Bayesian Experimental Design of a multi-channel interferometer for Wendelstein 7-X *

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Bayesian Experimental Design (BED) is a framework for the optimisation of diagnostics basing on probability theory. In this work it is applied to the design of a multi-channel interferometer at the Wendelstein 7-X stellarator experiment. BED offers the possibility to compare diverse designs quantitatively, which will be shown for beam-line designs resulting from different plasma configurations. The applicability of this method is discussed with respect to its computational effort.

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I. DIAGNOSTIC DESIGN USING PROBABILITY THEORY

Future fusion experiments like ITER or Wendelstein 7–X are characterised by a complex interaction of scientific and technical requirements. In result, a combination of physical questions of interest and technical boundary conditions has to be taken into account for the design of the diagnostic set-up.

To assure the gain of maximum information about the physical parameters of interest, a method for the optimisation of plasma diagnostic has to take into account these conditions. A probabilistic approach is given by *Bayesian Experimental Design (BED)*, which offers a consistent mathematical ansatz independent of the diagnostic type and the physical question to be addressed.

Figure of merit is a *utility function* $U(D, \eta)$ (see [2] for discussion), which depends on the data D of future experiments and the design parameters η. These design parameters describe the diagnostic unit to be optimised, e.g. the geometry of the line-of-sight configuration of an interferometer. Because the future data is unknown, the utility function is marginalised over the expected data space, the *Expected Utility (EU)* reads as

$$EU(\eta) = \int \mathrm{d}D\, U(D, \eta)\, p(D|\eta). \quad (1)$$

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Here, $p(D)$ is a probability density function, describing a weight function for future D from a given diagnostic design η. The marginalisation theorem applied here, i.e. the integration over the expected data space, is part of the Bayesian probability theory.

A useful choice for the utility function itself is the *Kullback-Leibler distance*

$$U_{KL}(D,\eta) = \int \mathrm{d}\theta\, p(\theta|D,\eta)\, \log\left(\frac{p(\theta|D,\eta)}{p(\theta)}\right), \quad (2)$$

which describes the information gain about the parameters of interest, θ. The knowledge about θ before the measurement $p(\theta)$ is compared to the state of knowledge after the measurement, $p(\theta|D,\eta)$, where the data D and a design configuration η is given. The parameters of interest are the physical quantities one wants to estimate, e.g. the parameters describing an spacial electron density distribution.

Using Bayes' theorem the EU from equation (1) reads

$$EU(\eta) = \int \mathrm{d}D \int \mathrm{d}\theta\, p(D|\theta,\eta)\cdot p(\theta) \log \frac{p(D|\theta,\eta)}{p(D|\eta)} \quad (3)$$

and describes the expected information gain for the parameters of interest θ by future measurements, depending on the design configuration η. The EU in this version is an absolute measure for the information gain, it is expressed in *bit* if the base–2 logarithm is used. Bayesian Experimental design is the maximisation of the EU with respect to η.

The basic principle of *BED* is proposed by Lindley [1], a more detailed explanation of the EU is given, e.g., in [3].

II. BEAM-LINE DESIGN FOR DIFFERENT PLASMA CONFIGURATIONS IN W7–X

For the interferometer diagnostic at W7–X an infrared system based on a CO_2 laser ($\lambda = 10.6\,\mu m$) is considered in a plane at a toroidal angle of $\Phi \approx 195°$ [4]. In earlier work case studies about the effect of the error statistics on the beam-line design have been presented [5], and the beam-line designs for different physical questions have been compared [6]. These analyses were made for the standard magnetic configuration at W7–X. In this work, the influence of different plasma configurations and of plasma outside the last closed magnetic surface (LCMS) will be studied.

The physical question of interest to be used here as the design criterion is the occurrence of "hollow" electron density profiles, where the maximum density is not localised in the plasma centre. This effect may occur in case of Core Electron Root Confinement (CERC) scenarios. Parameters of interest are the maximum density, the width and the depth of the "hollow" part (fig. 1).

To measure these parameters of interest, a four channel interferometer was optimised taking into account the technical boundary conditions at W7–X, i.e. the port system and the fixed positions for retro-reflectors at the inner wall of the plasma vessel, which are restricted by the structure of the inner wall. Due to the realisable vacuum flange positions at the end of the ports and limited number of possible positions for retro-reflectors, 101 lines of sight were available for probing beam-lines in the interferometry plane of W7–X.

As an additional line of sight, a beam-line congruent to the Thomson scattering diagnostic beam at a toroidal angle of $\Phi \approx 187°$ can be realised. Because this line of sight turns out to be very informative [6], it was assumed as given for the following analysis, so only three beams were located at the interferometry plane.

Four different plasma configurations were analysed for this study: the standard magnetic configuration of W7–X, a high-ι configuration ($\iota = 1.2$ at the LCMS), a set-up with a significant kinetic pressure of the plasma ($\beta = 0.04$) and finally a plasma with a non-vanishing electron density outside the separatrix. For the latter case, an exponential decay of the plasma density was assumed outside the LCMS with a decay length of $5\,cm$ in magnetic coordinates.

Figure 2 shows the results for the optimal beam-line design of the respective plasma configuration with respect to the cross section of the plasma. The beam-line designs for the standard configuration and the high-ι case turn out to be identical, the beam geometry for the other cases varies slightly. In all cases, the beam-lines cross the "bulky" part of the plasma cross section, covering the region where the effect of "hollowness" occurs.

Because only one geometry can be realised in the final diagnostic set-up, one has to estimate the beam geometry which is most beneficial for all cases. For this, an important feature of BED can be applied: Because the expected information gain of a design can be estimated, the optimal set-up can be found by quantitative comparison of the different EU's from the various designs.

Table I shows the values of the EU for two cases: the optimal design for the respective plasma configuration as shown in figure 2, and the EU for the different plasma configurations when the beam-line design for the standard configuration (fig. 2 (a)) is used. In case of the high-ι plasma the EU's are identical (because the beam-line configuration is the same), for the high-β plasma and the case of plasma outside the separatrix the difference is in the order of one percent of the EU. Concluding this, one can state that the beam-line design for the standard magnetic configuration is also a near-to-optimum design for the other plasma configurations analysed here.

III. COMPUTATIONAL EFFORT

Central element of BED is the calculation of the Expected Utility function (eq. (1)). For this, the necessary probability density functions need to be estimated first: $p(\theta)$ describes the *area of interest*, i.e. the range of the parameters one is interested in. In the case presented here, the three parameters of interest (maximum density, depth

	optimal design	CERC design
high-ι plasma	6.741 ± 0.005	6.741 ± 0.005
high-β plasma	6.822 ± 0.005	6.748 ± 0.005
plasma outside LCMS	6.847 ± 0.005	6.839 ± 0.005

TABLE I: *EU* values (in *bit*) for different beam-line designs and different plasma configurations: optimal beam-line design for the respective plasma configuration (centre column), *EU* for CERC beam-line set-up (right column).

and width of the hollow part) have been varied uniformly within certain limits, $p(\theta)$ is therefore a three-dimensional uniform distribution. The limits of this distribution are then the limits of the integration over θ.

The likelihood function $p(D|\theta, \eta)$ includes the forward function, a mathematical model function describing the measurement process. It also describes the error statistics of the future measurement. For the interferometer at W7–X, a constant Gaussian background error was assumed. Because of four interferometry channels the likelihood turns out to be a four-dimensional Gaussian, the integration over D in equation (1) is performed within $[-\infty; \infty]$.

As the final element, the evidence function $p(D|\eta)$ is calculated from the likelihood function and the "interest" function by marginalisation:

$$p(D|\eta) = \int \mathrm{d}\theta \; p(\theta) \, p(D|\theta, \eta). \tag{4}$$

In general, the multi-dimensional integral in equation (1) cannot be solved analytically. The results presented here have been calculated numerically using a simple sampling Monte-Carlo algorithm. The computing was done on a multi-CPU blade cluster: two Intel Xeon CPUs (3.06 GHz) with hyperthreading capability located on one blade, eleven blades were used at the same time. The *EU* was then computed by calculating 100×100 samples in parallel.

For the optimisation of the interferometer, only one beam-line was optimised first by calculating its *EU* for all 101 possible beam positions. Then, the beam-line was fixed at the position with the highest *EU* value, the next beam-line was estimated the same way, and so on.

Although the parallel computing algorithm was applied, the calculation of the optimal beam-line configuration turned out to be time consuming. As a case study, typical calculation times for one *EU* value using this method are given in table II with respect to the number of parameters of interest and the number of beam-lines. The computation time in particular depends on the number of parameters of interest, because of the calculation of the evidence function (eq. 4). This function needs to be computed for every MC step of the *EU* calculation, so the dimension of θ has a significant impact on the computation time. The optimisation of one four channel interferometer, where three beam-lines were varied (as presented in this work), finally took about 3-5 days in total. It has to be mentioned that these

# par. of interest	# beam-lines	computation time
3	2	6 min
3	3	10 min
3	4	16 min
4	4	130 min
5	4	285 min
5	8	660 min

TABLE II: Computation time for the optimal beam-line configuration of a multi channel interferometer at W7–X with respect to the number of parameters of interest (dimension od θ) and to the number of beam-lines (dimension of D).

computation times strongly depend on the problem to be solved. In addition to the dimension of θ and D, the forward function, which is an essential part of the likelihood, also plays an important role. In case of the interferometer one has to deal with an integration along the line of sight, resulting in further computational effort. For other diagnostics, e.g. if a linear forward function can be applied, this effort could be reduced significantly.

IV. CONCLUSION

Bayesian Experimental Design as a method for diagnostic design combines several advantages: As a general approach, it is independent of the diagnostic to be optimised. The physical question of interest is implemented directly into the design process by the parameters of interest and their probability density distribution. And the quantitative nature of the Expected Utility offers the possibility of direct comparison of different designs, which was used in this work to study the impact of different plasma configurations on the beam-line configuration of a four channel interferometer.

Besides this advantages, it was shown that in case of non-trivial design problems the computation time may be significant. For a detailed analysis of a physical problem like the one discussed in this work, the overall computation time may take weeks or even months. In such cases, the application of BED will depend on a cost-benefit analysis.

For fusion experiment like W7–X, which is designed to be a long-run experiment, the optimisation of the diagnostics using BED can be recommended to assure the maximum gain of information from future measurements.

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[1] Lindley, D. V.: Bayesian Statistics - A Review, 1972, SIAM (Philadelphia)

[2] K. Chaloner et al., Stat. Sci. **10**, pp. 273-304 (1995)

[3] R. Fischer, AIP Conference Proc. **735**, pp. 76-83 (2004)

[4] P. Kornejew et al., Rev. Sci. Instrum., **77**, 10F128 (2006)

[5] H. Dreier et al., Fusion Sci. Technol. **50**, pp.262-267 (2006)

[6] H. Dreier et al., AIP Conference Proc. **993**, pp. 183-186 (2008)

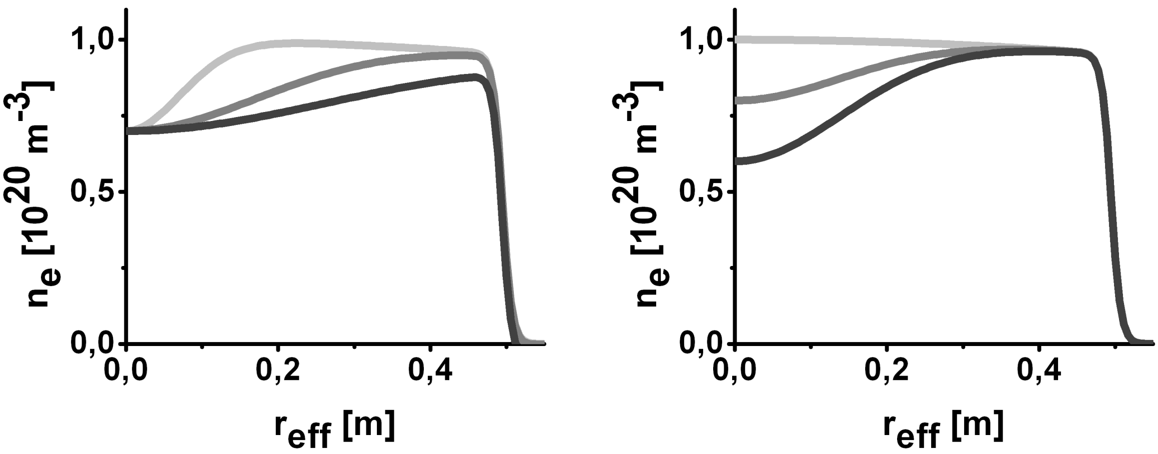

FIG. 1: Assumed variations for two parameters of interest θ: width (left) and depth (right) of the hollow part of the electron density distribution. The maximum density is varied from $0.1 - 1.0 \cdot 10^{20}\, m^{-3}$ (not shown).

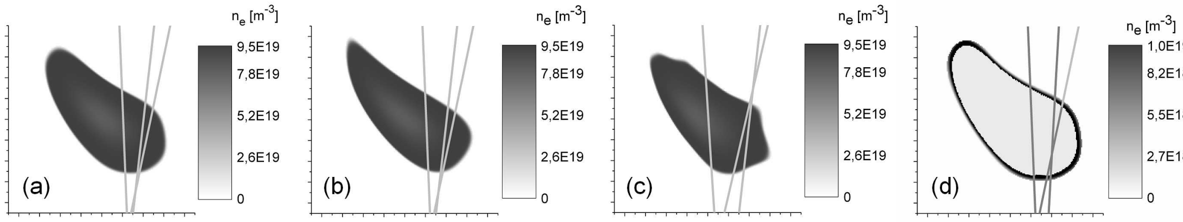

FIG. 2: Plasma cross section in the interferometry plane at W7—X with optimal beam-line geometry for the measurement of Core Electron Root Confinement density effects at different plasma configurations: standard configuration for W7–X (a), high-ι plasma (b), high-β (c), and a plasma with an assumed exponential decay of electron density outside the LCMS (d). For the latter case, the plasma inside the LCMS is shadowed to accentuate the edge plasma.