Behavior of zonal flows and transport in the high-ρ_s-regime

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Introduction

We have been studying numerically the interaction between drift waves and zonal flows in a nonlinear self-consistent sheared-slab resistive drift wave system based on the Hasegawa-Wakatani equations in order to examine how plasma zonal flows behave in this type of environment, specifically concerning the details of their formation above a certain threshold parameter ρ_{crit} (with ρ being defined as the ratio between the ion sound Larmor radius ρ_s and the length scale of maximal drift wave growth at a given parallel shear length marked by $L_\perp$) and their transport bifurcations which we have observed for the first time in such a system. These bifurcations correlate with density corrugations which are related to an asymmetry in the shape of the zonal flows, yielding flattened positive flows (those in the ion diamagnetic drift direction) and steepened negative flows.

Drift wave turbulence is not only crucial within the high gradient tokamak edge (close to the outer stretches of the internal transport barrier called the H-mode), but also in a number of other systems, most prominently in the atmospheres of gas giants where geostrophic modes act as a drift wave analagon in the case of planetary turbulence.

The basis of our work is provided by a turbulent cold-ion resistive sheared-slab Hasegawa-Wakatani drift-wave system which is simulated using NLET, a two-fluid code developed at IPP [3]:

$$d_t n = d_t \nabla_\perp^2 \phi \qquad (1)$$

$$\rho^{-3} d_t \nabla_\perp^2 \phi = -\partial_\parallel^2 (\phi - n) \qquad (2)$$

Transport scales

With eqns. (1) & (2), the general growth rate of the drift modes in the shearless, non-adiabatic case can be determined as $\gamma = \Im(\omega) \propto \left[k_\perp^2 + k_\parallel^2 \left(\frac{1}{k_\perp k_y} + \frac{k_\perp}{k_y}\right)^2\right]^{-1}$ which, when approximated by $\gamma = \omega *^2 / \omega_\parallel = k_\perp^2 / \left(k_\parallel^2 / (\rho^{-3} k_\perp^2)\right) = \rho^{-3} k_\perp^4 / k_\parallel^2$, can be used to come up with a mixing length estimate, Prandtl-style.

The resulting heat diffusion coefficient $D = \gamma / \vec{k}_\perp^2$ includes $k_\perp$. There exist two potential ways of determining this quantity, either via ρ_s or via $L_\perp$. Thus, e.g. in the ρ_s-dominated high-ρ-regime, $D_{L_\perp}|_{k_\perp \hat{=} \rho_s} = \frac{\gamma_{L_\perp}}{k_{L_\perp}^2}|_{k_\perp \hat{=} \rho_s} \propto \rho^{-2}$ and $D_{\rho_s}|_{k_\perp \hat{=} \rho_s} = \frac{\gamma_{\rho_s}}{k_{\rho_s}^2}|_{k_\perp \hat{=} \rho_s} \propto \rho^0$ are found.

Similar results can be gained in the $L_\perp$-dominated low-ρ-regime as well. Notably, a transition regime has to exist as well, located roughly around $\rho \approx 0.15 - 0.20$. This coincides very well with the main onset of zonal flow formation. A very simple relation between the two scales can be derived:

$$\frac{D_{L_\perp}}{D_{\rho_s}} = \rho^{-2} \tag{3}$$

Notably, this correlates with an absolutely flat D_{ρ_s}-plateau in the high-ρ-regime - at least in theory.

High-ρ-regime

In numerical experiments, it has been proven problematic to achieve the high-ρ-regime necessary for fully developed zonal flows as the resource requirements increase with ρ^3 (since transport times scale with $k^{-2}D^{-1}$ and thus ρ^4 while the drift wave scale is merely proportional to ρ). In addition, much higher resolutions have been required due to convergence issues. These simulations have been performed on the HPC-FF and Helios clusters (in addition to the local Bob cluster), and have finally yielded the expected behavior of transport in the zonal flow regime: A near-perfect plateau in the diffusion coefficient - and thus of turbulent transport - in units of ρ_s, as numerous sets of parameter scans over ρ have shown.

Transport bifurcations and wavelength-dependency

Our results are of no little importance as they show the first example of transport bifurcations - bifurcations being defined as a forking of one steady state into two above a certain critical parameter, even though both states may be realized at the same time at different locations - with two stable density gradients within self-consistent nonlinear drift wave simulations, while also yielding a qualitative explanation for these findings.

These bifurcations are most easily perceptible as density corrugations - correlating with two different stationary transport states with regions of low diffusivity and high gradients close to the sharply concentrated negative flows (those in the electron diamagnetic drift direction) and broad low-gradient-regions with increased diffusivity close to the flows in the ion diamagnetic drift direction. Or, in other words: The bifurcations co-exist with a pronounced zonal flow asymmetry.

Typical time scales for the emergence of this flow structure come to lie around $\sim O(10^1)$ for $\rho \approx 0.28$, growing by approx. one order of magnitude for every doubling of ρ). This is quite possibly the reason why these drift-wave-based zonal flows have not been found in earlier studies [1] (in addition, the advances in technology and subsequent increases in resolution will have helped as well).

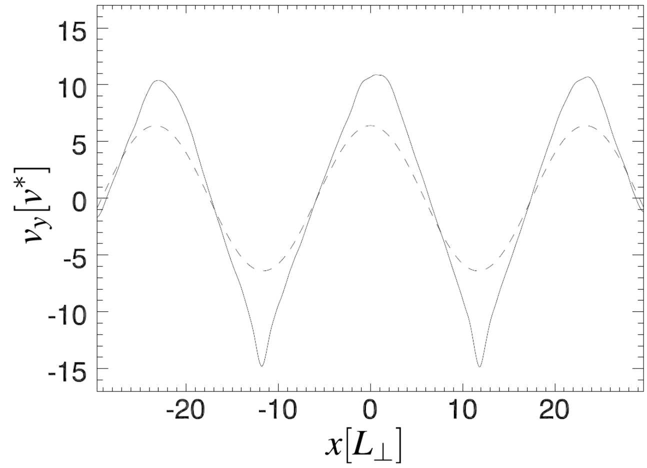

Figure 1: Artificially prescribed sinusoidal zonal flow pattern (dashed line) and resulting final asymmetric same-wavelength state (solid line) in units of $L_\perp/t_\perp$ plotted vs. radius in units of $L_\perp$.

There is one crucial difference between these drift wave zonal flows and their ITG-based counterparts: Drift wave zonal flow wavelength is not prescribed by the system, but rather randomly chosen, meaning that it is possible to enforce any zonal flow wavelength without risking subsequent countermeasures by the system - as shown above.

Qualitative mechanism

It is possible to derive an analytical term for the interaction between the drift waves and the zonal flows utilizing the general drift wave action invariant N for the wave packet intensity in the form introduced in [2],

$$\partial_t N_{\vec{k}} = -\nabla_{\vec{x}} \left(N_{\vec{k}} \cdot \vec{v}_{gr,\vec{k}} \right) - \nabla_{\vec{k}} \left(N_{\vec{k}}(x) \cdot \dot{\vec{k}}(\vec{x},\vec{k}) \right) \tag{4}$$

Here, the second term comes from the local influence of the shear flow on the wave number [5] via $\dot{\vec{k}} = -\vec{\nabla}_x \vec{v} \cdot \vec{k}_0$). From this follows directly that the turbulence repulses negative flows, with the opposite holding true for their positive counterparts. Thus, the flows act in pretty much the same way as forcefields, being capable of changing the radial drift wave wavenumbers, thereby reducing transport (and thus turbulence levels) around the negative flows.

But in any equilibrium, the transport balance $\partial_x \Gamma(x) = 0$ has to still be maintained, requiring heightened gradients around the negative flows in order to counterbalance such a drift-wave-related reduction of transport. Similarly, gradients in the positive flow area are reduced, leading to the formation of stepped density gradients (with an increased drift mode generation rate near the flow minima due to the steepened gradients). Since the drift waves are repelled by the negative flows, they exhibit Reynolds stresses fueling, in return, the flow. The associated drift wave carry-off causes not only a deepening of the negative flows but also a broadening of their positive counterparts, leading to the flow asymmetry described above.

Zonal flow transition

Interestingly, the zonal flow regime transition can be traced to two factors of influence, a drift wave repulsion effect (which is independent of ρ) as well as the resonant surfaces, which lead to both repulsion and amplification of drift waves, with the repulsion growing weaker for higher values of ρ.

Generally, the drift waves are accelerated up the flow gradient due to the aforementioned repulsion effect. Now, for low values of ρ, strong repulsion by the resonant surfaces (together with dispersion broadening) leads to a longer growth time for the drift waves at the low-v_y-side of the resonant surfaces than those at the high-v_y-side, as the former are forced to stay fore a more extensive period of time in close vicinity to the resonant surfaces. This results in predominantly negative Reynolds stress and thus zonal flow damping.

The opposite is true for high values of ρ. While the dispersion broadening effect remains strong, acceleration up the flow gradient is now dominant over the resonant surface effect, leading to predominantly positive Reynolds stresses and thus an excitation of zonal flow growth. The transition between both cases occurs around $\rho \approx 0.5$.

Summary

We have analyzed sheared-slab drift wave turbulence systems, finding both a robust plateau in turbulent transport within the high-ρ zonal flow regime and a transport bifurcation for the resulting drift wave zonal flows. These two distinctly different transport states correlate with an asymmetry in the zonal flow pattern - a robust finding that has been verified within a considerable parameter range as well as having been explained qualitatively. In addition, both the distinct differences between the zonal flow and flowless states have been commented on and a peculiar characteristic of drift wave based zonal flows has been found, exhibiting a complete lack of any pre-described flow wavelength.

References

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