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Filtered Gyro-kinetic Simulations: an attempt to reduce computational costs

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Gyro-kinetic solvers have made significant progresses during the last decade and are now able to describe a large panel of gyro-kinetic turbulence features [1, 2] Nevertheless, these simulations of micro-turbulence remain extremely demanding in terms of computational efforts.

The present work explores a possible way to reduce this computational cost. It is based on the application of a filtering operator to the gyro-kinetic equations. This approach is strongly inspired by a similar methodology that has been developed for fluid equations: the so-called large-eddy simulations (LES). In LES for fluid turbulence [3, 4], the filtering operator is applied to separate the large, energy containing scale from the small scales at which dissipative processes take place. The principle of LES is quite simple: The large scales are computed numerically while the influence of the small scales is modelled. However, designing an accurate and robust model for the small scales is far from being a simple task. The application of such a scale separation technique to a kinetic equation will be referred to as filter-gyro-kinetic simulation (FGS). It obviously requires a number of adaptations. In principle, the filtering could be applied both in physical space and in the velocity space. In the following, it will be systematically assumed that the filtering is acting only in the physical space.

In this preliminary study, we first analyse the influence of the application of a filtering to the gyro-kinetic simulation without any attempt to model the small scales. Such a strategy simply consists in analysing the effect of a lack of resolution on the gyro-kinetic results. Although fairly simple, this analysis provides very important information on the expected effect of the small-scales modelling.

Filtered Gyro-kinetics

We present in this section the filtered gyro-kinetic equations. Our starting point is the equations solved in the GENE code (Gyro-kinetic Electromagnetic Numerical Experiment) in its local version associated to a flux tube geometry. For clarity, we chose the simplified circular concentric magnetic flux surfaces geometry usually referred as the $\hat{s}-\alpha$ model. The space coordinates are $(x, y, z, v_{\|}, \mu)$ with (x, y) giving the plane perpendicular to the magnetic field, z being the coordinate along the field line, $v_{\|}$ the velocity coordinate parallel to the field and the magnetic moment $\mu = m_i v_\perp^2 / 2B_0$ is chosen as the coordinate associated to the module of perpendicular velocity.

The first step of Large Eddy Simulation method consists in applying a filter to the equations. The fluxtube assumption allows us to consider any equilibrium quantity to be constant along the radial box, so that we can assume that the filtering do not act on these quantities. Under the assumption of electrostatic fluctuations and fluxtube $\hat{s}-\alpha$ geometry with α set to zero, the filtered gyro-kinetic Vlasov equation reads for the ion distribution function $f_k = f(k_x, k_y, z, v_\parallel, \mu, t)$:

$$\partial_t \overline{f_k} = \overline{Z_k} + \mathscr{L}[\overline{f_k}, \overline{\phi_k}] + \mathscr{N}[\overline{f_k}, \overline{\phi_k}] + SGT, \qquad (1)$$

where the overline symbolizes action of the filtering operator, which consists in a convolution product in direct space between any function and a filter kernel. In Fourier space, this amounts to a simple product: $\overline{f_k, \phi_k} = G_k(f_k, \phi_k)$.

The terms introduced in the right hand side are $\overline{\mathscr{Z}}$ which is independent of f_k and ϕ_k, a linear contributions $\overline{\mathscr{L}}$, a nonlinear quadratic term $\overline{\mathscr{N}}$ due to the $\mathbf{E} \times \mathbf{B}$ velocity and the Sub Grid Term (SGT) which comes from the filtering of the nonlinear term as given below:

$$\overline{\mathscr{Z}} = \frac{v_{Ti}^2 (2v_\parallel^2 + \mu B_0)}{2\omega_{ci}} \left[\omega_{ni} + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \omega_{Ti} \right] K_x \delta_{k_{x,y},0} G_k \qquad (2)$$

$$\begin{aligned} \overline{\mathscr{L}[f_k]} &= \overline{\langle \mathscr{C}(f_k) \rangle} - \left[\omega_{ni} + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \omega_{Ti} \right] F_{0i} i k_y \overline{\mathscr{J}_{0ik} \phi_k} \\ &- \frac{v_{Ti}^2 (2v_\parallel^2 + \mu B_0)}{2\Omega_{ci}} \left(K_x \overline{\Gamma_{k_x i}} + K_y \overline{\Gamma_{k_y i}} \right) + v_{Tj} \left[\frac{\mu \partial_z B_0}{2} \partial_{v_\parallel} \overline{f_{kj}} - v_\parallel \overline{\Gamma_{kiz}} \right] \end{aligned} \qquad (3)$$

$$\overline{\mathscr{N}[g_{kj}]} = \sum_{k'_\perp} (k'_x k_y - k_x k'_y) \overline{\mathscr{J}_{0ik'_\perp} \phi(k'_\perp)} \, . \, \overline{f(k_\perp - k'_\perp)} \qquad (4)$$

$$SGT = \overline{\sum_{k'_\perp} (k'_x k_y - k_x k'_y) \mathscr{J}_{0ik'_\perp} \phi(k'_\perp) f(k_\perp - k'_\perp)} - \overline{\mathscr{N}[g_{kj}]} \qquad (5)$$

Since $\mathscr{L}$ is a linear function of f_k and ϕ_k, applying the filtering operator to it does not introduce unknown terms:

$$\overline{\mathscr{L}[f_k, \phi_k]} = \mathscr{L}[\overline{f_k}, \overline{\phi_k}] \qquad (6)$$

On the contrary, the application of the filtering to the quadratic term $\mathscr{N}$ leads to an unknown term:

$$SGT = \overline{\mathscr{N}[f_k, \phi_k]} - \mathscr{N}[\overline{f_k}, \overline{\phi_k}] \qquad (7)$$

This term must be modeled as a function of $\overline{f_k}$ and $\overline{\phi_k}$ in order to close the equations.

The Bessel function $\mathscr{J}_{0ik_\perp} = \mathscr{J}_0\left(\frac{v_{Tj}}{\Omega_{cj}} k_\perp \sqrt{B_0\mu}\right)$ is the Fourier transform of the gyro-average, which is not affected by the filtering. Other abbreviations used are:

$$\Gamma_{k_{x,y}} = ik_{x,y} f_k + ik_{x,y}\frac{q_i F_{0i}}{T_{0i}}\mathscr{J}_{0ki}\phi_k \quad \Gamma_z = \partial_z f_k + \tfrac{q_i F_{0i}}{T_{0i}}\partial_z \mathscr{J}_{0ki}\phi_k \tag{8}$$

$$F_{0i} = \pi^{-3/2} e^{-(v_\parallel^2+\mu B_0)} \qquad \omega_{(n,T)i} = \partial_x \ln(n,T)_i \tag{9}$$

$$K_y = -\frac{\cos z + \hat{s} z \sin z}{R} \qquad K_x = -\tfrac{\sin z}{R} \tag{10}$$

We introduced the ion thermal velocity $v_{Ti}^2 = 2T_{i0}/m_i$, and the cyclotron pulsation $\Omega_{ci} = q_i B_0/m_i$. The electrostatic potential is given by the quasi neutrality assumption, valid for small values of $k_\perp^2 \lambda_D^2$. We assume adiabatic electrons with small electrostatic fluctuations:

$$q_i n_{i0}\left[1-\Gamma_0(b_j)\right]\frac{q_i}{T_{i0}}\phi_k + q_e n_{e0}\frac{q_e}{T_{e0}}\left[\overline{\phi_k} - <\overline{\phi_k}>\right] = \pi B_0 q_i n_{i0}\int d\mu dv_\parallel \overline{\mathscr{J}_{0ki} f_k} \tag{11}$$

Where $<\phi_k> = (\int dz\phi_k/B_0)/(\int dz/B_0)$ represents the flux surface average of the electrostatic potential, the jacobian being the inverse of the magnetic field. The function $\Gamma_0(b_j) = \Gamma_0(v_{Ti}^2 k_\perp^2/\Omega_{ci}^2)$ appears due to the integration of the gyroaverage operator along the magnetic moment.

In order to get the final Filtered Gyro kinetic Model, all the work consists of modeling the SGT. It is thus very important to measure its effects. Two approaches can be used to this purpose:

- ⋆ An under resolution analysis, which consists in performing several simulations with the same set of parameters, but with different grid resolutions. For each run, relevant quantities have to be computed, and the comparison of the values obtained will provide us information on how the under resolution could alter the performances of the simulation. In particular we will measure the moments of the distribution function and the electrostatic fluctuations.

- ⋆ Another kind of information could be obtained by post-processinga full resolution simulation. By applying a filter the the full fields f_k and ϕ_k, we can then have a direct measure of the SGT.

In this work, only the first approach has been used. In practice, this is achieved by applying a filter without modeling SGT ($SGT = 0$).

The present analysis is based on the Cyclone Base Case set of parameters, which allows to study the Ion Temperature Gradient turbulence. We will focus in the latter on the comparison between different resolutions and not on the results themselves.

Figure (1) represents the electrostatic potential (top) and ion density (bottom) spectrum along k_x. It corresponds to the Fourier transform of the radial coordinate in the flux tube approximation. The usual Cyclone Base Case is represented by the black curve (with the grid $n_x = 64$ points, $n_y = 16$ points). We used three different factors to decrease resolution along both k_y and k_x: $3/4$ (dashed red), 2 (blue) and 4 (red). As we can see, the decrease of resolution has a simple cutoff effect for large values of the wave vector, associated to the small scales neglected by the under resolution. But it does not affect seriously the large scales, especially the location of the maximum remains unchanged. The loss due to under resolution at the maximal value is between one half (factors 2 and 4) and one third (factor $3/4$) for the modulus of the electrostatic fluctuations.

These results clearly show the need of a model that would take into account the effect of the under resolved scales on the large scale physics.

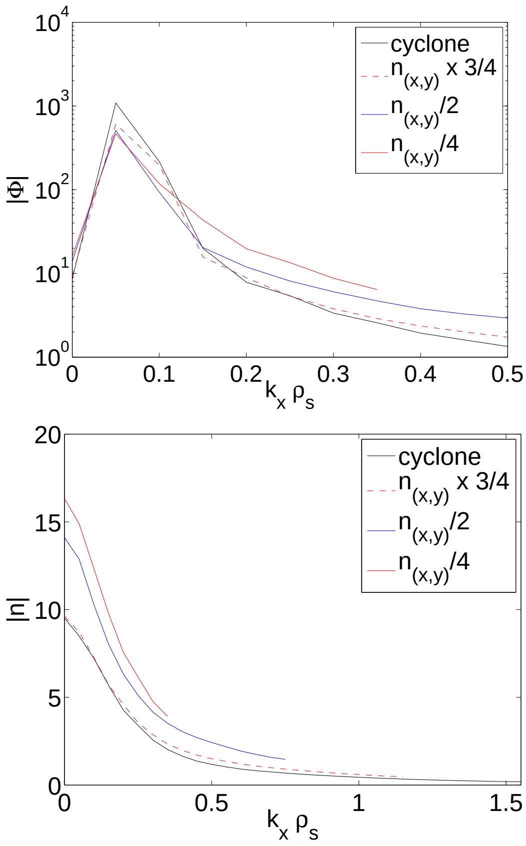

Figure 1: k_x dependence on the number of grid points in perpendicular plane (x,y). $\omega_n = 2.22R^{-1}$, $\omega_T = 6.97R^{-1}$, $r = 0.18R$, R being the major radius.

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