Continuous spectrum of shear Alfvén waves inside magnetic islands.

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Introduction

Shear Alfvén waves (SAW) are electromagnetic plasma waves propagating with the characteristic Alfvén velocity $v_A = B/\sqrt{4\pi\rho}$ (B is the magnetic field and ρ the mass density of the plasma). Shear Alfvén fluctuations can be resonantly excited by energetic particles and are, therefore, particularly dangerous for a burning plasma [1, 2]. One of the main damping mechanisms of shear-Alfvén modes in nonuniform plasmas is continuum damping [3]. Meanwhile, due to nonuniformities along the magnetic field lines in toroidal geometry, gaps appear in the SAW continuous spectrum [4]. Discrete Alfvén Eigenmodes (AE), having the frequency inside continuum gaps, are practically unaffected by continuum damping [1, 2]. For this reason, the importance of understanding the continuous spectrum structure is clear when addressing the stability problem of a tokamak and its potential impact on reaching the ignition condition.

We analyze the problem of the nonlinear modification of the SAW continuous spectrum due to the presence of a magnetic island [5]. We find that inside the islands there is a continuous spectrum very similar to that calculated for the whole tokamak plasma [6]. In particular, we find a very large ellipticity induced AE [7] (EAE) gap due to the strong eccentricity of the island's cross section, dubbed here as magnetic island induced AE (MiAE) gap. Moreover, we find that the continuous spectrum of modes with the same helicity of the magnetic island have SAW continuum accumulation points at the O-point of the island (MiO-CAP) and that the beta induced AE [8, 9, 10] (BAE) CAP frequency is modified due to the presence of the island.

Equilibrium and model equations

We derive model equations for the SAW continuum structure in the presence of a finite size magnetic island in finite-β plasmas (with β the ratio of plasma and magnetic pressures), keep-

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ing into account only toroidicity effects due to geodesic curvature, which are responsible for the BAE gap in the low frequency part of the SAW continuous spectrum [8, 9, 10]. A fluid description based on linear ideal MHD is adopted. The magnetic island is treated as stationary, due to the separation ot the time scales of SAW and magnetic island dynamics, and modelled as a straight flux tube with length $Z_0 = \gamma q_0 R_0$ (q_0 is the safety factor at the island position and R_0 the major radius) and with a non-circular cross section. Here $\gamma = \sqrt{1+\varepsilon_0^2/q_0^2}$ and $\varepsilon_0 = r_0/R_0$ is the inverse aspect ratio. The local differential equation describing the nonlinear SAW continuum structure [8] is:

$$\frac{\omega^2}{\omega_A^2}\nabla_\perp^2\phi + Z_0^2\nabla_\parallel\nabla_\perp^2\nabla_\parallel\phi - \frac{\omega^2_{BAE-CAP}}{\omega_A^2}\nabla_\perp^2\phi = 0 \tag{1}$$

where $\omega_A = v_A/Z_0$. The BAE-CAP frequency $\omega_{BAE-CAP}$, defined in [8, 9, 10], can be formally treated just as constant (in space) upward shift in the continuous spectrum, by introducing the normalized frequency $\Omega^2 = (\omega^2 - \omega^2_{BAE-CAP})/\omega_A^2$.

Results for modes with helicity different from that of the magnetic island

Equation 1 is solved numerically with a shooting code, in the whole spatial range of interest as well as analytically, near the O-point and at the island separatrix. Along the translational symmetry direction ζ, the modes are not coupled and the corresponding scalar potential can be assumed as $\phi = \hat{\phi}(x,\theta)\exp(-in\zeta)$. The result for the general case of modes with helicity different from that of the magnetic island, is shown in Fig. 1. In the case $e \ll 1$, we consider continuum modes with $n = 10$. Here $e = 1 - Mn_{isl}^2\gamma^2/s^2$ is the eccentricity of the flux surfaces

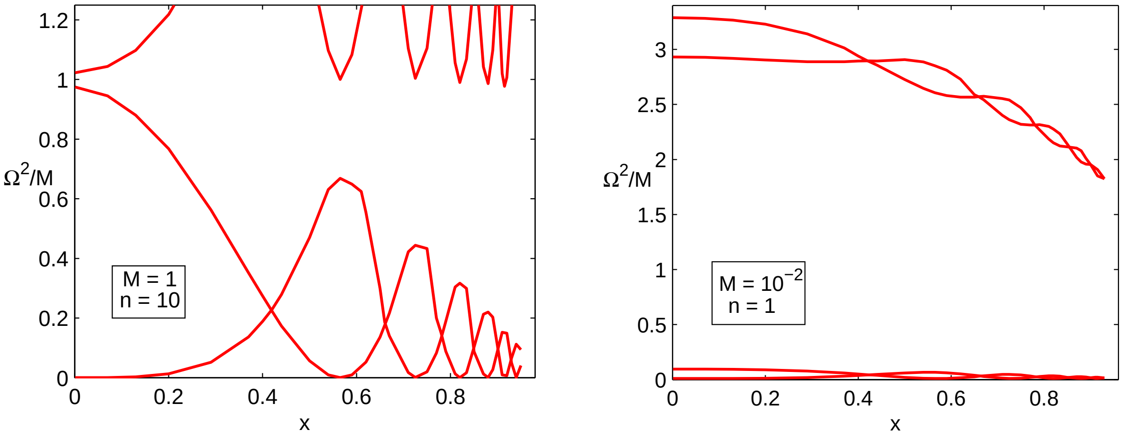

Figure 1: Continuous spectrum for modes with $n \neq 0$, for the small eccentricity case, $e \ll 1$, corresponding to $M \simeq 1$, and for a typical size magnetic island, $e \simeq 0.99$, corresponding to $M = 10^{-2}$. Typical values of the parameters have been chosen and $n_{isl} = 1$. The O-point is at $x = 0$ and the separatrix at $x = 1$. The MiAE gap is found at frequencies $\Omega^2 \simeq Mn_{isl}^2$.

near the O-point of the magnetic island, with $M = (q_0|s|/n_{isl})(B_{isl}/B_{pol,0})$, s is the magnetic shear, and the magnetic island amplitude B_{isl} and the poloidal field $B_{pol,0}$ are calculated at rational surface $q = q_0$ (island location). We choose typical values for the equilibrium parameters, $q_0 = 2$, $s = 1$, $\varepsilon_0 = 0.1$, $n_{isl} = 1$. In this case with $M \simeq 1$ ($e \ll 1$), analytical solution shows that near the O-point, $x \simeq 0$, a gap is formed at $\Omega^2 \simeq M n_{isl}^2$ due to small but finite ellipticity, involving essentially nearest neighbor poloidal harmonics (m=9,11, with m the poloidal mode number inside the island). On the other hand, for $x \sim 1$, non-circular cross section of flux surfaces becomes more pronounced and, therefore, many modes with different m-numbers are coupled. This mechanism creates a gap in the spectrum analogous to the ellipticity induced Alfvén Eigenmode (EAE) gap in tokamaks [7], dubbed here as magnetic island induced AE (MiAE) gap [6]. The case with $e \ll 1$ ($M \simeq 1$) serves only as example, for it corresponds to a "large" magnetic island. We repeated the calculation for a magnetic island with a typical size, $M = 10^{-2}$, using the same equilibrium parameters. This corresponds to a magnetic island half-width $W_{isl} \simeq \sqrt{1-e}\; r_0 = 0.1\, r_0$, which is the order of magnitude of a saturated magnetic island in tokamaks. For this case, we considered modes with $n = 1$: the continuous spectrum structure is the same as for $M = 1$, but we obtain a much wider MiAE gap. This is due to the fact that flux surfaces of a typical magnetic island have a high eccentricity, which strongly couples modes with different m-numbers. The MiAE gap central frequency is proportional to the magnetic island half-width:

$$\Omega_{MiAE} = \sqrt{M} n_{isl} = \frac{q_0\, s\, n_{isl}}{2} \frac{W_{isl}}{r_0} \qquad (2)$$

Results for modes with the same helicity of the magnetic island

When modes with the same helicity of the magnetic island are considered ($n = 0$) [11], the problem reduces to a two dimensional problem. In this case, depicted in Fig. 2, three main features describe the continuous spectrum. 1) The continuous spectrum branches Ω_j^2 have continuum accumulation points at the O-point, named here MiO-CAPs. 2) Even and odd eigenfunction have frequencies which are substantially different inside the island, due to the non-uniformity of the magnetic field intensity along the field line. 3) At the separatrix, the continuum frequencies converge to two different BAE-CAP. The first inner odd eigenfunction has frequencies converging to the usual linear

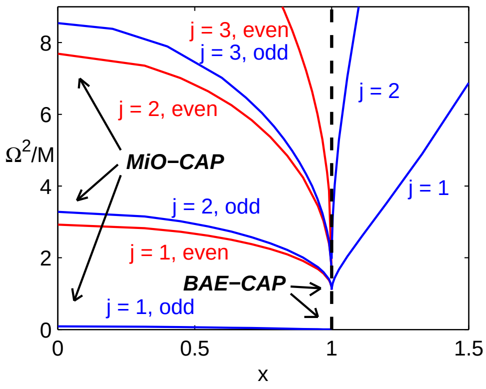

Figure 2: Continuous spectrum for $n = 0$ and $M = 10^{-2}$. The MiO-CAPs are shown at the O-point ($x = 0$), and the BAE-CAP at the separatrix ($x = 1$).

BAE-CAP [8, 9, 10], whose value is the same as in an equilibrium without islands, correspondent to $\Omega^2 = 0$. All other branches converge to a nonlinearly modified BAE-CAP, whose value is $\Omega_{nlBAE-CAP} = \Omega_{MiAE} = \sqrt{M} n_{isl}$.

Conclusions

We have found that there exists a SAW continuous spectrum within a magnetic island, qualitatively similar to that calculated in tokamak equilibria, and characterized by a magnetic island induced AE (MiAE) gap. Note that MiAE can exist as bound states within the island, essentially free of continuum damping, provided that plasma equilibrium effects and free energy sources can drive and bind them locally. We have also shown that the BAE-CAP frequency is nonlinearly modified by the presence of the magnetic island. Consequently, the BAE frequency is expected to be modified by the presence of a magnetic island as well, consistently with the BAE-CAP frequency nonlinear modification [12, 13, 14]. This suggests the possibility of using the MiAE and BAE frequency scalings as novel magnetic island diagnostics.

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