Ray Tracing Calculations for the W7-X Stellarator Using the BRT Code

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Abstract. A new ray tracing code is developed for electron cyclotron and electron Bernstein wave studies in arbitrary 3D magnetic configurations, with emphasis on heating, current drive and ECE diagnostics. For the tracing equations, the weakly relativistic, non-relativistic, or “cold” dielectric tensor can be used. Absorption and emissivity can be calculated for an arbitrary electron distribution function. Quantities such as the absorption and emissivity are calculated separately for passing and trapped electrons. For calculations of ECCD efficiency, the adjoint approach is used in either collisional or collisionless limits. For ECE spectrum simulations, the theoretical limit of resolution is also calculated.

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DESCRIPTION OF THE BRT CODE

The abbreviation BRT (Beam/Ray Tracing) is the working name of the new code, which has to incorporate advantages of both the ray- and the beam-tracing techniques. At the present moment, only the ray tracing part is finished. The main goal of this paper is to demonstrate appropriate modelling of the different scenarii of ECR/EBW heating and as an ECE diagnostic for the W7-X stellarator (under construction in Greifswald, Germany).

The magnetic configuration provided by the 3D equilibrium code VMEC is converted to Boozer co-ordinates and rapidly interpolated by a specially developed highly optimized package. The code is written to be platform independent and to work as a stand-alone program and as a module inside a 1D transport code [1]. The code is used through an especially designed graphical user interface, which allows the preparation of input parameters and viewing simulations results in convenient (2D and 3D) form. The aim of this interface is to make the code suitable for any interested user.

The ray tracing equations are the standard Hamiltonian ones (see, e.g. Ref. [2]). For the Hamiltonian, $\mathcal{H}$, the most general form suggested by Tokman and Westerhof [3, 4] is adopted. Strictly speaking, the code includes both models, of Westerhof, $\mathcal{H}^W = N^2 - N_\parallel^2 - (\Re N_\perp)^2$, and of Tokman, $\mathcal{H}^T = \Re(D_{ij}^H e_i^* e_j)$, which, being based on the same physics, produce almost identical results (here, $N_\perp$ is the corresponding root of the complete dispersion relation, $\det D_{ij} \equiv \|N^2\delta_{ij} - N_i N_j - \epsilon_{ij}\| = 0$, $\epsilon_{ij} = \epsilon_{ij}^H + i\epsilon_{ij}^{aH}$ is the weakly relativistic dielectric tensor, expanded to the Hermitian, $\epsilon_{ij}^H$, and anti-Hermitian, $\epsilon_{ij}^{aH}$, parts, and $e_i$ is the $i$-th component of the dimensionless polarization vector). With the weakly relativistic dielectric tensor, the model of tracing includes...
those kinetic effects which become a significant in the vicinity of the EC resonance, leading to “anomalous” dispersion effects and possible bending of the rays there [3, 4]. Generally, the “cold”, “warm” non-relativistic or weakly relativistic dielectric tensor can also be used in the Hamiltonian. For example, in contrast with the “cold” approach (which should be sufficient for many cases), the weakly relativistic model can give a quite different result in the case of quasi-vertical launch with the ray trajectories almost tangential to the resonance line, i.e. in the vicinity of the resonance, but along a direction with small $\nabla B$.

Absorption and emissivity are calculated for an arbitrary electron distribution function, $f_e$, from the anti-Hermitian part of the fully relativistic dielectric tensor, $\epsilon_{ij}^R (f_e)$, and from the micro-current correlation tensor, $G_{ij} (f_e)$, respectively (see, e.g. [5]). Assuming, that the deviation from the Maxwellian distribution function, $f_M$, is not so large, it is sufficient to restrict the treatment to a linear approach, i.e. to use $f_e$ only for calculations of $\epsilon_{ij}^H (f_e)$ and $G_{ij} (f_e)$, while the wave power flux has to be calculated with $\epsilon_{ij}^H (f_M)$. In the code, the electron distribution function used for absorption, current drive, and emissivity calculations can be Maxwellian, bi-Maxwellian or arbitrary (numerically given), thus allowing future coupling with a Fokker-Planck solver. At present, the adjoint approach is used to calculate the CD efficiency [6, 7]. By analysing the value of the magnetic moment during the integration along the resonance curve, the wave-particle interaction is modelled separately for passing and trapped electrons.

The code also incorporates modules from the ART code [8] to deal with the ordinary-extraordinary-Bernstein (OXB) mode conversion. The “warm” non-relativistic dielectric tensor, $\epsilon_{ij} (f_M)$, is sufficient to capture the EBWs and OXB physics in W7-X in most cases. For the simulations the “standard” Hamiltonian is chosen, $\mathcal{H}^s = \Re (\mathcal{D}^H)$, with the absorption coefficient $\alpha = -2 \Im (\mathcal{D}^a H) / / \partial \Re (\mathcal{D}^H) / / \partial \mathbf{k} \partial$, (here $\mathcal{D}^H \equiv \det D_{ij}^H$ and $\mathcal{D}^a H \equiv \det D_{ij}^a H$, respectively).

For simulation of the ECE spectrum, special options are included to estimate the theoretical limit of the spatial resolution (width of the emission line) and to identify the energy range of electrons dominating in the emission (this might be used to diagnose fast electrons).

The code was partly benchmarked against the “old” W7-AS code [7] and the TOR-BEAM code (using only the reference ray data) [9]. The code is now routinely exploited in modelling heating at various harmonics of the ordinary and extraordinary mode (O1, O2, X2 and X3) in different magnetic configurations. The code is also supporting the design of ECRH launcher components for W7-X [10, 11].

**O2-SCENARIO: HIGH-FIELD-SIDE LAUNCH**

As an example for W7-X, the scenario of the quasi-perpendicular high-field-side (HFS) launch of O2-mode near the “triangular” plane is simulated. The main features of this cross-section are that the minimum of $B$ is situated there (for the “standard”
configuration, used in simulations, $B_{mae}/B_{min} \simeq 1.09$ on axis), and that $\nabla B$ has an inverted sign giving the unique possibility to produce the HFS launch from the outer side of the device. Since the density cut-off for $f=140$ GHz is about $2.4 \cdot 10^{20}$ m$^{-3}$, the O2-mode is candidate for high-density operation, planned for W7-X.

The simulation is performed for $B_0 \equiv \langle B \rangle_{\text{axis}} = 2.59$ T, $n_e(0) = 1.5 \cdot 10^{20}$ m$^{-3}$, and $T_e(0) = 3$ keV with almost flat core profiles. The Gaussian beam with an initial width of 10 cm is discretized by 80 rays. Because of high density, refraction of the launched beam is significant and does not allow the focusing of the beam on axis (Fig.1, left). Since the plasma is “optically gray” for O2-mode, the deposition profile, $p(r_{\text{eff}}) \equiv dP_{\text{abs}}/dV$ (absorbed power per unit volume), is quite broad (Fig.1, middle), and the power shine-trough is about 17% (Fig.1, right).

The most important result (and even somewhat surprising for the O-mode) is that about 60% of power is absorbed by ripple trapped electrons (Fig.1, right, triangles). Furthermore, the shapes of the deposition profiles for trapped and passing electrons ($p_{tr}(r_{\text{eff}})$ and $p_{pa}(r_{\text{eff}})$, respectively, with $p = p_{tr} + p_{pa}$) are quite different (Fig.1, middle). The damping along the ray becomes significant starting from $r_{\text{eff}}/a \simeq 0.7$ with the cyclotron interaction velocity range (not shown) $v/v_{th} \sim 3$, where the absorption by trapped electrons is dominant. In the region $r_{\text{eff}}/a < 0.4$, where the interaction velocity range is decreased up to $v/v_{th} \sim 1.5$, mainly passing electrons are responsible for damping. The optical depth is still not so high there, $\tau \sim 1 - 1.5$ (plasma is gray), and the rays contain quite enough power to heat also the passing electrons. For comparison, simulation for X2-scenario (almost the same launch conditions, but reduced density, $n_e(0) = 5 \cdot 10^{19}$ m$^{-3}$, and magnetic field increased up to $B_0 = 2.68$ T), gives quite similar $p_{tr}(r_{\text{eff}})$ and $p_{pa}(r_{\text{eff}})$, and the damping by passing electrons is dominating. For the X2-mode, the plasma is optically thick with well localized damping, and the conditions within the absorption region are almost unchanged.

Note, that this O2 feature is specific for the W7-X configurations and for the HFS launch scenario. Nevertheless, it demonstrates a flexibility of the W7-X, where the fraction of trapped particles in the “triangular” plane can be varied in configuration scans. This adds to the flexibility of the launcher.
SIMULATION OF OXB-SCENARIO

For overdense plasma heating, EBWs can be utilized, as their propagation doesn’t suffer any upper density limit. These quasi-longitudinal waves have a very high absorption near the cyclotron harmonics, $\omega \approx n\omega_{ce}$. The most typical example of EBW heating is the OXB-scenario [12, 13]. This consists of an oblique launch of O-mode, followed by a conversion into slow X-mode at the O-cutoff, and, finally, a conversion into EBW in the vicinity of the upper-hybrid resonance. The EBE diagnostic [13] is based on the same physics.

For simulations, the beam was launched with an optimal angle (with O-X conversion efficiency more than 99%) near the “bean-shaped” plane. In this cross-section the magnetic field reaches its maximum, hence the number of trapped particles is close to zero. The frequency and the averaged magnetic field on axis are $\Omega = 140$ GHz and $B_0 = 2.2$ T, respectively, corresponding to $\omega \approx 2\omega_{ce}$ as the lowest (and dominating) resonance for damping. Reasonable plasma core parameters were chosen as $n_e(0) = 3 \times 10^{20}$ m$^{-3}$ and $T_e(0) = 1$ keV. The resulting ray trajectory is shown in Fig.2, along with the evolutions of $N_\perp$, $N_{||}$ and the pitch between the wave vector and the group velocity. Just to illustrate the features of EBW propagation, the ray trajectory shown is significantly prolonged after its complete absorption. The expected behavior of the refractive index is observed: after the conversion into EBW $N_\perp$ grows up to large values, $N_\perp \gg 1$, while $N_{||} \sim 1$ (Fig.2, top right). Note also that the pitch between the wave vector and the group velocity inverts its sign (Fig.2, bottom right), which is the sign of the “backward propagation” of EBWs. The power is absorbed near the axis, producing well localized deposition profile (not shown here).

LFS AND HFS ECE OBSERVATIONS

The code can also be applied to EC emission spectrum calculations and possibly gives more insight into non-thermal electrons. An important advantage of the code is the possibility to get information about the spatial location of the emission line together with its spatial width, which is the theoretical limit of the spatial resolution [14, 15]. The energy range of electrons, contributing in emission and reabsorption,
together with the relativistic and Doppler broadening, is calculated. This option is important for the problem of distinguishing the thermal and non-thermal contributions in an ECE spectrum, and can be useful for quantitative estimations of the deviation of the electron distribution function from a Maxwellian. To illustrate this, the results of simulations for both HFS and LFS observations, which are considered for the W7-X ECE diagnostic [15], are shown and discussed.

The beam collected by the ECE antenna is modelled by a bundle of rays, the intensity of the beam is distributed among the rays according to a Gaussian antenna pattern. For each frequency, $f$, the results are averaged over the $f \pm \Delta f/2$ range, where $\Delta f$ is the width of the corresponding channel in the radiometer (here $\Delta f = 0.3$ GHz is used).

Simulations are performed for $n_e(0) = 2 \times 10^{19}$ m$^{-3}$ with an almost flat profile near the axis, and for a peaked $T_e$ profile with $T_e(0) = 5$ keV. The magnetic field averaged on axis is taken to be $B_0 = 2.4$ T. The port for the main LFS ECE antenna is situated near the “bean-shaped” plane. It is assumed, that the antenna beam has a very slow divergence, of $1.2^\circ$, and the (beam) cross-section size at the antenna (initial beam waist) is 1 cm. The HFS antenna is assumed to be located at the opposite position. For operation at moderate densities, the refraction effects are almost negligible, and both sightlines should coincide.

The results of simulation are shown in Fig.3. One can see there the frequency range which contains the distinguishable information about the non-thermal electrons, and the theoretical limit of the spatial resolution of the ECE measurements. For LFS the emission line locations are very close to the “cold” resonance position (straight line $R_{ee} = R_{cb}$), and the spatial resolution, which is limited by relativistic broadening, is sufficient (about 5%). For the HFS measurements the situation is quite different: due to down-shifted emission, the same frequency gives the emission far from the “cold” resonance. The resolution is also not as high in comparison with the LFS case. Nevertheless, the (down-shifted) HFS emission contains the information about the non-thermal electrons (see Fig.3, right), and being complementary to the LFS one, provide a tool to estimate the weight of the non-Maxwellian fraction. Note, that in contrast with the radiative temperature, $T_{ee}$, the spatial locations of the emission lines with their width (resolution) and the velocity range of emitting electrons are not so sensitive to the electron distribution function. Numerical simulations with a bi-Maxwellian distribution function (5% of the supra-thermal fraction with $T_{eT}/T_{e0} = 3$) confirm it: Maxwellian and bi-Maxwellian distributions give quite different $T_{ee}$ from almost coincident emission line locations [15].
SUMMARY

The applications of the BRT code presented in this paper cover a broad area of problems in both heating (ECRH/ECCD/EBW) and diagnostics (ECE/EBE). Thanks to a general model adopted for the Hamiltonian in the tracing equations, the kinetic effects which lead to anomalous dispersion near the resonance are taken into account. Absorption and emissivity can be calculated for arbitrary electron distribution functions, being necessary for future coupling of the BRT code with a Fokker-Planck solver.

Macroscopic quantities such as the deposition profile or the radiative temperature can be decomposed in the contributions from trapped and passing electrons. This approach provides a better understanding of trapped particle effects on heating, emission, and CD in stellarators, not only in interpreting the experiments but also in preparing suitable plasma targets and magnetic configurations. The spatial width of the emission line, calculated together with the ECE spectrum, gives the theoretical upper limit to the spatial resolution of the ECE diagnostic. The energy range of electrons contributing to the absorption and emission is also calculated. Future applications include Cotton-Mouton polarimetry and predictions of the stray radiation level in case of incomplete plasma absorption and multiple reflections from the walls.

A graphical user interface front-end to the tracing code has been developed to assist the user in parameter set up, running of the code, and post processing of the data. The interface contains 2d and 3d plotting tools that display the results of the simulation.

References

[10] H.P. Laqua et al, “ECRH heating scenarios and in-vessel components at W7-X”, this workshop