Comparative studies to the design of the interferometer at W7-X with respect to technical boundary conditions

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Abstract. For the optimisation of the beam line configuration of the multi-channel interferometer at the Wendelstein 7-X stellarator the probabilistic approach of Bayesian experimental design (BED) is applied. Parameters of physical interest are directly implemented as design criteria. The quality of the design is analysed according to its expected information gain (*expected utility*) about the parameters of interest, which enables one to compare different diagnostic configurations quantitatively. The focus of this work lies on the comparison of different technical approaches for the interferometer diagnostic. Different physical problems (high confinement regimes, neoclassical predictions) and their effect on the density distribution are applied as optimisation goals. The influence of the port system and the in-vessel components (retro-reflectors) is discussed. For this, the design was done with and without technical restrictions, the resulting expected utilities are compared and analysed. Furthermore, the impact of an additional beamline at a different toroidal position (congruent to the Thomson scattering diagnostic) is examined.

Keywords: Bayesian experimental design, multi-channel interferometer, Wendelstein 7-X, diagnostic optimisation, information measure

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INTRODUCTION: BAYESIAN EXPERIMENTAL DESIGN

Fusion experiments like the Wendelstein 7-X stellarator are designed and optimised according to selected physical questions. It is obvious, that when the experiment is running later, it has to be checked whether these criteria are fulfilled. For this, the diagnostic units to be applied at the experiment have to be designed according to the same physical questions of interest.

The approach of Bayesian experimental design (BED) offers the possibility to implement physical parameters of interest directly as design criteria. As a utility function for the optimisation process an information measure, namely the Kullback-Leibler distance, is used:

$$U_{KL}(D,\eta) = \int d\theta \; p(\theta|D,\eta) \log\left(\frac{p(\theta|D,\eta)}{p(\theta)}\right)$$

This expression gives an absolute measure for the information gain from the measurement. It compares the probability density function (PDF) $p(\theta)$, describing the knowledge about the parameters of interest θ before the measurement, with $p(\theta|D,\eta)$, the PDF expressing the state of knowledge about θ when data D from an experiment with the design parameters η are available. The information gain is given in *bit*, if the base-2 logarithm is used.

To take into consideration all possible data expected from the future experiment, the utility function is integrated (marginalised) over the data space expected from the future experiments, which is encoded with $p(D|\eta)$:

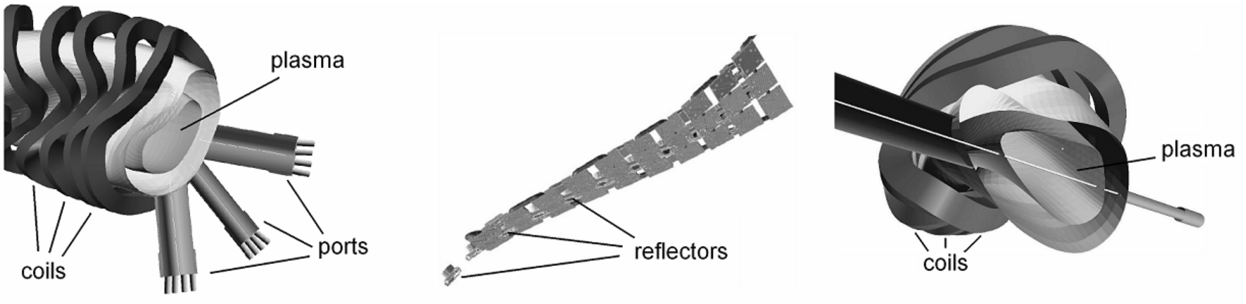

FIGURE 1: Technical boundary conditions for the interferometer at W7-X: port system at $\varphi = 194°$ (left), retro-reflectors at the opposite wall (centre), additional beam line congruent to the Thomson scattering diagnostic at $\varphi = 187°$ (right).

$$EU(\eta) = \int dD\, U_{KL}(D,\eta)\, p(D|\eta)$$

This expected utility function (EU) only depends on the design parameters. Bayesian experimental design means maximising $EU(\eta)$ with respect to η.

For a multi-channel interferometer, e.g., the parameters of interest θ may be the describing the electron density distribution, the data D are the measured phase shifts from the different channels, the design parameters η may be the measuring frequency of the probing beams or the geometrical parameters of the beam line configuration.

The method of BED was introduced by Lindley [1], a review over different approaches and comparison with “classical” design can be found in [2]. First applications in plasma physics are given in [3], for the Wendelstein 7-X stellarator, case studies for the interferometer are shown in [4] and [5].

In this work, the design of a four-channel interferometer at W7-X will be discussed with respect to the given technical boundary conditions. The possibility to compare different designs in a quantitative way, which is an important feature of BED, will be used to point out the impact of these boundary conditions. Different design approaches for the beam line configurations will be analysed.

INTERFEROMETRY AT W7-X

The interferometer at W7-X is supposed to be part of the set of "start-up" diagnostics. It will be realised as a multi-channel configuration for the detection of electron density distributions [6]. A four-channel diagnostic is planned for the start-up phase, which will be extended later on. For vibration compensation, a two-colour set-up will be applied, using probing frequencies of 9.3 µm (CO_2 laser) and 5.4 µm (CO laser).

Three ports at W7-X are dedicated to the interferometry system at a toroidal angle of approximately 194° (fig. 1 left), allowing different beam line configurations from vertical to horizontal optical paths. Because no opposite ports are available, the probing beams have to be reflected by corner cube retro-reflectors mounted at the opposite wall (fig. 1 centre). These reflectors have to fit the structure of the in-vessel components, 11 fixed positions can be realised. Taking into account the diameter of the vacuum flanges necessary to couple the probing beams into the ports, the number of realisable beam lines is given with 101.

An additional extra beam line can be applied at a toroidal angle of 187° (fig. 1 right). This line of sight is complemetary to the probing beam of the Thomson scattering diagnostics (TS), which allows an absolute calibration of TS measurements. In addition, this beam is characterised by a long path length inside the plasma, leading to a good signal-to-noise ratio (SNR).

COMPARISON OF DIFFERENT INTERFEROMETER DESIGNS

Two physical questions of interest have been selected as design criteria for this study:

1. the occurence of hollow density profiles, a feature of the Core Electron Root Confinement (CERC) plasma scenario as predicted from neoclassical theory [7];
2. the variation of the density profiles at high confinement regimes (optimal confinement (OC), H-mode and high density H-mode (HDH), as observed at W7-AS).

The resulting variations of the density distribution, which were used as the design criteria for the optimisation of the interferometer, are shown in figure 2.

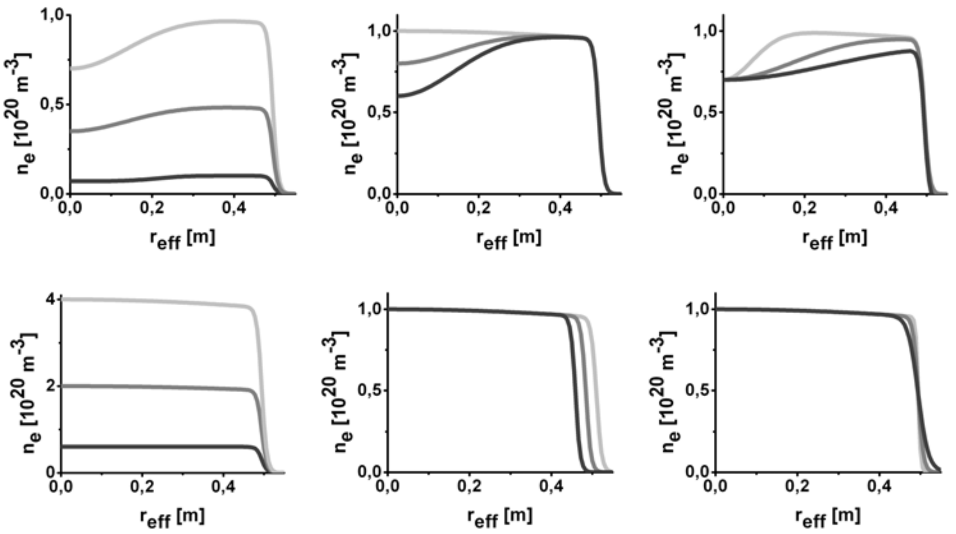

FIGURE 2: Parameter of interest for the design of a multi-channel interferometer. In the upper row, the density variations for the case of hollow profiles are displayed (from left: variation of maximum density, depth and width of the indention of the density distribution). In the lower row, the parameters are varied according to high confinement regimes (from left: maximum density, position of the steepest gradient, steepness of the edge gradient).

The effect of hollow profiles is located near the core region of the plasma (fig. 2, upper row), whereas the high confinement regimes change the density distribution mainly at the plasma edge (lower row). One would expect that the optimal beam line configuration of the interferometer corresponds to these criteria.

Figure 3 shows the design results for the four-channel interferometer with respect to the detection of hollow profiles in the interferometry plane. The beam lines are plotted with respect to the density distribution in this plane. As expected, the beam lines cover the central part of the plasma, where the density effect is localised. For the configuration with all four beam lines lying in the interferometry plane, the EU is given with EU= 6.43 ± 0.01 *bit* (fig. 3 left), and EU= 6.86 ± 0.01 *bit* if one line of sight is congruent to the TS probing beam (fig. 3 centre). So, the configuration including the extra beam is more informative than the design with four beams in the interferometry plane.

For comparison, a line of sight design without technical restrictions was also calculated (fig. 3, right). It shows that the optimal beam lines to measure hollow profiles would cross the plasma on long path lengths, which corresponds to a good SNR. The unconstrained design leads to EU= 8.71 ± 0.01 *bit*. This means that about 25% of the possible information gain is lost due to the limited access to the plasma vessel.

The result of the four-channel design with respect to high confinement regimes is displayed in figure 4: the configuration without (left) and including (centre) the extra beam congruent to TS, and the design calculated without restrictions (right). As expected, one or more lines of sight are located at the plasma edge, because for the physical problem analysed here the variation of the density distribution takes place in this region. But in the configurations limited by the port system only edge beams with very short path lengths inside the plasma can be realised.

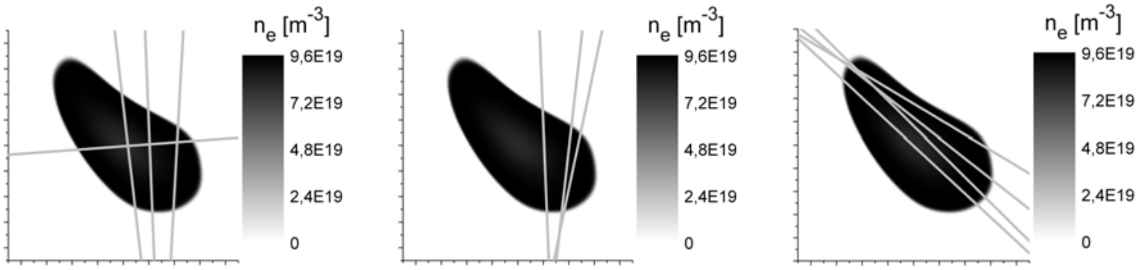

FIGURE 3: Four beam line design result for the measurement of hollow density profiles: Four beam lines in interferometry plane (left), including the beam line congruent to Thomson scattering (centre), four beam configuration without technical constraints (right).

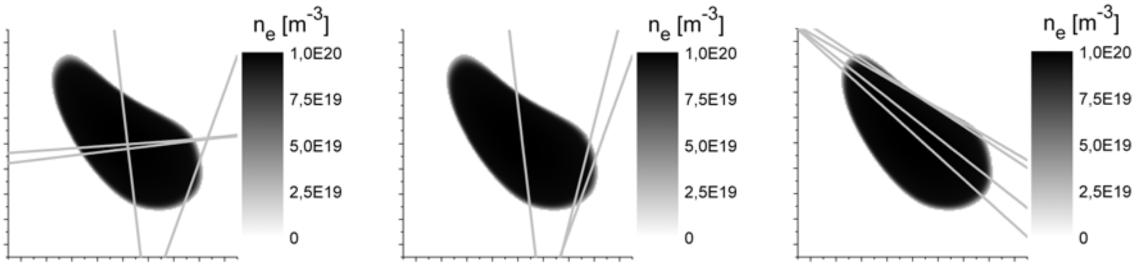

FIGURE 4: Design result for four interferometry chords with respect to the measurement of high confinement regimes: Four beam lines in interferometry plane (left), including the beam line congruent to Thomson scattering (centre), four beam configuration without technical constraints (right).

The EU's are calculated with EU= 8.53 ± 0.01 *bit* for four beams in the interferometry plane and EU= 8.85 ± 0.01 *bit* with one beam congruent to TS. As in case of hollow profiles, the latter configuration shows a higher EU and is therefore more informative. For the unconstrained design one obtains EU= 28.3 ± 0.2 *bit*, meaning that about 70% of the information gain is lost due to the technical boundary conditions. This tremendous lost can be explained by the beam line configuration of the unconstrained design (fig. 4 right): Two lines of sight are localised at the very edge of the plasma, these beam lines are therefore very sensitive to the density changes in this region. Furthermore, they are passing the plasma on a very long path, which leads to a good SNR. Both aspects cannot be realised in the case where the port system has to be taken into account. One has to conclude here, that under these conditions the multi-channel interferometer is not a sufficient diagnostic to measure the density variations at the plasma edge as expected for high confinement scenarios.

CONCLUSION

In this paper, Bayesian experimental design was applied to the multi-channel interferometer of W7-X. Two different physical problems were analysed. In both cases, the design configurations without technical boundary conditions lead to a more informative design. Whereas a loss of 25% of the expected information gain due to the limitations in case of the measurement of hollow profiles seems to be acceptable, the loss of about 70% for the problem of high confinement scenarios is too large. In conclusion, the interferometer may not be sufficient as a plasma edge diagnostic for the analysed cases at W7-X.

Nevertheless, it turned out that the implementation of a beam congruent to the TS diagnostic led to the more informative design in both analysed cases. This seems to be the result of the beams long path length inside the plasma, which leads to a good SNR. So the installation of this line of sight is recommended.

Summarising, BED turns out to be a valuable method for the comparison of different design approaches. The loss of information gain due to boundary conditions can be quantified as well as the information gain from additional effort like the extra beam.

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