ISSN 1063-780X, Plasma Physics Reports, 2009, Vol. 35, No. 7, pp. 529–534. © Pleiades Publishing, Ltd., 2009.
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Pressure Insensitivity of a High-β Quasi-Isodynamic Stellarator[1]

M. I. Mikhailov[a], V. D. Shafranov[a], and J. Nührenberg[b]

[a] *Russian Research Centre Kurchatov Institute, Moscow, Russia*
[b] *Max-Planc-Institut für Plasmaphysik, EURATOM Association, D-17491 Greifswald, Germany*

Received November 28, 2008

Abstract—The sensitivity of a high-β quasi-isodynamic stellarator equilibrium with respect to changes of the plasma pressure is investigated. It is shown that a plasma boundary near to the high-β boundary exists with the property that the magnetohydrodynamic and drift-kinetic properties of the associated low-β plasma are similar.

PACS numbers: 52.55.s, 52.55.Hc, 52.55.Dy, 52.65.Cc

DOI: 10.1134/S1063780X09070010

Integrated physics optimization of a quasi-isodynamic (qi) stellarator with poloidally closed contours of the magnetic field strength has led to a promising high-β ($\langle\beta\rangle \sim 0.1$) stellarator configuration, [1]. In this context it is worthwhile to investigate whether the high-β plasma state can be reached from a low-β plasma state without major configurational changes. The demonstration of this useful property is the purpose of this brief communication.

To this end, the following procedure is adopted. The high-β configuration is completely specified by its boundary shape and its pressure profile; there is no net toroidal current on its magnetic surfaces since the bootstrap current has been optimized to be negligible (for the relation between quasi-isodynamicity and bootstrap current see, e.g., the Appendix). To find a corresponding low-β equilibrium, a small β has been selected ($\langle\beta\rangle \sim 0.025$) and a re-optimization has been undertaken to establish similar plasma properties as in the high-β case. The result is that such properties are obtained at a boundary shape close to that of the high-β case which proves the assertion.

The main objective of the re-optimization has been to establish a low-β equilibrium which avoids low-order rational values of rotational transform (i.e., in the situation considered here, $\iota = 6/6$ and $\iota = 6/7$) and, simultaneously, avoidance of these values for the same boundary considered as a vacuum-field flux surface.

In the following, the result of this re-optimization is described in some detail. Figure 1 shows a comparison of the two plasma boundaries. The main differences are slightly stronger indentation and slightly less triangularity of the high-β case. The rotational transform profiles are seen in Fig. 2 and show that plasma boundaries quite close to each other keep the rotational transform in the interval [6/7, 6/6] in the whole range of β values considered, $0 < \langle\beta\rangle < 0.09$. The physical situation underlying this property can be seen in Fig. 3. Although the aspect ratio of the configuration is ≈12, the toroidal effect seen in the strength of B, namely $B_{1,0}/B_{0,0}$, and in the Jacobian $\sqrt{g}$, namely $(\sqrt{g})_{1,0}/(\sqrt{g})_{0,0}$, expressed in magnetic coordinates corresponds to $A > 30$ and $A > 40$, respectively.

So, it remains to be shown that the MHD-stability and the neoclassical properties of the low-β case are favorable, too. As to the MHD stability, local-ballooning stability holds at half the normalized toroidal flux, which has to be compared to the slightly unstable situation in the high-β case (see Fig. 11 of [1]). As for the neoclassical properties, the structural characteristics of the magnetic field strength of the high-β case are well preserved. The contours of B on magnetic surfaces are poloidally closed (see Fig. 4). The surfaces of constant B = const (see Fig. 5) do no longer enclose a true local minimum of B (see Fig. 4 of [1]) but except for a small fraction of very deeply reflected particles, most reflected particles see concave surfaces of B = const. Accordingly, quasi-isodynamicity, as judged from the contours of the second adiabatic invariant (see Fig. 6), prevails in the plasma core. This situation is reflected in the collisionless α-particle confinement which is compared in Fig. 7. The loss occurring in the low-β case, although significantly larger than in the high-β case, is still less than 4% in 1 s flight time. Only a small fraction of it occurs as prompt loss and the contours of the second adiabatic invariant (see Fig. 6) show that this loss consists of the very deeply trapped particles not considered in Fig. 6, because these particles are not confined in a true local minimum of B as in the high-β case. From the point of view of energetic particles impinging on material surfaces this prompt loss is of course irrelevant, because the plasma does not have to be in fusion conditions in this type of configuration at such a low β value. In keeping with this still remark-

[1] The article was translated by the authors.

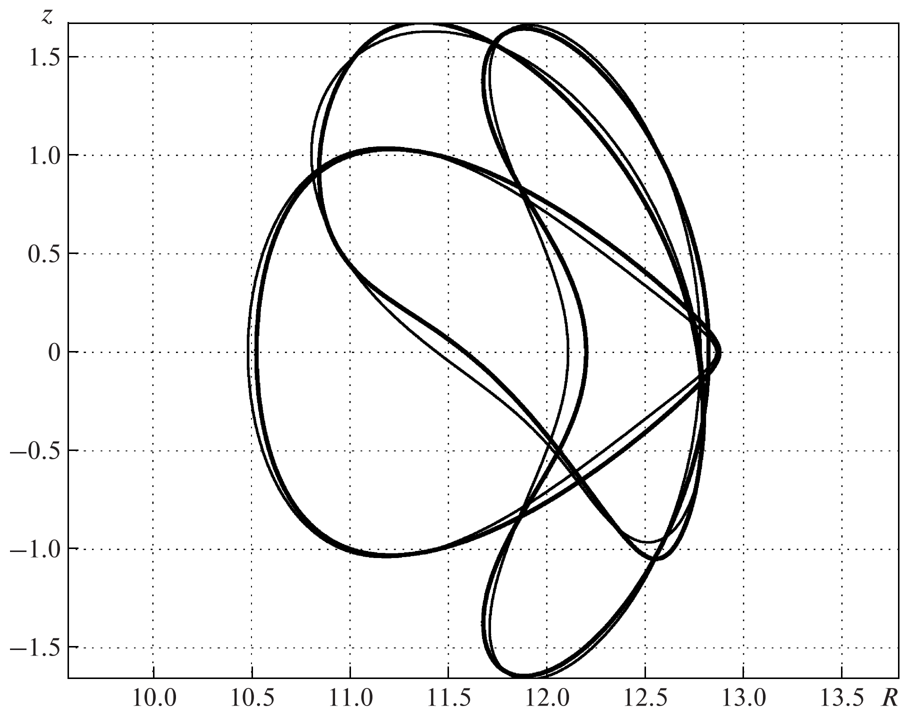

Fig. 1. Cross-sections of flux surfaces in the two symmetry planes half a period apart and in between these two at a quarter of a period. Heavy lines show boundaries of the high-β case, thin lines those of the low-β case.

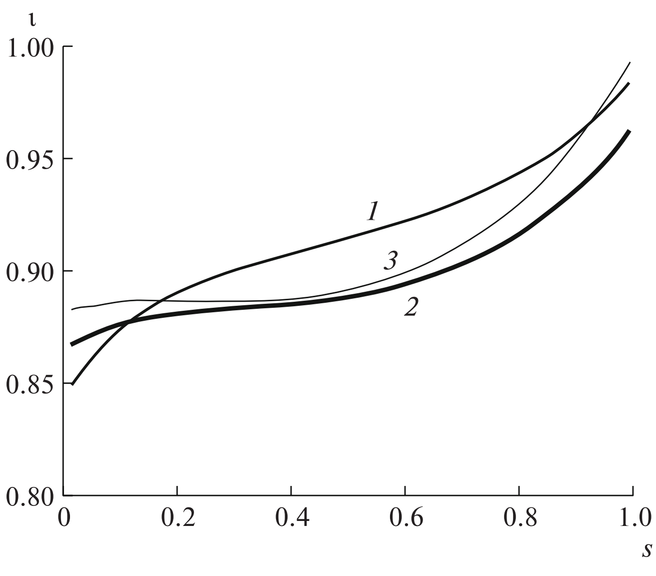

Fig. 2. Rotational transform profiles vs. toroidal-flux label for the high-β (curve *1*), low-β (curve *2*) cases, additionally the low-β boundary evaluated at zero β (curve *3*).

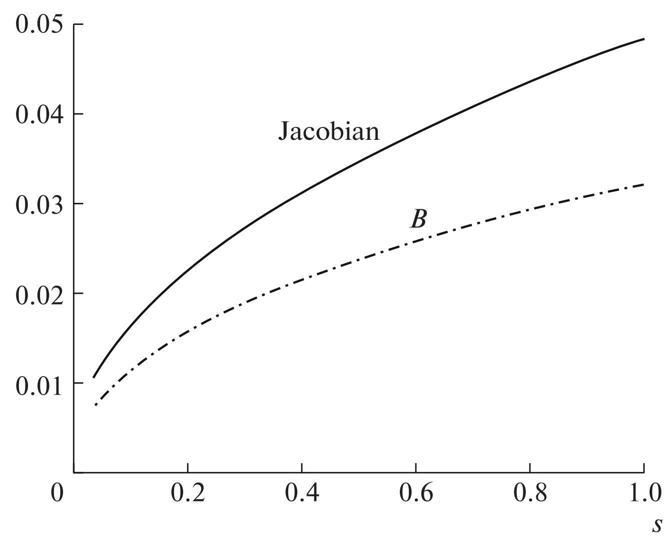

Fig. 3. Normalized Fourier coefficients in magnetic coordinates corresponding to toroidal curvature: for $B_{1,0}$ of B, $-B_{1,0}/B_{0,0}$, and for $(\sqrt{g})_{1,0}$ of $\sqrt{g}$, $(\sqrt{g})_{1,0}/(\sqrt{g})_{0,0}$.

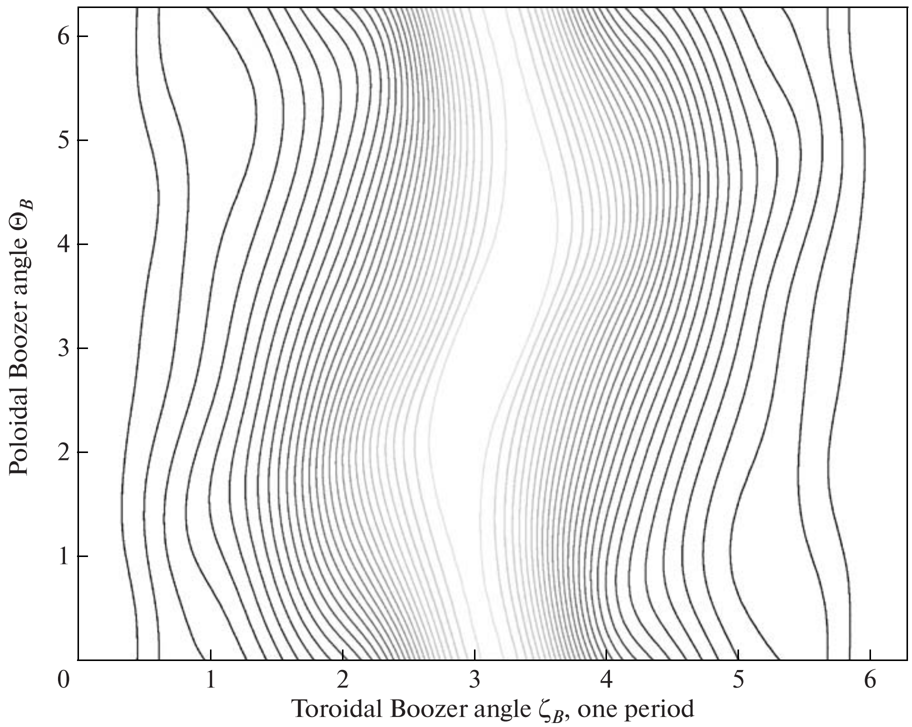

Fig. 4. Contours of B on the magnetic surface at half the plasma radius.

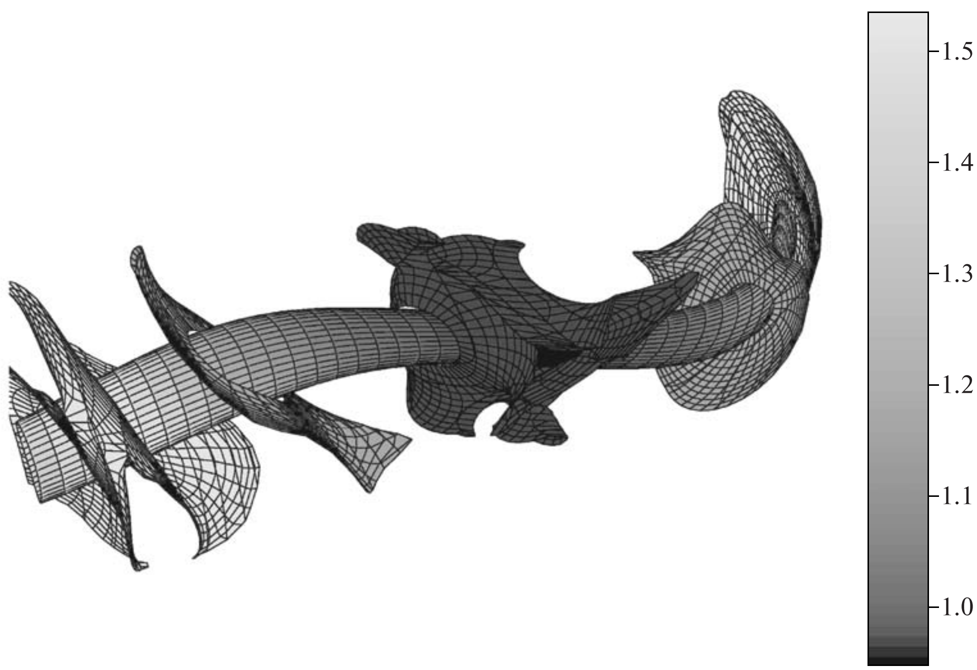

Fig. 5. Surfaces of constant magnetic field strength and the plasma column at a third of the average plasma radius.

able collisionless confinement behavior, the neoclassical transport as characterized by the equivalent ripple is similar in the outer part of the confinement volume and even smaller in the plasma core in the low-β case, see Fig. 8. Finally, Fig. 9 shows that the property of very small bootstrap current is preserved, too.

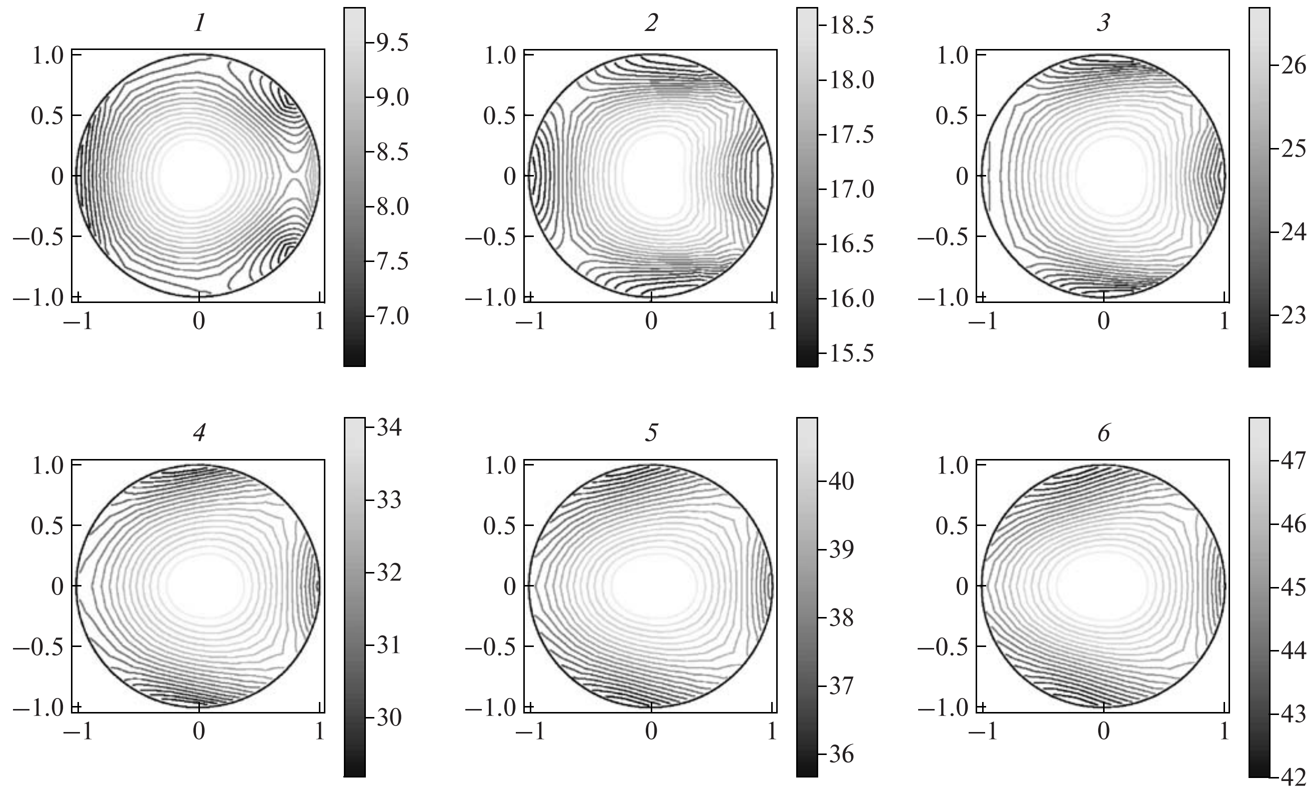

Fig. 6. Contours of J in the low-β configuration obtained for different values of B, B_{ref}, at which trapped particles are reflected; *1*—near the minimum of B, *6*—near the maximum of B.

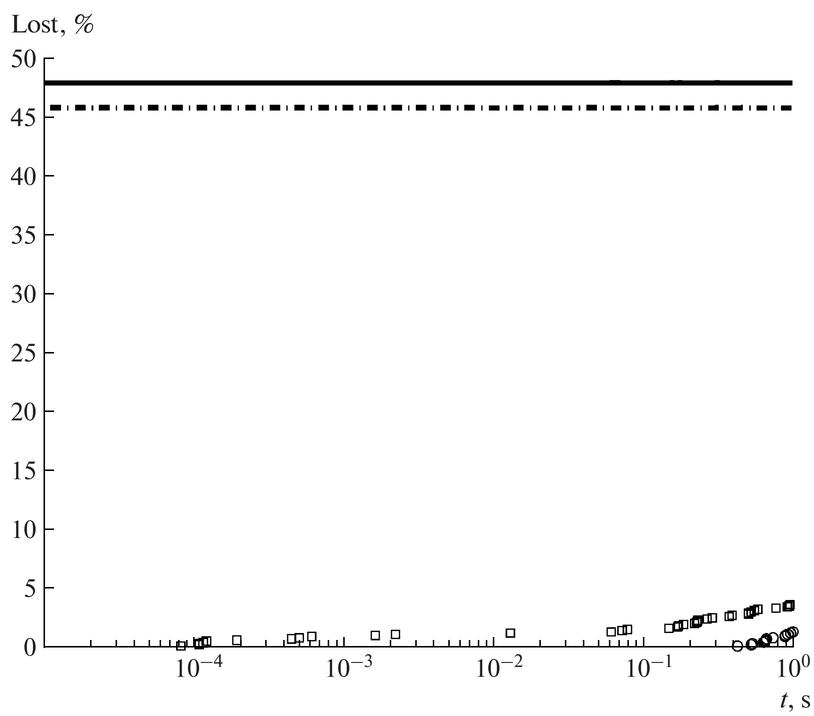

Fig. 7. Loss histories of 1000 α-particles started at half of the plasma radius and randomly distributed in the angular variables and the pitch angle. Normalization: plasma volume 10^3 m^3, magnetic field 5 T. Each symbol marks the loss of one particle. The straight lines indicate the fractions of reflected particles. Circles and broken line refer to the high-β, squares and thick line to the low-β case.

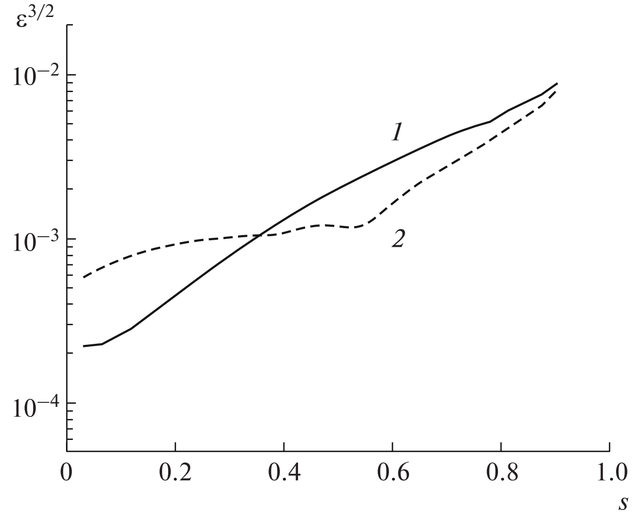

Fig. 8. Profiles of the Equivalent ripple (the 3/2 power of it) for the high-β and low-β cases (curves *2* and *1*, respectively).

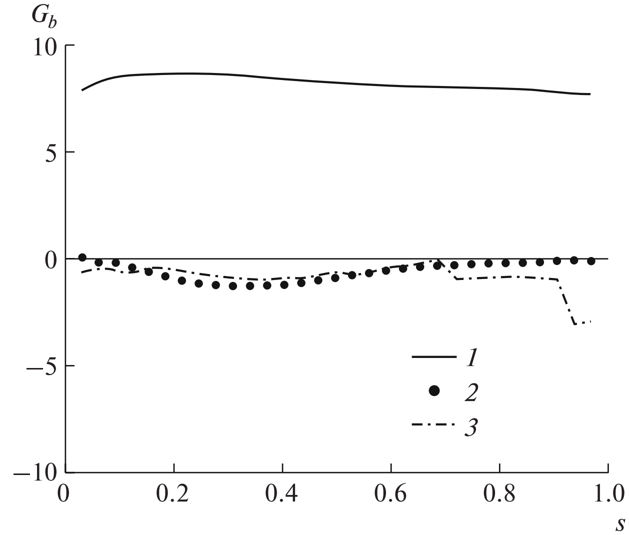

Fig. 9. Comparison of structural factors of the bootstrap current for Wendelstein 7-X (see, e.g., [2]) (*1*), in which the bs current is already strongly reduced as compared with quasi-symmetric configurations (see, e.g., [1]), and the two configurations discussed here, low-β case (*2*) and high-β case (*3*).

ACKNOWLEDGMENTS

This work was supported in part by the Russian Foundation for Basic Research, project no 09-02-01420-a.

APPENDIX

Here, a more complete proof of the relation between quasi-isodynamicity and bootstrap current is given than in [1]. For convenience, the first part of the appendix in [1] is repeated. Consider a net current free MHD equilibrium with poloidally closed contours of the field strength B and a single minimum and a single maximum of B per period. It seems that such an equilibrium is most intuitively described by the non-periodic invariant coordinate system F_t, θ_0, B with F_t the toroidal flux labeling the magnetic surfaces and θ_0 labeling the fieldlines. Then, $\mathbf{B} = \nabla F_t \times \nabla\theta_0 = \frac{1}{\sqrt{g}}\mathbf{r}_{,B}$, so that $B\sqrt{g} = \frac{\partial l}{\partial B}$ with l the length along the field line and $\mathbf{B}\cdot\nabla B = \frac{1}{\sqrt{g}} = B/\frac{\partial l}{\partial B}$. The increment of θ_0 in one period comparing two equivalent space points is $\Delta\theta_0 = \iota_{\text{period}}$.

With J the second adiabatic invariant and the assumption $J = J(F_t)$ for all reflection values $B_{\text{ref}} = B^+_{\text{ref}} = B^-_{\text{ref}}$ one sees from

$$J \propto \int_0^{l^+}\sqrt{B^+_{\text{ref}} - B}dl + \int_0^{l^-}\sqrt{B^-_{\text{ref}} - B}dl$$

$$= \int_{B_{\min}}^{B^+_{\text{ref}}}\sqrt{B^+_{\text{ref}} - B}\partial_B l^+ dB + \int_{B_{\min}}^{B^-_{\text{ref}}}\sqrt{B^-_{\text{ref}} - B}\partial_B l^- dB$$

(with l^-, l^+ the lengths from $B_{\min}$ to B^-_{ref} and B^+_{ref}) by computing $\partial_{\theta_0}J$

$$\partial_{\theta_0}(dl^- + dl^+) = 0 \qquad (*)$$

equivalent to the omnigenity condition found in [3]. Hence, in particular, $l^+(B, \theta_0) + l^-(B, \theta_0) = L(B)$ with $L(B)$ the length between equal values of B.

Also,

$$\partial_{\theta_0}\left(\int_0^{l^+_{\max}} Bdl + \int_0^{l^-_{m\max}} Bdl\right) = 0$$

with $l^+_{\max}$, $l^-_{\max}$ the lengths from $B_{\min}$ to $B^+_{\max}$ and $B^-_{\max}$.

In addition, because of $J_t = 0$, on each rational fieldline (with, e.g., n toroidal turns before closure on itself);

$$nN_p\int_{B^-_{\max}}^{B^+_{\max}} Bdl = nN_pJ_p$$

with N_p the number of periods and J_p the poloidal current per period, so that $\int_{B_{max}^-}^{B_{max}^+} Bdl = J_p$. On the other hand, for $J_t = 0$, the orthogonals to the fieldlines in the magnetic surfaces are closed and $J_p = \int \mathbf{B} \cdot \mathbf{dl}$ between any two points on the same magnetic surface on two orthogonals one period apart. Starting the integration from an intersection $\theta_{0,1}$ of an orthogonal to $\mathbf{B}$ with a B_{max} line one sees that, after one period, the corresponding orthogonal must intersect the corresponding B_{max} line at $\theta_{0,1} + \iota_{period}$, too. Repeating this process shows a B_{max} line to be an orthogonal to the fieldlines.

In the evaluations of $j_{\|}$ from MHD equilibrium and of the bootstrap current from driftkinetics, functions X occur which obey magnetic differential equations of the type

$$\mathbf{B} \cdot \nabla X = (\mathbf{B} \times \nabla F_t) \cdot \nabla H(B).$$

This equation takes the form

$$\frac{\partial X}{\partial B} = -\dot{H}(B)\mathbf{B} \cdot \frac{\partial \mathbf{r}}{\partial \theta_0}.$$

Because of the vanishing longitudinal current and the poloidally closed contours of B

$$0 = \oint \mathbf{B} \cdot \mathbf{dl} = \int \mathbf{B} \cdot \frac{\partial \mathbf{r}}{\partial \theta_0} d\theta_0,$$

so that

$$\mathbf{B} \cdot \frac{\partial \mathbf{r}}{\partial \theta_0} = \frac{\partial \tilde{g}}{\partial \theta_0}.$$

Since the current density lies in the magnetic surfaces the loop integral connecting neighboring fieldlines (separated by $d\theta_0$) via the contours of B on a magnetic surface at equal values of B

$$0 = \oint \mathbf{B} \cdot \mathbf{dl} = \left(\mathbf{B}^+ \cdot \frac{\partial \mathbf{r}^+}{\partial \theta_0} - \mathbf{B}^- \cdot \frac{\partial \mathbf{r}^-}{\partial \theta_0}\right) d\theta_0$$

(where quasi-isodynamicity leads to cancellation of the contributions parallel to $\mathbf{B}$), so that

$$\frac{\partial \tilde{g}^+}{\partial \theta_0} = \frac{\partial \tilde{g}^-}{\partial \theta_0} = \frac{\partial \tilde{g}}{\partial \theta_0}.$$

Therefore, choosing the boundary condition $X(B_{max}) = 0$,

$$X^+ = X^- = X,$$

with

$$X = \int_B^{B_{max}} \dot{H}(B') \frac{\partial \tilde{g}}{\partial \theta_0} dB' = \partial_{\theta_0} \int_B^{B_{max}} \dot{H}(B') \tilde{g} dB'.$$

So, the increment of X vanishes. In particular, the increment of $j_{\|}/B$ vanishes so that the current density lines are closed within one period between the B_{max} lines.

Similarly, because of

$$\frac{\partial F_t}{dt} = -\frac{1}{qB^2}\left(\frac{mv_{\|}^2}{B} + \mu\right)(\mathbf{B} \times \nabla F_t) \cdot \nabla B$$

the increment per period of F_t for passing particles. ΔF_t, vanishes, too, so that the drift surfaces of passing particles and the magnetic surfaces osculate at the B_{max} contours.

The volume average of X between neighboring surfaces is proportional to $\iint X \frac{\partial l}{\partial B} \ln B d\theta_0$.

Because of

$$\iint X^+ \frac{\partial l^+}{\partial B} d\ln B d\theta_0 + \iint X^- \frac{\partial l^-}{\partial B} d\ln B d\theta_0 = \iint X \frac{dL(B)}{dB} d\ln B d\theta_0$$

this integral vanishes. Thus the volume averages of X and $XF(B)$ with an arbitrary function $F(B)$ vanish. As a consequence the bs current in the lmfp regime (see, e.g., [4]) vanishes.

REFERENCES

1. A. A. Subbotin, M. I. Mikhailov, V. D. Shafranov, et al., Nucl. Fusion **46**, 921 (2006).
2. H. Maassberg, W. Lotz, and J. Nuehrenberg, Phys. Fluids **5**, 3728 (1993).
3. J. R. Cary and S. G. Shasharina, Phys. Plasmas **4**, 3323 (1997).
4. J. L. Johnson, K. Ichiguchi, Y. Nakamura, et al., Phys. Plasmas **6**, 2513 (1999).