the ablated and ionized matter.

2.3 Magnetic diffusion model

The model considered here can be described as follows: The pellet is initially in direct contact with the surrounding plasma and thus the ablated particles are ionized within a time \(\Delta t_i = \Delta t_i (n_{c0}, T_{c0})\), where subscript "o" denotes the
surrounding plasma \((n_{eo} = n_0, \; T_{eo} = T_0)\). Considering collisional ionization, this time is much shorter than the ablation time in all cases of practical interest. As time goes on, the ablated particles form a blanket around the pellet, whose temperature is different from the unperturbed plasma temperature: \(T_1 < T_0\). The ionization front moves some distance away from the pellet surface \((r_1 > r_p)\). Some of the electrons in the blanket recombine, and, since the characteristic recombination times are also much shorter than the ablation times (owing to the high densities), the electron density in the blanket \(n_{e1}\) may be approximated to a fairly good degree of accuracy by the equilibrium (Saha) density value corresponding to the temperature \(T_1\). The particles ablated leave the pellet with a random velocity whose magnitude lies in the range given by equ. (0.7), and are ionized after a time \(\Delta t_1 = \Delta t_1 (n_{e1}, T_1)\), where subscript "1" denotes parameter values in the blanket around the pellet. The ionization front while moving outward displaces the magnetic flux lines. If field diffusion could be neglected, the magnetic flux originally imbedded in the pellet would be distributed over the blanket of radius \(r_1\). However, as will be shown, the penetration depth of the magnetic field \(\Delta l\) is comparable in some cases to the blanket radius or is even larger. The penetration depth is given by

\[
\Delta l = \left(\frac{\Delta \tau}{\sigma_1 \mu_0}\right)^{1/2}.
\]

The field penetration is complete if the field diffusion velocity is greater than or equal to the outward directed flow velocity, i.e. if
\[ v_D = \frac{\Delta t}{\Delta T} = (\mathcal{G}_1 \mu_0 \Delta t)^{-1/2} > v_1 = \frac{\Delta r}{\Delta T}; \Delta r = r_1 - r_p, \]

i.e. if
\[ R_m = \mathcal{G}_1 \mu_0 v_1 \Delta r < 1, \tag{2.22} \]

where \( R_m \) is the magnetic Reynolds number based on the blanket parameters. Thus the \( R_m = 1 \) condition represents the limit between magnetized and diamagnetic ablation plasmas.

It is assumed that the electrons heating the pellet can only move along the magnetic flux lines. Energy is thus transported only along those flux lines which either pierce the blanket or are at a distance from it not larger than the electron gyro-radius. The effective area exposed to the thermal flux \( \phi_0 = 0.5 n_e v_e o k T_0 \) is thus given by
\[ a_1 = 2\pi \left( \frac{r_p^2}{r_1^2} + \left[ \frac{r_1^2 - (r_1 - \Delta l)^2}{(r_1 + r_e c)^2 - r_1^2} \right] + \frac{(r_1 + r_e c)^2 - r_1^2}{(r_1 + r_e c)^2 - r_1^2} \right), \]
where the first and second terms of the r.h.s. account for the magnetic flux lines that were originally imbedded in the pellet and those diffused into the blanket, while the third term accounts for the gyrating electrons, whose trajectory dips into the sides of the blanket. The energy carried by the gyrating ions is accounted for simply by doubling the corresponding electron flux term. As can readily be seen, for magnetic field strengths of practical interest \((B \lesssim 3 \, \text{T})\), this contribution is altogether negligible relative to the fluxes corresponding to the first two terms. There is of course a limit on \( a_1 : a_1 \lesssim 2\pi r_1^2 \).
A quasi-steady approximation is used, the time-dependent build-up of the blanket around the pellet not being considered here. The shock kinetics discussed in the previous sections as well as the phenomena associated with the finite penetration depths of the incident particles in the blanket are also left out of consideration. It is assumed that the energy flux incident on the pellet surface causes only vaporization, and that the radial velocity component of the vaporized particles \( v_{\text{vap}} \) can be approximated by the sonic velocity at the pellet temperature. The major part of the energy is transferred to the particles at the blanket-plasma interface. It is assumed, furthermore, that the vapor velocity does not change appreciably over the ionization length \( \Delta r \). The motion remains spherically symmetric up to the ionization radius \( r_1 \), and the charged particles then lose their radial momentum and leave the blanket region along the magnetic flux lines.

The system of equations defining the model can thus be written in the following form:

\[
2\pi r_1^2 n \Phi_0 = G (\frac{1}{2} m a v_1^2 + \frac{3}{2} kT_1) + G_e (\epsilon i + \frac{3}{2} kT_1),
\]

\[
4\pi r_p^2 \Phi_1 = G \epsilon_{\text{vap}},
\]

\[
r_1 - r_p = \Delta t_{\text{vap}} v_{\text{vap}}; \quad \Delta t_1 = (n_{e_1} S_{c_1})^{-1}; \quad S_{c_1} = S_c (n_{e_1}, T_1)
\]

where

\[
G = G_a + G_e, \quad G_a = 4\pi r_1^2 n a v_1, \quad G_e = 2\pi r_1^2 n e v_1
\]
\[ \eta = \min (\eta_0, 1), \quad \Delta \ell = \Delta r / \text{Rm}^{1/4} r_1 / \text{Rm}_1^{1/4}, \quad \text{Rm}_1 = 6 \mu \nu_1 r_1, \quad \text{and} \]

\[ \eta_0 = \left( \frac{r_p}{r_1} \right)^2 + \text{Rm}_1^{-1/2} (2 - \text{Rm}_1^{-1/2}) + 2 \left( \frac{r_{ec}}{r_1} \right) \left( 2 + r_{ec}/r_1 \right). \]

In the present estimates, the ionization coefficient values tabulated by Duchs\(^9\) on the basis of the analysis of Bates, Kingston, and McWhirter\(^10\) have been used. The above equations are supplemented by the Saha relation

\[ n_{e1} = \alpha n_1, \quad \alpha^2/(\alpha - 1) = f_s / n_1, \]

where \( n_1 = n_a + n_e \), and \( f_s = \text{const.} \quad T_1^{3/2} \exp(-E_i/\kappa T_1) \) (2.26)

The energy flux \( \phi \) is defined by equ. (0.3). We thus have four equations (2.23) to (2.26) with five unknowns: \( n_1, n_{e1}, T_1, v_1 \), and \( r_1 \).

**Sonic approximation.** To make the above system of equations closed, we shall assume, as Rose\(^5\) and Chang\(^7\) did, that the ablation plasma leaves the blanket with sonic velocity:

\[ v_1^2 = c_1^2 = \gamma p_1 / \rho_1; \quad p_1 = n_a k T_1 + 2 n_{e1} k T_1; \quad \rho_1 = n_1 \mu_a. \]  (2.27)

With this assumption the above set of equations reduces to (MKS system):

\[ \eta \phi_0 = n_1 c_1 k T_1 \{(1 - \alpha/2) [\gamma(1 + \alpha) + 3] + \alpha (\epsilon_i / \kappa T_1 + 3/2) \}, \]  (2.28)

\[ \frac{r_1}{r_p} = \left( \frac{74.92}{(\alpha + 1)^{3/2}} \right) \left( \frac{\alpha k T_1}{2 - \alpha \epsilon_{vap}} \right)^{1/2}, \]  (2.29)

\[ r_1 - r_p = v_{vap} \Delta t_i, \]  (2.30)
\[ n_{e1} = n_e(n_1, T_1)_{\text{Saha}}, c_1 = \left[ \frac{1}{2} \left( 1 + \alpha \right) \frac{kT_1}{m_a} \right]^{1/2} \]  

(2.31)

which can be solved iteratively for any given values of \( \phi_0 \), \( r_p \), and \( v_{vap} \) \( (v_{vap} \approx (\gamma kT_c / m_a)^{1/2} \approx 262 \text{ m/s}) \).

Results calculated on the basis of this approximation are displayed in Table 2 for \( \phi_0 \) flux values ranging from present low-temperature boundary layer ablation experiments up to future reactor conditions. A discussion of the results follows below.

**Approximation with transverse momentum balance.** As is known from laser-plasma calculations \(^{11, 12} \), the assumption of sonic flow is a rather crude approximation for spherical flows with energy influx. It is thus of interest to check in what way and to what extent the ablation parameters change if the sonic constraint is removed. Since the velocity \( v_1 \) is now an unknown quantity, an additional condition is introduced in the form of the transverse momentum balance:

\[ (p + \frac{\gamma}{2} v^2 + B^2/2\mu_0) r \leq r_1 = p_0 + B_0^2/2\mu_0 = B_{\text{vac}}^2/2\mu_0. \]  

(2.32)

Since the magnetic field penetrates into the blanket to a depth \( \Delta l \) and the ionization radius \( r_1 \) is thus located in the region of the surrounding magnetic field, there is no field discontinuity at \( r = r_1 \) and equ. (2.32) reduces to

\[ (1-\alpha)\frac{\gamma}{2} v_1^2 + (1+\alpha)n_1kT_1 = 2n_0kT_0. \]  

(2.33)

For checking the effect of the magnetic pressure on the ablation parameters, some calculations were performed also by retaining the \( B^2/2\mu_0 \) term in the above equation and assuming that
\[ \pi r_{B_0}^2 = \pi (r_1 - \Delta \lambda)^2 B_1, \] (2.34)

which is an additional equation defining \( B_1 = B(r \leq r_1) \).
In the momentum balance approximation equ. (2.28) is replaced by

\[ \eta \phi_0 = n_1 c_1 k T_1 M_1 \left\{ (1-\alpha/2) [(1+\alpha) \gamma M_1^2 + 3] + \alpha (\epsilon_i/k T_1 + 3/2) \right\} \] (2.35)

and is supplemented by an equation that can be obtained from equ. (2.33):

\[ (1+\alpha) \left\{ 1 + (1-\alpha) \gamma M_1^2 \right\} n_1 k T_1 = 2 n_0 k T_0. \] (2.36)

The system becomes somewhat more complicated if equ. (2.33) is replaced by equs. (2.32) and (2.34).

An iterative solution to the above system of equations is somewhat tedious, but possible. Results based on the approximation with equ. (2.33) (or eq. 2.36) are displayed in Table 3.

Discussion of the results.

The parameters \( r_1, n_1, T_1, M_1 = v_1/c_1 \), and the ablation rate \( G \) are displayed in Tables 2 and 3 as functions of the input parameters \( \phi_0(n_0, T_0) \) and \( r_p \). The last two rows in both tables correspond to two different pellet sizes at the same flux value (reactor plasma conditions). The ablation time \( \tau = N_p/G \) is also shown in Table 3.

The results corresponding to the two different approximations display a common characteristic: as the flux intensity \( \phi_0 \) increases, the blanket radius \( r_1 \) decreases, but, at the same time, the blanket density \( n_1 \) increases, and so the blanket temperature \( T_1 \) remains practically constant (of the order of 1 eV), over 5 orders of magnitude of \( \phi_0 \)-variation. Hence the
### TABLE 2  MAGNETIC DIFFUSION MODEL, SONIC APPROXIMATION \((M_1 = 1)\)

<table>
<thead>
<tr>
<th>(\phi_o \left( \frac{W}{m^2} \right))</th>
<th>(n_o \left( \text{cm}^{-3} \right))</th>
<th>(T_o \text{ (eV)})</th>
<th>(r_p \text{ (mm)})</th>
<th>(r_1 \text{ (mm)})</th>
<th>(n_1 \left( \text{cm}^{-3} \right))</th>
<th>(T_1 \text{ (eV)})</th>
<th>(G \text{ (s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 \times 10^7)</td>
<td>(1.2 \times 10^{13})</td>
<td>(1.0 \times 10)</td>
<td>0.31</td>
<td>43.8</td>
<td>(2.0 \times 10^{14})</td>
<td>1.80</td>
<td>(4.17 \times 10^{22})</td>
</tr>
<tr>
<td>(10^9)</td>
<td>(2.0 \times 10^{13})</td>
<td>(9.7 \times 10)</td>
<td>0.60</td>
<td>16.1</td>
<td>(1.0 \times 10^{17})</td>
<td>1.05</td>
<td>(3.07 \times 10^{24})</td>
</tr>
<tr>
<td>(10^{11})</td>
<td>(2.0 \times 10^{13})</td>
<td>(2.0 \times 10^{3})</td>
<td>0.61</td>
<td>3.15</td>
<td>(1.4 \times 10^{19})</td>
<td>0.99</td>
<td>(2.11 \times 10^{25})</td>
</tr>
<tr>
<td>(10^{13})</td>
<td>(1.0 \times 10^{14})</td>
<td>(1.5 \times 10^{4})</td>
<td>0.60</td>
<td>1.00</td>
<td>(1.6 \times 10^{21})</td>
<td>0.97</td>
<td>(1.76 \times 10^{26})</td>
</tr>
<tr>
<td>(10^{13})</td>
<td>(1.0 \times 10^{14})</td>
<td>(1.5 \times 10^{4})</td>
<td>5.10</td>
<td>6.35</td>
<td>(1.7 \times 10^{21})</td>
<td>0.92</td>
<td>(7.45 \times 10^{27})</td>
</tr>
</tbody>
</table>

### TABLE 3  MAGNETIC DIFFUSION MODEL, MOMENTUM BALANCE \(^{a)}\)

<table>
<thead>
<tr>
<th>(\phi_o \left( \frac{W}{m^2} \right))</th>
<th>(r_p \text{ (mm)})</th>
<th>(r_1 \text{ (mm)})</th>
<th>(n_1 \left( \text{cm}^{-3} \right))</th>
<th>(T_1 \text{ (eV)})</th>
<th>(M_1)</th>
<th>(G \text{ (s}^{-1} )</th>
<th>(\tau \text{ (\mu s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 \times 10^7)</td>
<td>0.33</td>
<td>42.8</td>
<td>(4.8 \times 10^{14})</td>
<td>2.50</td>
<td>1.70</td>
<td>(1.75 \times 10^{22})</td>
<td>433</td>
</tr>
<tr>
<td>(10^9)</td>
<td>0.60</td>
<td>36.8</td>
<td>(9.7 \times 10^{14})</td>
<td>1.38</td>
<td>3.90</td>
<td>(4.85 \times 10^{23})</td>
<td>93</td>
</tr>
<tr>
<td>(10^{11})</td>
<td>0.59</td>
<td>17.5</td>
<td>(1.7 \times 10^{15})</td>
<td>1.35</td>
<td>15.3</td>
<td>(7.7 \times 10^{23})</td>
<td>56</td>
</tr>
<tr>
<td>(10^{13})</td>
<td>0.61</td>
<td>9.5</td>
<td>(3.3 \times 10^{15})</td>
<td>1.36</td>
<td>55.4</td>
<td>(1.50 \times 10^{24})</td>
<td>31</td>
</tr>
<tr>
<td>(10^{13})</td>
<td>5.15</td>
<td>6.9</td>
<td>(1.7 \times 10^{15})</td>
<td>1.20</td>
<td>54.3</td>
<td>(4.25 \times 10^{25})</td>
<td>678</td>
</tr>
</tbody>
</table>

\(^{a)}\) The \(n_o\) and \(T_o\) values corresponding to a particular \(\phi_o\) are the same as in Table 2.
ablation process seems to be self-regulating: the constant
(flux-independent) blanket temperature provides an effective
shielding also at high flux intensities. Owing to the flow
restriction, the sonic approximation yields rather high
blanket densities at high flux values. Removal of the sonic
constraint yields Mach numbers of the order of unity at low
flux values, but rather high Mach numbers at reactor con-
ditions. The corresponding ablation rates vary accordingly:
at low flux values the two approximations yield results of
the same order of magnitude, which are also in agreement
with experimental observations\textsuperscript{1,2}; at high flux values,
owing to the rather high inherent blanket densities, the
sonic approximation yields ablation rates that are of 2 orders
of magnitude higher than those obtained with the momentum
balance. Considering experimental installations of the type
of ASDEX in Garching ($\phi_0 \approx 10^{11}$) or larger, the ablation
times corresponding to the momentum balance approximation
would require pellet injection speeds of the order of $10^3$
to $10^4$ m/s.

It should be noted that the assumptions leading to the
ablation rates displayed in Tables 2 and 3 are rather
optimistic: computations performed with vapor velocities
$v_{\text{vap}}$ greater than the sonic velocity at the pellet temperature
yield higher ablation rates. Diamagnetic effects may cause
an additional increase of the estimated rates. For example,
if in the momentum balance approximation equ. (2.33) is
replaced by equs. (2.32) and (2.34), for the case with
$\phi = 10^{11}$ W/m$^2$, $B_{\text{vac}} = 3$ T, and $r_p = 0.52$ mm (ASDEX-conditions)
the following set of ablation parameters is obtained:
\[ r_1 = 5.6 \text{ mm}, \quad n_1 = 3.2 \times 10^{17} \text{ (cm}^{-3}\text{)}, \quad T_1 = 1.1 \text{ eV}, \quad \Delta \xi = 4.4 \text{ mm}, \]
\[ M_1 = 4.7, \quad G = 5.4 \times 10^{24} \text{ s}^{-1}, \quad \text{and} \quad \tau = 5.3 \mu\text{s}. \]
The functional relation among \( \phi_0, n_1, T_1, \) and \( r_1 \) is qualitatively the same in all these approximations.

3. CONCLUSIONS

a) An alalysis of the kinetics of ablation waves requires a two-wave approximation: the effect of the shock wave preceeding the ablation wave cannot be generally neglected. The two-wave model has one free (unknown) flow parameter, which should be chosen in accordance with experimental observations.

b) The magnetic Reynolds number based on the parameters of the blanket surrounding the pellet plays an important role in the ablation process: it determines whether the blanket remains diamagnetic or becomes magnetized and thus affects the magnitude of the energy influx.

c) The field diffusion model presented here indicates that the ablation process may be self-regulating: higher energy influx is compensated by smaller blanket radius and higher blanket density, the blanket temperature of the order of 1 eV being practically independent of the energy flux value.
d) A self-consistent analysis of the ablation process should include the consideration of such phenomena as the interaction of the incident plasma particles with the blanket (penetration depths, energy deposition rates, etc.); the ionization, recombination and charge exchange processes in the blanket, the blanket-magnetic field interaction, and the ablation wave kinetics in the pellet. A simultaneous account for all these effects is likely to require properly posed numerical models.

REFERENCES


10  D.R. Bates, A.E. Kingston, and R.W. McWhirter,


12  S.J. Gitomer, R.L. Morse, and B.S. Newberger,
Fig. 1 TWO-PHASE ABLATION MODEL

Fig. 2 TWO-PHASE ABLATION MODEL: PARAMETER VARIATIONS VS INCIDENT ENERGY FLUX $\Phi$ (w/m²). $T_{ref} = 10^5K$, $v_{ref} = 262m/s$
Fig. 3 THREE-PHASE ABLATION MODEL

Fig. 4 ABLATION PARAMETER VARIATIONS AS FUNCTIONS OF THE COMPRESSION RATIO, a:
FOR $\Phi = 10^{11}$ W/m², $T_{ref} = 10^6$ K, $u_{ref} = 262$ m/s,
$\rho_{ref} = 167$ kg/m³
Fig. 5 ABLATION PARAMETER VARIATIONS CORRESPONDING TO \( v_1 = 0 \) AND \( \alpha = \langle \alpha \rangle \). 
\( u_{\text{ref}} = 262 \text{ m/s}, \varphi_{\text{ref}} = 167 \text{ kg/m}^3, T_{\text{ref}} = 10^9\text{K} \). THREE PHASE ABLATION MODEL.

Fig. 6 PELLET ABLATION IN MAGNETIC FIELD