Non-Gaussian beam tracing in inhomogeneous anisotropic plasmas

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Abstract. *The propagation of non-Gaussian beams is described in terms of the beam tracing asymptotic technique. The need to study non-Gaussian beams emerges in scenarios involving injection of waves in the plasma where the beams entering the plasma are not Gaussian or modify during the propagation in the plasma. The sequence for tracing arbitrary beams is established, involving the formulation of the decomposition of arbitrary electric field profiles into Gaussian-Hermite modes and the generalization of the beam width parameter. The effect of the phase-shift of the modes is analyzed within beam tracing and included in the description of the beam. We apply the above to the propagation of multi-mode beams in a simplified plasma geometry, where a comparison with an exact solution is possible.*

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INTRODUCTION

The propagation of EC waves in plasmas is described by Maxwell equations, a full solution of which is in general very hard. In case the wavelength is small compared to the scale length of inhomogeneity of the plasma, a simplification is achieved by using asymptotic methods: geometrical optics (ray tracing), complex eikonal (quasi-optics) or paraxial WKB (beam tracing). The state of the art in applications is mainly based on the simple, lowest-order Gaussian beams. However, the beam that enters the plasma might not always be Gaussian e.g. due to deformation of a steering mirror by extreme heat load, or it could be set up on purpose for reduction of the power density on the diamond window of the launching system. In this work, the beam tracing method is applied to the propagation of non-Gaussian beams and the sequence for tracing arbitrary beams is described. The simplified description through characteristic parameters (eg. width) is retained by generalizing the parameters existing for Gaussian beams. As an application, the propagation of a multimode EC beam in simplified geometry is studied, with a validation of the results based on the exact solution.

REVIEW OF ASYMPTOTIC METHODS

The pioneer asymptotic method is geometrical optics [1], where the wave field is described by the eikonal form $\mathbf{E}=\mathbf{A}e^{i\kappa s}$, with s the eikonal ($\mathbf{k}=\kappa\nabla s$) and $\kappa=\omega L/c$ for the short wavelength limit. For each ray one can determine the wave field by Hamiltonian ODEs, obtained by exploiting an asymptotic expansion of the solution sought, $\mathbf{A}=\Sigma_i \mathbf{A_i} k^{-i}$, and separating terms of different order. The zero-order terms result to the ray equations

$$\frac{d\mathbf{r}}{d\tau}=\frac{\partial H}{\partial \mathbf{k}}, \qquad \frac{d\mathbf{k}}{d\tau}=-\frac{\partial H}{\partial \mathbf{r}}, \tag{1}$$

where the dispersion relation plays the role of the Hamiltonian. The 1st-order gives the amplitude evolution along the ray

$$\frac{d|A_0|^2}{d\tau}+\left(\nabla\bullet\mathbf{v_g}+2\gamma\right)|A_0|^2=0, \tag{2}$$

where γ is the absorption coefficient. A limitation of ray tracing is that wave phenomena (eg. diffraction) are not taken into account, and in situations where these cannot be neglected, e.g. for focused beams, the result is inconsistencies near foci or caustics. Methods that refine geometrical optics, taking into account wave effects, have been developed. The complex eikonal method [2,3] considers a complex phase $\overline{s}=s+i\varphi$, with the imaginary part related to the beam profile. The corresponding complex solutions can be obtained by two different approaches (for more details see [2,3]). A significant simplification is achieved by using the pWKB beam tracing method [4], where the electric field has the same form as in quasi-optics but the amplitude expansion contains the intermediate order $k^{-1/2}$

$$\mathbf{A}=\Phi_{mn}\mathbf{A_0}-\frac{\partial\Phi_{mn}}{\partial x_j}\frac{iA_1^j}{\kappa^{1/2}}-\frac{\partial^2\Phi_{mn}}{\partial x_i\partial x_j}\frac{A_2^{ij}}{2\kappa}-\Phi_{mn}\frac{iA_3}{\kappa}. \tag{3}$$

The function $\Phi_{mn}(\xi_1,\xi_2)=H_m(\xi_1)H_n(\xi_2)e^{-\delta_{ij}\xi_i\xi_j/2}$ describes the transverse beam profile in a system of dimensionless coordinates (τ,ξ_1,ξ_2) associated with the beam, where τ is along and (ξ_1,ξ_2) are across the propagation, which can be identified as a geometrical-optics ray. Around the reference ray, the complex phase of the wave field is expanded in Taylor series and the coefficients are determined by an ODE emerging from the terms of order κ^{-1} in Eq. (3)

$$\frac{d\overline{s}_{\alpha\beta}}{d\tau}=\frac{\partial^2 H}{\partial x_\beta\partial k_\gamma}\overline{s}_{\alpha\gamma}+\frac{\partial^2 H}{\partial x_\alpha\partial k_\gamma}\overline{s}_{\beta\gamma}+\frac{\partial^2 H}{\partial k_\gamma\partial k_\delta}\overline{s}_{\alpha\gamma}\overline{s}_{\beta\delta}-\frac{\partial^2 H}{\partial x_\alpha\partial x_\beta}. \tag{4}$$

The coefficients $s_{\alpha\beta}=\mathrm{Re}(\overline{s}_{\alpha\beta})$ relate to the curvature radius ($\propto R^{-1}$) while $\varphi_{\alpha\beta}=\mathrm{Im}(\overline{s}_{\alpha\beta})$ relate to the beam width ($\propto W^{-2}$). The amplitude transport equation is similar to ray tracing, however here absorption is calculated on the central ray but refers to the whole beam. Based on the above, the general solution for the electric field is expressed as superposition of partial solutions

$$E_{mn}=\frac{C_{mn}}{\sqrt{\pi 2^{m+n} m!n!}}H_m(\xi_1)H_n(\xi_2)e^{i\kappa s-\kappa\varphi-i\Theta_{mn}}. \tag{5}$$

The advantages of beam tracing over the other asymptotic methods are worthy. With respect to ray tracing, diffraction is taken into account with a less number of equations. In comparison with quasi-optics, the beam is described by ODEs instead of PDEs and always in real space, which makes applications more straightforward.

SEQUENCE FOR TRACING NON-GAUSSIAN BEAMS

The sequence for the analysis of non-Gaussian beams amounts to first assigning proper initial conditions to all the beam-tracing variables, and then solve for the reference ray, the wave-front curvature and the beam width, as well as for the amplitude of each mode. For the first step, no difficulty appears as the beam-tracing variables are calculated on the reference ray and thus are common for all modes. For the initial amplitudes $C_{mn}|_{\tau=0}$, the initial field is decomposed into a series of Gaussian-Hermite modes $E(\xi_1,\xi_2)|_{\tau=0}=\Sigma_{mn}C_{mn}|_{\tau=0}\Phi_{mn}(\xi_1,\xi_2)$, where the expansion coefficients are the initial amplitudes

$$C_{mn}\big|_{\tau=0} = \int_{-\infty}^{\infty} E(\xi_1,\xi_2)\Phi_{mn}(\xi_1,\xi_2)d\xi_1 d\xi_2 \quad . \tag{6}$$

As an example, we consider the decomposition of a real electric field with a 1-D square profile. For this case, $C_n|_{\tau=0}$ is non-zero only for even values of n. In Fig. 1(a) we show the profile of the square beam, reconstructed according to the results of a decomposition using N=50 modes. The approximation is very successful, and increasing the number of modes makes it even better. In Fig.1(b) the coefficients of the even modes are plotted as a function of the indices 2n. The higher-order modes (n>25) have very small coefficients and do not play an important role.

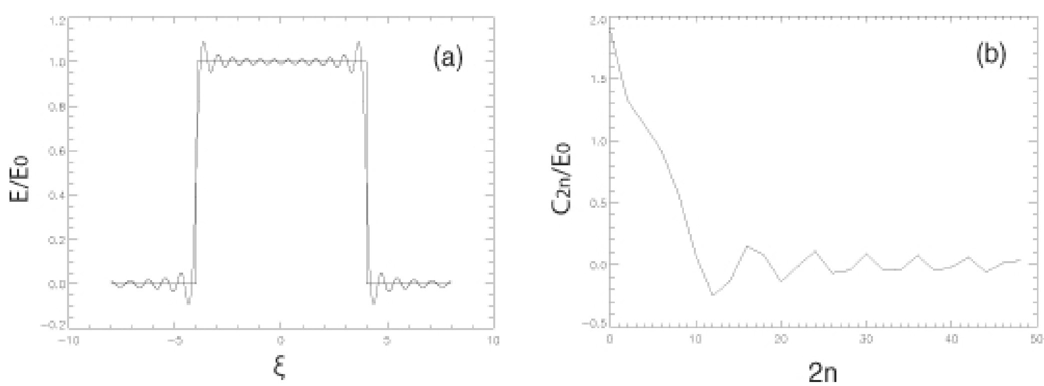

FIGURE 1. (a) Profile of the square beam, reconstructed according to the results of a decomposition using N=50 modes. (b) Coefficients of the even modes as a function of the indices 2n.

For the solution of the beam tracing equations and the amplitude transport, the formalism in [4] is mainly followed. An issue here is that the equations for the amplitudes are not the same for all modes, because the absorption coefficient depends on the wave-vector which is different for each mode, $\mathbf{k_{mn}}=\mathbf{k}-d\Theta_{mn}/d\tau\nabla\tau$, with $\mathbf{k}$ the wavenumber on the reference ray. Another issue is that the formalism [4] has met successful application to the case of a Gaussian beam, however for arbitrary beams a generalization is needed [5]. This results to a new form for the phase shift

$$\Theta_{mn} = \left(m+\frac{1}{2}\right)\int_0^\tau \mu\mathbb{N}d\tau' + \left(n+\frac{1}{2}\right)\int_0^\tau \nu\mathbb{N}d\tau' , \tag{7}$$

where $\mu,\nu,\mathcal{N}$ are functions of τ (more on the relation between $\mu,\nu,\mathcal{N}$ and the choice of beam coordinates is presented in [5]).

PARAMETERIZATION OF ARBITRARY BEAMS

A Gaussian beam can be described by few parameters, the most important of which are the width and the curvature radius. The width is usually defined as the distance from the maximum where a decrease of a factor of 1/e in the amplitude occurs, and the curvature radius is the radius of the spherical curve defined by the wavefront. In the case of non-Gaussian beams, these definitions usually fail in properly describing the beam and it is necessary to revise. This can be done by generalizing the parameters already defined for Gaussian beams, based on the moments of the electric field distribution, $\left\langle x^m y^n \right\rangle = \int_{-\infty}^{\infty} x^m y^n \left|E(x,y)\right|^2 dxdy$. The width is accurately described by a 2-D symmetric matrix $\hat{w}$, which is in general non-diagonal [6]. However, an "aligned" coordinate system can be found where $\hat{w}$ is diagonal, $\hat{w}^2=[\delta_{ij}w_{qi}^2]$, with $w_{qi}^2=2(\langle q_i^2\rangle-\langle q_i^1\rangle^2)/\langle q_i^0\rangle^2$ the generalized width per direction. For the case of a superposition of the lowest-order Gaussian (main mode) and a sum of higher-order modes, $\mathbf{E}=\mathbf{E_{00}}+\Sigma_{mn}\mathbf{E_{mn}}$, the width matrix in the frame (ξ_1,ξ_2) is diagonal and normalized to the principal (Gaussian) widths. The 1st-order moment for ξ_1 is

$$\left\langle \xi_1^1 \right\rangle = \frac{\sum\limits_{mnkl} |r_{mn}||r_{kl}|\delta_{nl}\cos\left(\theta_{mn}-\theta_{kl}\right)\left(\frac{m}{2}\delta_{m,k+1}+\frac{k}{2}\delta_{m,k-1}\right)+2\sum\limits_{mn}|r_{mn}|\cos\theta_{mn}\sqrt{\frac{m}{2}}\delta_{m,1}\delta_{n,0}}{1+\sum\limits_{mn}|r_{mn}|^2}, \quad (8)$$

where $r_{mn}=C_{mn}/C_{00}$ is the ratio of the amplitude of the mode (m,n) over the main mode and $\theta_{mn}=\Theta_{mn}-\Theta_{00}-\arg(r_{mn})$ is the total phase-shift of the mode (m,n). The result for ξ_2 can be obtained by interchanging m with n and k with l. The second order moment for ξ_1 and accordingly for ξ_2 by interchange, is

$$\left\langle \xi_1^2 \right\rangle = 1+\frac{\sum\limits_{mnkl} |r_{mn}||r_{kl}|\delta_{nl}\cos\left(\theta_{mn}-\theta_{kl}\right)\left(\sqrt{m(m-1)}\delta_{m,k+2}+\sqrt{k(k-1)}\delta_{m,k-2}\right)+2\sum\limits_{mn}|r_{mn}|\cos\theta_{mn}\sqrt{m(m-1)}\delta_{m,2}\delta_{n,0}}{1+\sum\limits_{mn}|r_{mn}|^2}. \quad (9)$$

NON-GAUSSIAN EC PROPAGATION IN A PLASMA SLAB

We study the perpendicular cold-plasma propagation of a multimode EC beam in a simplified geometry (slab). The plasma is confined within $-\alpha\leq x\leq\alpha$ and magnetized along z, and all the plasma properties are functions only of x. The beam is launched at $x_0=\alpha$ in the negative direction $\{k_{x0}<0,\ k_{y0}=k_{z0}=0\}$, with electric field the superposition of a Gaussian (0,0) and a higher-order mode (m,n). The analytic solution of the beam tracing equations for this problem has been obtained in [7]. The simplification here is that the dispersion relation reduces to a quadratic form $H=(D_{ij}k_ik_j-1)/2$ (i,j=x,y,z) with a diagonal dispersion tensor $D=[\delta_{ij}D_{ij}^M]$ (M=O,X refers to the wave polarization). The Hamiltonian does not depend on y,z and thus k_y, k_z remain constant, while k_x is fully determined by the dispersion relation, so the only ray-tracing equation of interest is the one for $dx/d\tau$

$$\frac{d\mathbf{r}}{d\tau}=\frac{\partial H}{\partial \mathbf{k}}, \qquad \frac{dx}{d\tau}=\frac{\partial H}{\partial k_x}=\sqrt{D_{xx}^M}, \quad (10)$$

In this context, τ is a function only of x and the beam coordinates are y/W_y, z/W_z, with W_y, W_z the Gaussian widths. The Gaussian widths are expressed in terms of the initial widths and curvatures

$$\left(\frac{W_j}{W_{j0}}\right)^2 = \left(1+\frac{l_j}{\kappa R_{j0}}\right)^2 + \left(\frac{l_j}{\kappa W_{j0}}\right)^2 , \tag{11}$$

with $c l_j/\omega = -\int_{x_0}^{x} D^M_{jj}(x')/D^M_{xx}(x')dx'$. Under the choice $\arg(r_{mn})=0$, Eq. (7) yields for the total phase shift

$$\theta_{mn} = \frac{c^2}{\omega^2}\left(m\int_a^x \frac{D^M_{yy}}{W_y^2}dx' + n\int_a^x \frac{D^M_{zz}}{W_z^2}dx' \right), \tag{12}$$

and the generalized width in the ξ_1-direction reads

$$w_1^2 = 1 + 2m\frac{|r_{mn}|^2}{1+|r_{mn}|^2} + \frac{2|r_{mn}|}{1+|r_{mn}|^2}\cos\theta_{mn}\delta_{n,0}\left(\sqrt{m(m-1)}\delta_{m,2} - m\frac{2|r_{mn}|}{1+|r_{mn}|^2}\cos\theta_{mn}\delta_{m,1} \right) , \tag{13}$$

with the result being interchangeable for ξ_2. The only cases where θ_{mn} has an effect on w are (m,n)=(1,0),(2,0),(0,1),(0,2). In all the other cases, the generalized width is proportional to the principal width (r_{mn} does not vary along the propagation in the absence of absorption), increases with (m,n) and does not depend on θ_{mn}. In the simulations, we use a=1m, $\omega/2\pi$=140GHz ($1st O or 2nd X), $\omega_p^2/\omega^2=0.9-0.4(x/\alpha)^2$, W_0=3.02cm and R_0=-82cm. We specify r_{mn} in terms of the ratio of the power density of the mode over the total power density, $\varepsilon_{mn}=|r_{mn}|^2/(1+|r_{mn}|^2)$. In Fig. 2 the generalized widths are compared to the Gaussian ones for O-mode propagation and (a) (m,n)=(2,1), (b) (m,n)=(0,2), (in both cases ε_{mn}=0.1). In Fig. 2(a), w_q is constantly larger from W_q in both directions. The two widths in this case are actually proportional, because the generalized width does not depend of the phase shift. In the case of Fig. 2(b), the width in the y-direction coincides with W_y because m=0, while in the other direction the effect of the phase shift is obvious.

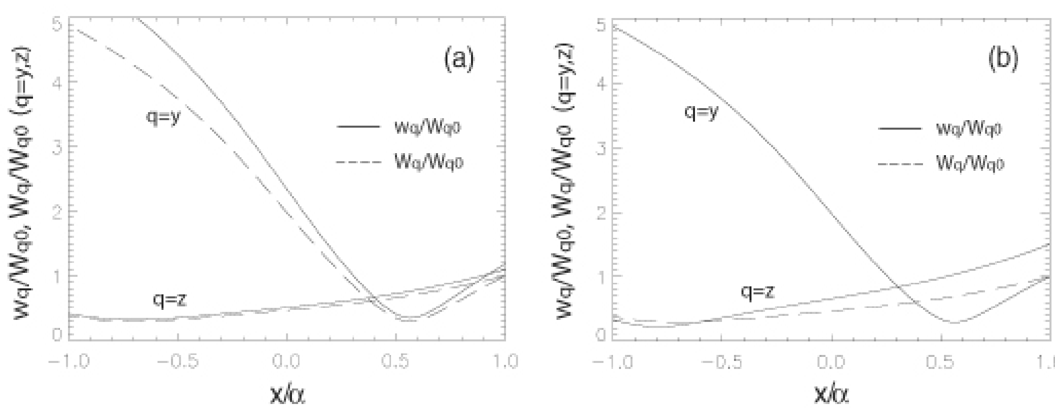

FIGURE 2. The generalized widths in comparison with the Gaussian ones for O-mode propagation and (a) (m,n)=(2,1), (b) (m,n)=(0,2) (in both cases ε_{mn}=0.1).

In Fig. 3(a) the evolution of θ_{mn} is shown for both O- and X-mode with (m,n)=(0,2), ε_{mn}=0.1. The variation is significant in both cases. The phase shift exhibits a different behaviour for the two cases, which is due to the dielectric tensor and the Gaussian widths having different functional forms. The problem of wave propagation considered above is addressed in simplified plane geometry that allows a direct solution of the electric field without resorting to asymptotic techniques. The numerical implementation is done by the code ECPROP [8], which calculates the propagation for nearly perpendicular injection in cold plasma. In Fig. 3(b) the results of our code are compared to ECPROP, and the agreement between the results is very good. Notice here that using Eq. (7) without generalizations, the disagreement is significant.

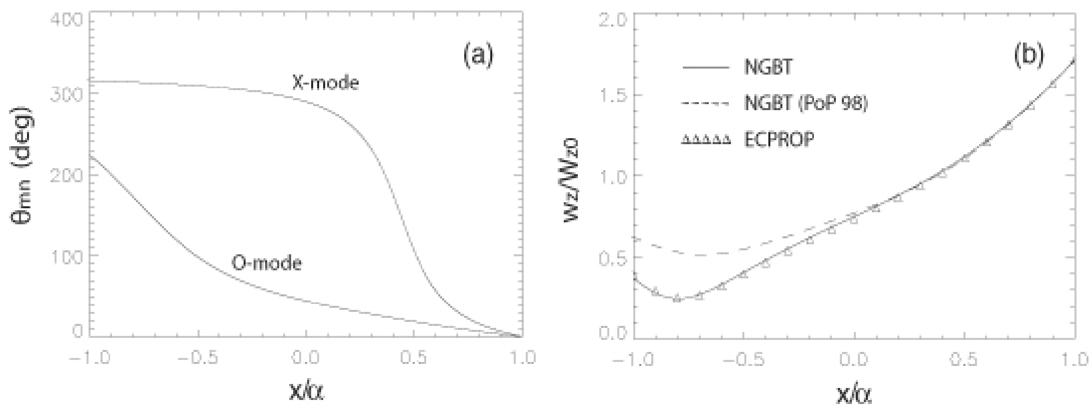

FIGURE 3. (a) Total phase-shift for O-, X-mode with (m,n)=(0,2). (b) Generalized width in the z-direction, as calculated by NGBT (based also on the formalism of [4]) and ECPROP, for O-mode with (m,n)=(0,2) and ε_{mn}=0.2.

CONCLUSIONS

The propagation of non-Gaussian beams was formulated in terms of the pWKB beam-tracing asymptotic method. The need to study non-Gaussian beams emerges from the fact that, in ECRH/ECCD scenarios, the beams entering the plasma might not always be Gaussian or might be modified in the plasma. We applied the method to the propagation of the simplest non-Gaussian EC beam in a plasma slab. The results were benchmarked against the code ECPROP and good agreement was found in all cases. The work presented here opens the way for the construction of a theory for the description of modifications in the beam profile due to localized, asymmetric or inhomogeneous absorption.

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