Monte Carlo delta-f simulations of neoclassical flows and currents in the pedestal of an H-mode tokamak plasma

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In the edge of a high-confinement mode (H-mode) plasma in present large tokamaks the width of the transport barrier region (pedestal) with large density and temperature gradients can be as small as a poloidal ion gyro radius [1]. The strong pressure gradient does not only create a large bootstrap current, but would also cause a large toroidal plasma rotation if it was not compensated to a high degree by a strong radial electric field. The surface averaged parallel velocity is given by [2,3]

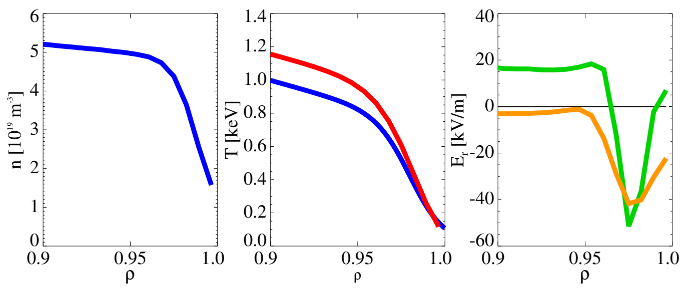
$$\langle n u_\parallel B\rangle = -\frac{RB_t}{e}\left(\frac{dp_i}{d\psi}\frac{1}{S} + k_i n\frac{dT_i}{d\psi}\frac{1}{S} + en\frac{d\Phi}{d\psi}\right) \tag{1}$$

where ψ is the poloidal flux. The factor S is unity in the standard neoclassical theory and was introduced to describe the effect of the orbit squeezing due to a strong gradient of the radial electric field on the plasma flow [3]. S is defined as $S = 1 + d^2\Phi/d\psi^2 (RB_t)^2/(\Omega B)$ $= 1 - dE_r/dr/(\Omega_p B_p)$, where B_p is the poloidal magnetic field and Ω_p is the cyclotron frequency for this field. Introducing the poloidal gyro radius, we can write $S \approx 1 + (\rho_{pi}/L_E)(|E_r|/B_p v_{Ti})$, where L_E is the gradient length of E_r. In the pedestal we have $L_E \lesssim \rho_{pi}$ and $E_r/B_p \lesssim v_{Ti}$, hence S can be considerably larger than unity.

With an inverse aspect ratio of $\epsilon \approx 0.3$ the scale length of the radial variations of density, temperature and electric field is of the order of the thermal ion banana orbit width, and a basic scaling assumption of the standard neoclassical theory is not valid. Furthermore, the strong electric field is highly localised, such that S also varys on the same radial length scale. Hence, the validity for this part of the plasma of the neoclassical theory and of more recent theories of the effect of orbit squeezing has to be examined. Therefore we studied the neoclassical physics in the plasma edge with guiding-centre particle simulations, which are well suited for capturing the effects due to deviation of the particle orbits from the flux surfaces. The delta-f code HAGIS [4] with a Monte Carlo model of Coulomb collisions [5] is employed. The distribution function is split into a local Maxwellian f_0 and the deviation from this Maxwellian, δf. The latter is represented by marker particles. The equations of motion of the particles are integrated along the particle orbits, which are the characteristic equations of the drift-kinetic equation,

$$\frac{d}{dt}f(\boldsymbol{r}, v_\parallel, \mu) = \frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f - \frac{\mu}{m}\nabla_\parallel B\frac{\partial f}{\partial v_\parallel} = C(f). \tag{2}$$

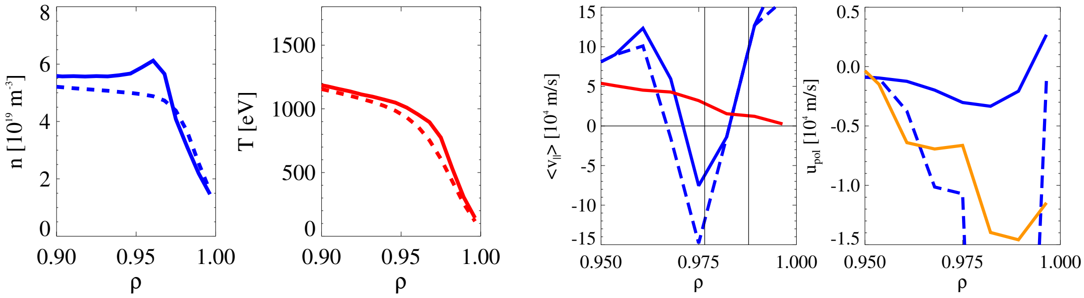
Here, $\boldsymbol{v} = v_\parallel \boldsymbol{b} + \boldsymbol{v}_d$ is the guiding-centre velocity with parallel component $v_\parallel$, $\boldsymbol{b} = \boldsymbol{B}/B$, and drift velocity $\boldsymbol{v}_d = \boldsymbol{v}_c + \boldsymbol{v}_B + \boldsymbol{v}_E = \boldsymbol{B}\times(mv_\parallel^2(\boldsymbol{b}\cdot\nabla)\boldsymbol{b} + \mu\nabla B + e\nabla\Phi)/eB^2$, $\mu = mv_\perp^2/2B$ is the magnetic moment, $\Phi(\boldsymbol{r})$ is the potential, and $C(f)$ is the collision operator. Pitch-angle scattering with the velocity dependent Coulomb collision frequency is applied and a correction term is added to the particle weights to ensure momentum conservation. For given radial profiles of the initial flux surface averages of density, temperature and electric potential, the distribution functions of ions and electrons are determined in two steps [5]. In the first step for obtaining the ion distribution function the ion-electron collisions can be neglected since the momentum loss caused by them is very small. Ion density and temperature (normally they deviate from those assumed for f_0, since this is not a solution to the drift-kinetic equation), and the parallel ion velocity are computed. In a second step including the friction between electrons and ions by electron-ion collisions the solution for the electrons is obtained and the bootstrap current is obtained.



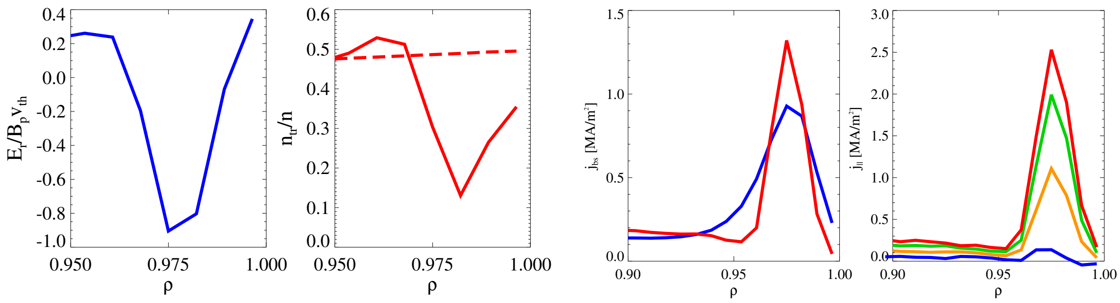
Figs. 1a–c: Profiles of density and temperature (blue: electrons) for f_0, and electric field (green) compared to $\nabla p/en$ (yellow), all versus $\rho = \sqrt{\psi/\psi_{\text{edge}}}$.

Profiles of density, temperature and electric field obtained from an ASDEX Upgrade discharge (#23227) are shown in Fig. 1 versus $\rho = \sqrt{\psi/\psi_{\text{edge}}}$. At the position of the steep gradient the electric field exceeds $\nabla p/en$. These data were taken for a simulation (density and temperature profiles for defining f_0). The profiles evolve away from the initial values (Fig. 2) since the simulation only contains neoclassical physics and since there are no sources included (duration about 10 collision times, the radial range covered is larger than shown in the figures, the temperature decreases at smaller radii). The flux surface averaged parallel ion velocity deviates from the result of neoclassical theory, Eq. (1), as shown in Fig. 3, even if the effect of orbit squeezing is accounted for (factor S in Eq. (1)). This could be related to the big size of the trapped ion orbit, which is indicated in the figure. The poloidal velocity (the guiding-centre part is obtained from the simulation and then the diamagnetic velocity is added) is considerably larger than according to the standard neoclassical theory, and closer to the predicted value with orbit squeezing taken into account [3],

$$u_p = -\frac{RB_tB_p}{enB^2}\frac{1}{S}\left[(1-S)\frac{dp_i}{d\psi} + k_i n\frac{dT_i}{d\psi}\right] \tag{3}$$



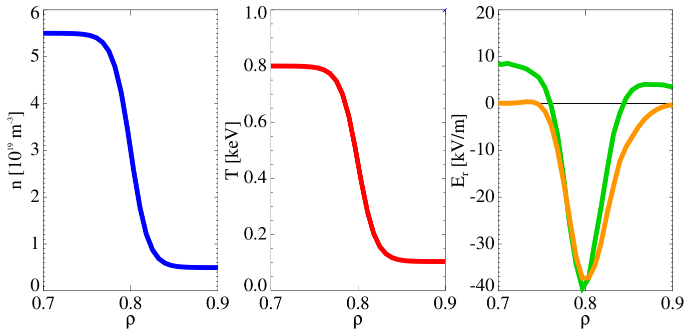
Figs. 2a,b (left): Evolution of the ion density and temperature (dashed line: initial values). **Figs. 3a,b (right)**: Parallel velocity (red) and poloidal velocity (yellow) compared with (blue) the corresponding neoclassical values, Eqs. (1,3), with (dashed) and without (solid) orbit squeezing effect. The orbit width is indicated by vertical lines.



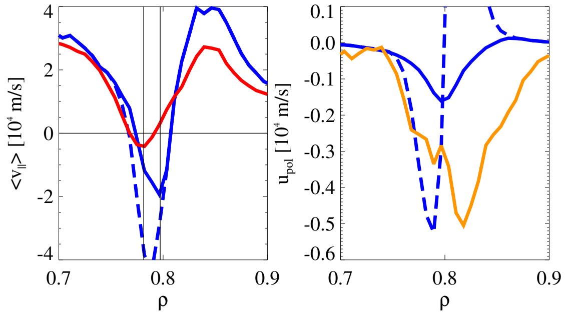
Figs. 4a,b (left): Parallel velocity of trapped ions at their (poloidal) turning point normalised to the ion thermal velocity, $E_r/B_p v_{th}$ (blue) and fraction of trapped ions (red, flux surface average) with (solid) and without (dashed) el. field. **Figs. 5a,b (right)**: Bootstrap current (red) compared with the neoclassical theory (blue), and variation of parallel current on the flux surface (in bins of width 0.2π) at four poloidal angles, 0 (red), 0.4π (green), 0.67π (yellow), π (blue).

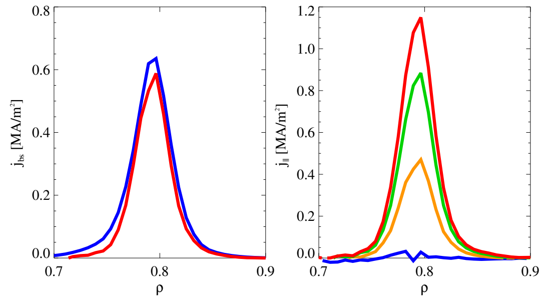
In presence of an electric field, the parallel velocity of trapped particles at their (poloidal) turning point is E_r/B_p. Here, at the position of the strong pressure gradient, this velocity is close to the ion thermal velocity (cf. Fig. 4a). Hence, the trapping region in phase space moves away from the bulk of the distribution function, and the fraction of trapped ions is strongly decreased as shown in Fig. 4b (the surface averaged fraction of trapped ions is shown; the parameter f_t of neoclassical theory, which gives the fraction of trapped particle orbits at the outer midplane, is about 0.7).

In Fig. 5a the result for the bootstrap current are shown. The current is different from the neoclassical value, but the deviation is not what is expected from the orbit squeezing effect, which is supposed to lead to a reduction of the current. Where the ion flow (Fig. 3a) is smaller (larger) than expected from Eq. (1), the bootstrap current is also smaller (larger) than in theory. However, a consistent picture is still missing, since these results were obtained with the standard version of HAGIS, where the marker particles are lost at $\rho = 1$. Due to the large pressure gradient, the poloidal variation of the parallel current (Pfirsch-Schlüter



Figs. 6a–c: Profiles of density and temperature (blue: electrons) for f_0, and electric field (green) compared to $\nabla p/en$ (yellow), all versus $\rho = \sqrt{\psi/\psi_{\text{edge}}}$.





Figs. 7a,b (left): Parallel velocity (red) compared with neoclassical theory (blue) with (dashed) and without (solid) orbit squeezing effect. The orbit width is indicated by vertical lines. Poloidal velocity (yellow) compared with neoclassical theory (blue) without (solid) and with (dashed) orbit squeezing effect. **Figs. 8a,b (right)**: Bootstrap current (red) compared with the neoclassical theory (blue), and variation of the parallel current on the flux surface (same colors as in Fig. 7b).

current) is large: the current ranges from close to zero at the inner midplane to twice the average at the outer midplane.

For comparison, we also made simulations with steep gradients further inwards in the plasma, such that orbit losses do not play a role. The results of such a simulation with (deliberately assumed) similar profiles, gradients and electric field are shown in figures 6-8. In this case there are no orbit losses. The plasma flow is closer to the neoclassical value than in the first case, but there is again a deviation and the expected effect of orbit squeezing is not present. However, this effect would be very localised as shown by the green line in Fig 7a. The parallel velocity has a local minimum similar to what is seen in the experiment. The poloidal velocity is again increased, the peak is shifted towards a larger radius compared to the neoclassical theory with orbit squeezing. The bootstrap current is close to the neoclassical result, in contrast to the result for the edge barrier case. This hints on a possible effect of the (too high) orbit losses on the result presented above.

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