# Two-dimensional and three-dimensional collisionless reconnection.

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## Introduction and model

Magnetic reconnection occurs when the equilibrium magnetic field lines break due to non ideal effects and reconnect with different magnetic topology [1]. Here, we consider the regime of high-temperature plasmas in the presence of a strong guide field, where reconnection is made possible by electron inertia and has higher growth rates than resistive reconnection [2, 3].

The evolution of a magnetic island is studied in a two dimensional periodic and three dimensional Harris-pinch equilibrium, in the presence of turbulence. The magnetic island grows unstable due to a current gradient in the background collisionless plasma. We adopt a gyrofluid model, obtained by taking the zeroth and first moments of the gyrokinetic equations [4, 5]. The gyrofluid model equations for the ions are:

$$\left(\frac{\partial}{\partial t}+\mathbf{u}_E\cdot\nabla\right)\tilde{n}_i = -B\nabla_{\|}\frac{\tilde{u}_{\|}}{B} \tag{1}$$

$$\hat{\beta}\frac{\partial \tilde{A}_{\|}}{\partial t}+\hat{\varepsilon}\left(\frac{\partial}{\partial t}+\mathbf{u}_E\cdot\nabla\right)\tilde{u}_{\|} = -\nabla_{\|}(\tilde{\phi}_G+\tau_i\tilde{n}_i) \tag{2}$$

and for the electrons:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_E\cdot\nabla\right)\tilde{n}_e = -B\nabla_{\|}\frac{\tilde{v}_{\|}}{B} \tag{3}$$

$$\hat{\beta}\frac{\partial \tilde{A}_{\|}}{\partial t}-\hat{\mu}\left(\frac{\partial}{\partial t}+\mathbf{v}_E\cdot\nabla\right)\tilde{v}_{\|} = -\nabla_{\|}(\tilde{\phi}-\tilde{n}_e) \tag{4}$$

where the tilde symbol denotes perturbed quantities, $\tau_i = T_i/T_e$, $n$, $u$ and $v$ are the fluid density and ion and electron velocities, and $\phi$ and $A$ are the scalar and vector potentials. The parameters $\hat{\beta}$, $\hat{\varepsilon}$ and $\hat{\mu}$ are defined by: $\hat{\beta} = \beta_e\hat{\varepsilon}$, $\hat{\varepsilon} = (qR/L)^2$ and $\hat{\mu} = (m_e/m_i)\hat{\varepsilon}$, where $\beta_e = 4\pi p_e/B^2$, and $qR$ and $L$ are the scale length along and perpendicular to the equilibrium magnetic field. The parallel gradient is calculated in the direction parallel to the total (equilibrium plus perturbed) magnetic field, and the gyro-averaged scalar potential is defined as $\tilde{\phi}_G = \Gamma_0^{1/2}(\tilde{\phi})$. The advection velocities $v_E$ and $u_E$ are the $E\times B$ drifts. Moreover, the polarization and induction equations are:

$$\Gamma_0^{1/2}\tilde{n}_i+\frac{\Gamma_0-1}{\tau_i}\tilde{\phi} = \tilde{n}_e\ ,\ \ -\nabla_\perp^2\tilde{A}_{\|} = \tilde{J}_{\|} = \tilde{u}_{\|}-\tilde{v}_{\|}$$

We also use the Padé approximant forms [4]: $\Gamma_0 = (1-\rho_i^2\nabla_\perp^2)^{-1}$, $\Gamma_0^{1/2} = (1-\rho_i^2\nabla_\perp^2/2)^{-1}$, where $\rho_i$ is the ion gyro-radius. Note that all quantities with symbol ˆ are normalized to $\hat{\varepsilon}$.

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**Two-dimensional configuration**

We start by studying a magnetic island dynamics in a two-dimensional periodic equilibrium, with the code *REC2* [6]. The equilibrium density and temperature are homogeneous, and isothermal dynamics is assumed. Here the physical time scales are the Alfvén velocity $v_A = B/\sqrt{n_i m_i}$ and sound speed $c_s = \sqrt{T_e/m_i}$, and the space scales are the ion-sound radius $\rho_s = c_s M_i c/(eB)$, the collisionless skin-depth $d_e = c/\omega_{pe}$, with $\omega_{pe}$ the plasma frequency, and the perpendicular equilibrium length $L$, that is the size of the simulation box in the x direction. Firstly, we study the linear growth rate $\gamma_L$ dependence on the ion gyro radius (see Fig. 1). We find that our results are in agreement with the analytical prediction [3], stating $\gamma_L \propto \rho_\tau^{2/3}$, where $\rho_\tau = \sqrt{\rho_i^2 + \rho_s^2}$, in the asymptotic limit of large $\rho_\tau$ given by $d_e\Delta' \gg (d_e/\rho_\tau)^{1/3}$. Here $d_e\Delta' = 2.03$. Secondly, we study the magnetic island nonlinear evolution for the case: $\beta_e = 0.005$, $L = 4\rho_s$, $\rho_i = \rho_s = 3d_e$ (see Fig. 2). We see that for such big values of $\rho_i$, the scalar potential reflects electron density structure.

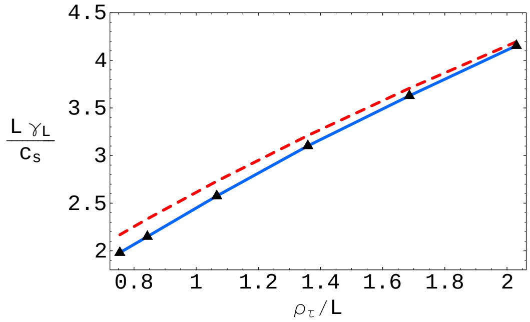


Figure 1: Linear growth rate vs $\rho_\tau$. Numerical results of REC2 are plotted as a blue line, and the scaling predicted by analytical theory as a red line (for $\rho_s = 3d_e$).

The energy conservation is a crucial issue in reconnection simulations. In fact, reconnection can be driven by non-physical effects such as numerical dissipation. To validate our results, we perform a total energy conservation check, where the total energy is given by $E_{tot} :=$

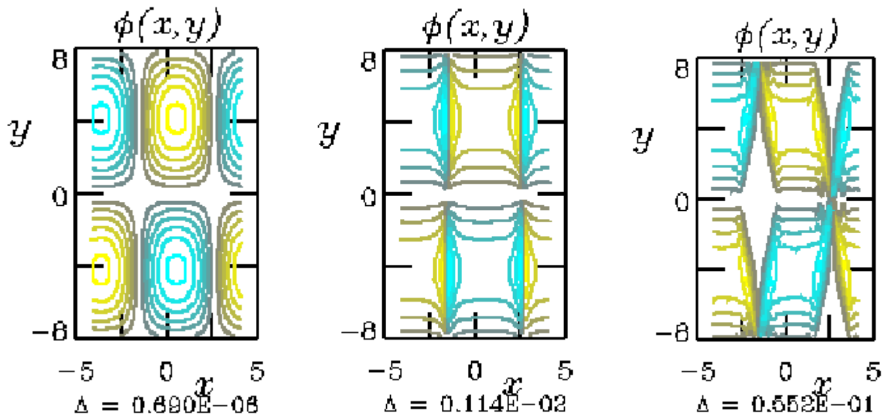


Figure 2: Scalar potential in the x-y plane (2D case). The plots are taken at times t=0.1 (left), where the initial perturbation is visible; t=2.8 (center), and t=4.3 (right), corresponding to the linear and nonlinear phases. Times are normalized to $L/c_s$, lengths to $\rho_s$.

$\int dV \frac{1}{2}[\delta|\nabla_\perp \phi|^2 + p^2 + \beta_e \delta|\nabla_\perp A_\parallel|^2 + \mu_e J^2 + u^2]$ (here $\delta = \rho_s/L$ and $\mu_e = m_e/m_i$). We find that the measured growth rate is $\gamma_{tot} := (dE_{tot}/dt)/(2E_{tot}) = 0.2 \cdot 10^{-9}$, to be compared with the magnetic energy growth rate which is of the order of one.

The role of Casimir invariants, defined as $G_\pm = \beta_e A_\parallel + \mu_e J_\parallel \pm \mu_e^{1/2}\Omega$, is also investigated. Here $\Omega = (n_i m_i c^2/B^2)\nabla_\perp^2 \phi$ is the vorticity. We consider the limit of vanishing ion Larmor radius, and our results confirm that these quantities are conserved by the Hamiltonian in the Lagrangian sense, consistently with published results [7].

**Three-dimensional configuration**

Here we study a magnetic island dynamics, in a three-dimensional Harris-pinch equilibrium, with the code *GEM* [8]. This includes also temperature fluctuations. The nonlinearity and the three-dimensionality of the model allows turbulence to affect the growth of the magnetic island. Typical values of the plasma parameters are chosen: the squared aspect ratio is $\hat{\varepsilon} = (qR/L)^2 = 18350$, and the normalized beta is $\beta_{norm} = \beta_e \hat{\varepsilon} = 1$.

A random bath is initialized at $t = 0$ with a magnetic island seed of length $\lambda_y = L_y$, with $L_y$ the size of the simulation box in the $y$-direction. For our plasma parameters, this corresponds to $k_y\rho_s = 0.025$. During the whole simulation, the magnetic island growth and nonlinear saturation is studied. The relevant result, is that the turbulence is observed to qualitatively modify the magnetic island dynamics. Three different phases can be recognized during the magnetic island growth:

1) t=0 - t=200: the initialized magnetic island ($k_y\rho_s = 0.025$) grows and is the dominant mode in

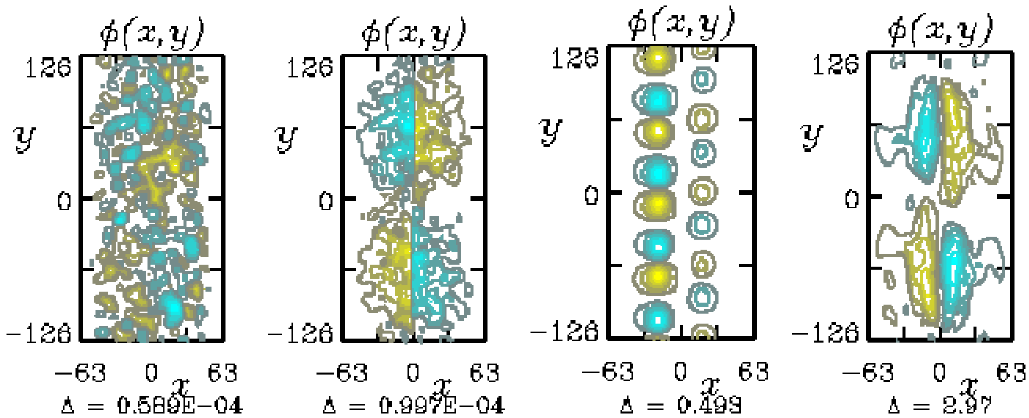


Figure 3: Scalar potential amplitude in the x-y plane, for the 3D case. The four plots correspond to the times t=10, where the random bath is visible; t=100, where the main harmonic is dominant reflecting the initial condition; t=360, where the fourth harmonic is dominant; t=480, where the main harmonic is dominant again and starts saturating.

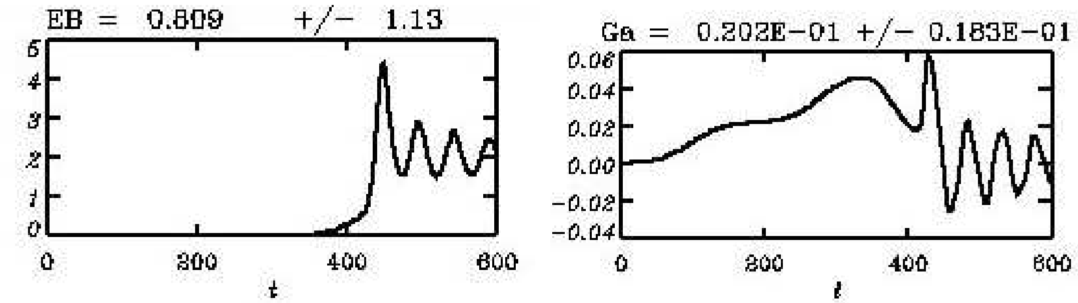


Figure 4: Total magnetic energy and correspondent growth rate for the 3D case. The total magnetic energy grows up to $t = 450$, then the nonlinear saturation occurs, where the mode corresponding to the main magnetic island ($k_y \rho_s = 0.025$) becomes the dominant harmonic.

the amplitude spectrum. 2) t=200 - t=400: the energy of the initialized magnetic grows slower than higher harmonics, where the dominant is the fourth harmonic, with $k_y \rho_s = 0.1$. 3) t $>$ 400: the main harmonic becomes the dominant one again, and saturates due to nonlinear dynamics.

Indications of nonlinear growth rate acceleration, due to the 3D interactions, are also found. A resolution check has been performed to validate our results, by running the same simulations with different grid sizes in the direction along the magnetic field. Detailed energetic analysis of nonlinear process is in progress.

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