Heat Propagation in Anisotropic Targets Irradiated by Laser Pulses

P. Bachmann, D. Hildebrandt, D. Sünder

Max-Planck-Institut für Plasmaphysik, Bereich Plasmadiagnostik Berlin, EURATOM

Introduction

Thermography is applied to magnetic fusion devices such as tokamaks and stellarators to supervise and diagnose the power loading of plasma facing components. 3D numerical codes are applied to derive power fluxes arriving at these internal components from the temporal evolution of the measured surface temperature distribution.

In order to check the accuracy of the evaluation procedure applied to the thermography system of the stellarator W7-AS accompanying laboratory experiments with well defined heat pulses from an infrared laser applied to the W7-AS divertor CFC target material have been carried out [1,2]. The measured surface temperature evolution of this material is compared with theoretical results which are obtained for Gaussian profiles and different duration of the laser pulse. The temperature variation measured at any surface point can be well described by the calculations.

Basic equations

The starting point is the time-dependent three-dimesional heat conduction equation for the temperature in Cartesian coordinates $T(x, y, z, t)$

$$c\rho\frac{\partial}{\partial t}T = \left(\frac{\partial}{\partial x}\kappa_x\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\kappa_y\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\kappa_z\frac{\partial}{\partial z}\right)T + f \tag{1}$$

with heat conduction coefficients $\kappa_i = \kappa_{i0}\kappa(T), i = x, y, z$, density ρ, heat capacity $c(T)$ and the source function f. Assuming $c\rho/\kappa = s(t)c_0\rho_0$ and introducing the temperature potential U and the time t^* determined by

$$\frac{dU}{dT} = \kappa(T),\ \frac{dt^*}{dt} = \frac{1}{s(t)} \tag{2}$$

we obtain the model equation

$$c_0\rho_0\frac{\partial}{\partial t^*}U = \left(\kappa_{x0}\frac{\partial^2}{\partial x^2} + \kappa_{y0}\frac{\partial^2}{\partial y^2} + \kappa_{z0}\frac{\partial^2}{\partial z^2}\right)U + f. \tag{3}$$

We consider a target with dimensions L_x, L_y and L_z, respectively, which is irradiated on the surface $x = 0$ by a laser pulse and is in contact with a water bath at $x = -L_x$ at the temperature potential U_W. The side walls $y = 0, y = L_y, z = 0, z = L_z$ are heat-insulated, $U(x, y, z, t^* = 0) = U_A$. This case will be described by the following linear boundary conditions:

$$-\kappa_{x0}\partial U/\partial x|_{x=0} = G_0(y, z, t^*), \quad -\kappa_{x0}\partial U/\partial x|_{x=-L_x} = h(U(x = -L_x, y, z, t^*) - U_W), \quad (4)$$

$$\kappa_{y0}\partial U/\partial y|_{y=0} = \kappa_{y0}\partial U/\partial y|_{y=L_y} = \kappa_{z0}\partial U/\partial z|_{z=0} = \kappa_{z0}\partial U/\partial z|_{=L_z} = 0. \quad (5)$$

Introducing dimensionless parameters and functions $\tau = t^*/t_0, \xi = x/L_x, \eta = y/L_y, \chi = z/L_z, d_x = t_0\kappa_{x0}/L_x^2 c_0\rho_0, d_y = t_0\kappa_{y0}/L_y^2 c_0\rho_0, d_z = t_0\kappa_{z0}/L_z^2 c_0\rho_0$ the solution without volume sources (f=0) is given in the form $U = U_W + U_1 + U_2$ with

$$U_1 = \sum_{m,n,p=-\infty}^{\infty} \frac{1}{g_m} cos(k_m\xi)cos(n\pi\eta)cos(p\pi\chi) \int_0^1 d\xi' d\eta' d\chi' (U_A(\xi', \eta', \chi') - U_W) \quad (6)$$
$$cos(k_m\xi')cos(n\pi\eta')cos(p\pi\chi')e^{-(k_m^2 d_x + n^2\pi^2 d_y + p^2\pi^2 d_z)\tau},$$

$$U_2 = \sum_{m,n,p=-\infty}^{\infty} \frac{t_0}{g_m L_x c_0\rho_0} cos(k_m\xi)cos(n\pi\eta)cos(p\pi\chi) \int_0^\tau d\tau' \int_0^1 d\eta' d\chi' G_0(\eta', \chi', \tau') \quad (7)$$
$$cos(n\pi\eta')cos(p\pi\chi')e^{-(k_m^2 d_x + n^2\pi^2 d_y + p^2\pi^2 d_z)(\tau-\tau')},$$

where k_m is the mth root of the transcendental equation (cp.[3])

$$h^* cos k_m = k_m sin k_m, \quad h^* = \frac{hL_x}{\kappa_{x0}} \quad (8)$$

and $g_m = 1 + sin2k_m/2k_m$ For the case $h^* = 0$ (heat-insulated at the bottom) we obtain the Neumann problem with $k_m = m\pi$. The Dirichlet boundary condition follows for large values h^* ($h^* \to \infty$), $U = U_W,\ k_m = (m + 1/2)\pi$.

Numerical results

In order to simplify the problem, we assume $U_A = U_W = 0, G_0 = \Gamma(\tau)F(\eta, \chi)$ with rectangular pulses and a Gaussian profile

$$\Gamma = \sum_{s=1}^{S} \Gamma^{(s)}\Theta(t - t_{s-1})\Theta(t_s - t), F = F_0 e^{-a_1(\eta-\eta_0)^2} e^{-a_2(\chi-\chi_0)^2}, \int_0^1 d\eta d\chi F = F_{00} \quad (9)$$

and obtain

$$U = \sum_{m,n,p=-\infty}^{\infty} U_{m,n,p}(\tau)A_{m,n,p} cos(k_m\xi)cos(n\pi\eta)cos(p\pi\chi) \quad (10)$$

$$U_{m,n,p} = \int_0^\tau d\tau' \Gamma(\tau') e^{-(k_m^2 d_x + n^2\pi^2 d_y + p^2\pi^2 d_z)(\tau-\tau')}, \tag{11}$$

$$A_{m,n,p} = \frac{C}{g_m}\int_0^1 d\eta d\chi e^{-a_1(\eta-\eta_0)^2} e^{-a_2(\chi-\chi_0)^2} cos(n\pi\eta) cos(p\pi\chi),\ C = \frac{t_0 F_0}{L_x c_0 \rho_0} \tag{12}$$

$U(\xi,\eta,\chi,\tau)$ is calculated numerically for parameters: $C = 1,\ F_{00} = 1,\ \Gamma^{(1)} = 1,\ \Gamma^{(2)} = 0,\ \Gamma^{(3)} = 1,\ \Gamma^{(4)} = 0,\ \tau_1 = 1, \tau_2 = 2,\ \tau_3 = 3,\ \tau_4 = 4,\ d_x = 1,\ d_y = 1,\ d_z = 3,\ \alpha_1 = 10,\ \alpha_2 = 10,\ \eta_0 = 0.25,\ \chi_0 = 0.5,\ h^* = 1$. Fig. 1 shows the U profile on the surface $x = 0$ at $\tau = 0.8$. The heat propagation in the maximum of the laser pulse ($\eta = 0.25, \chi = 0.5$) into the target is demonstrated in Fig. 2.

Experimental results

The temperature variation on a polished graphite target surface is measured by an infrared camera during and after a heat pulse with a duration of 4 ms applied by an infrared laser. The central power density was 50 MW/m^2. The data hase been taken from the center of the laser spot and is compared with theoretical results for $h^* = 0$, s=1, $\kappa = 1$ at $\xi = 0$ in Fig. 3.

Summary

The starting point is the time-dependent 3D heat conduction equation for the target temperature. The evaluation domain is a cube with a given heat influx with a Gaussian profile at the top, heat-insulated side walls and a water bath at the bottom as boundary conditions. The measured temperature excursion at surface points of a polished graphite target can be well described by the calculations. The theoretical model has to be improved for the description of the temperature propagation in graphit targets with unpolished surfaces.

References

[1] D. Hildebrandt, F. Gadelmeier, P. Grigul, D. Naujoks, D. Sünder, Journal of Nuclear Materials 313-316 (2003) 738

[2] D. Hildebrandt, D. Sünder, A. Herrmann, Proceedings of Inframation Conference 2003, Las Vegas (USA)

[3] Dean G. Duffy, Solutions of partial differential equations, 1986 TAB BOOkS Inc.

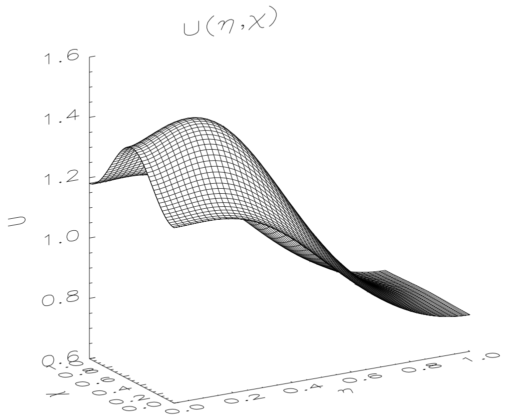

Figure 1: Temperature potential profile at the surface $\xi = 0$ at $\tau = 0.8$

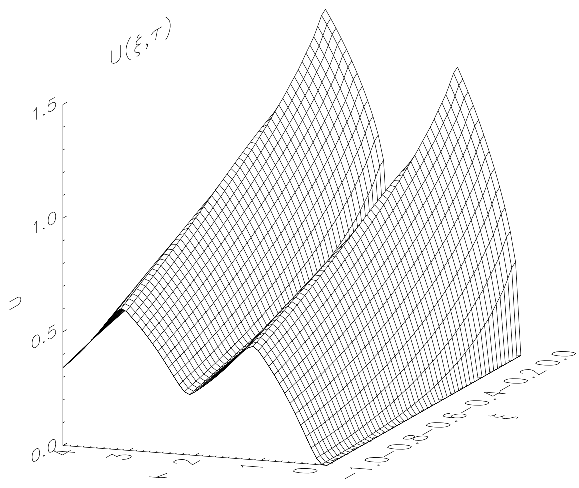

Figure 2: Heat propagation in the point $\eta = 0.25, \chi = 0.5$ into the target

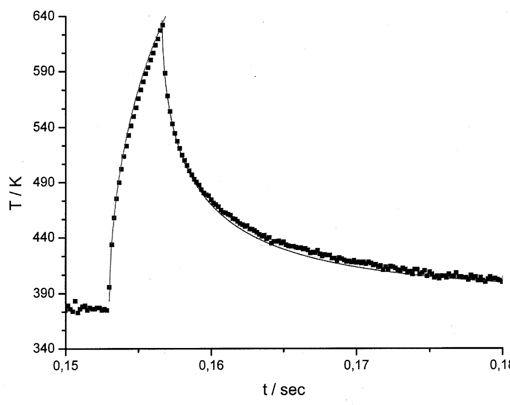

Figure 3: Comparision of the measured surface temperature excursion in the center of the laser spot with theoretical results