

Transport-driven toroidal rotation in the tokamak edge

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The interaction of passing-ion drift orbits with spatially-inhomogeneous but purely diffusive radial transport is demonstrated to cause spontaneous toroidal spin-up in a simple model of the tokamak edge. Physically, major-radial orbit shifts cause orbit-averaged diffusivities to depend on v_{\parallel} , including its sign, leading to residual stress. The resulting pedestal-top intrinsic rotation scales with T_i/B_{θ} , resembling typical experimental scalings. Additionally, an inboard (outboard) X-point enhances co- (counter-) current rotation.

Rotation patterns strongly affected by turbulent momentum transport are broadly observed in nature, for example in atmospheric flows, stellar interiors, and accretion disks [1]. Laboratory tokamak plasmas are observed to rotate toroidally in the absence of applied torque, with edge rotation directed with the plasma current (co-current), often proportional to plasma stored energy W over plasma current I_p , and reaching tenths of the ion thermal speed v_{ti} [2, 3]. Such intrinsic rotation is of practical as well as fundamental interest, since it stabilizes certain instabilities [4] and contributes to a sheared radial electric field E_r , believed to suppress turbulent transport [5]. Intrinsic rotation is of special importance for the next-generation tokamak ITER, since α -heating (nuclear fusion) applies no torque [6].

The intriguing experimental findings have triggered a broad theoretical search for the spontaneous rotation's physical origins. Although neoclassical (collisional transport) effects have been considered [7, 8], extensive experimental evidence indicates that turbulence dominates momentum transport [2, 9]. Numerical efforts have investigated turbulent momentum transport in the core, both linearly [10, 11] and nonlinearly [12]. Models for intrinsic rotation, also primarily core-focused, have treated quasilinear approximations [10, 11, 13–16], effects of inhomogeneity of the confining magnetic field \mathbf{B} [11, 14], nonresonant correlations between the fluctuating radial $\mathbf{E} \times \mathbf{B}$ drift $\tilde{v}_{E,r}$ and parallel velocity \tilde{v}_{\parallel} [17], and Stringer spin-up type effects [18]. Some scrape-off-layer (SOL) effects have been entertained [19, 20] without systematic consideration of the confined plasma's response. Symmetry arguments have strongly constrained gyrokinetic momentum transport in radially-local formulations [21].

Momentum transport in the tokamak edge presents particular challenges for theory. The turbulence is strong, with statistics very different from quasilinear estimates [22]. It is also strongly anisotropic, with parallel fluctuation length L_{\parallel} two orders of magnitude larger than the radial length scale L_{\perp} characterizing toroidal velocity and equilibrium plasma variation [23–25]. Since parallel fluctuation gradients $k_{\parallel} \sim 1/L_{\parallel}$ and the corresponding forces are accordingly weak, turbulently-accelerated \tilde{v}_{\parallel} and the resulting nondiffusive effects [15–17] are smaller than simple diffusive momentum transport by $k_{\parallel}L_{\perp} \ll 1$

for realistic edge parameters. Most of these effects are further reduced, actually proportional to a symmetry-breaking $\langle k_{\parallel} \rangle \ll \langle k_{\parallel}^2 \rangle^{1/2} \sim 1/L_{\parallel}$ [16, 21]. Since \mathbf{B} varies on the scale length of the major radius R_0 , the resulting momentum transport effects scale relative to simple diffusion as $L_{\perp}/R_0 \ll 1$ in the edge [11, 14]. Further, the interaction of edge and SOL makes the problem inherently radially nonlocal. For example, the amplitude of the (unnormalized) turbulent fluctuating potential decreases in the radial direction on a short length scale $L_{\phi} \sim L_{\perp}$ [26]. Given these experimental facts, the present work analyzes a simplified, purely-diffusive kinetic transport model, setting parallel acceleration identically to zero but retaining a model edge and SOL, passing-ion drift orbit excursions, and spatial variation of the diffusivity, finding differential transport of co- and counter-current ions to cause residual stress and consequent intrinsic rotation levels similar to those seen in experiment.

Analysis begins with a model axisymmetric drift-kinetic transport equation for the ions,

$$\partial_t f_i + v \partial_y f_i - \delta v^2 (\sin y) \partial_x f_i - D(y) \partial_x (e^{-x} \partial_x f_i) = 0, \quad (1)$$

representing an ensemble-averaged $L_{\perp}/a, 1/q, a/R_0, cE_r/B_{\theta}v_{ti} \ll 1$ reduction of [27] in a radially-thin simple-circular magnetic geometry with reference poloidal, toroidal and total magnetic field strengths B_{θ} , B_{ϕ} , and B_0 , minor radius a , safety factor $q \doteq aB_{\phi}/R_0B_{\theta}$, and positive toroidal field [28]. The ion parallel distribution function $f_i(x, y, v, t)$ is normalized to pedestal-top ion density over thermal speed $n_i|_{\text{pt}}/v_{ti}|_{\text{pt}}$ and the parallel velocity v is normalized to $v_{ti}|_{\text{pt}}$, positive for co-current motion. The radial position x , poloidal position y , and time t are respectively normalized to L_{ϕ} , a , and the ion transit time $aB_0/B_{\theta}v_{ti}|_{\text{pt}}$. Transport by turbulent fluctuations is modeled with an inhomogeneous turbulent diffusivity, normalized to $L_{\phi}^2 B_{\theta}v_{ti}|_{\text{pt}}/aB_0$ and assumed separable, with arbitrary poloidal dependence $D(y)$, decaying exponentially in x . Although phenomenological, the assumed purely-diffusive transport provides an important check on the common belief that nondiffusive transport is required for intrinsic rotation. The dimensionless parameter $\delta \doteq q\rho_i|_{\text{pt}}/L_{\phi}$, with ρ_i the thermal ion gyroradius, indicates the passing-ion orbit width relative to the radial turbulence inhomogeneity, taking values

around 1/4 for typical ASDEX-Upgrade (AUG) H-mode parameters [29]. Collisions are neglected, a reasonable approximation if pedestal-top ions escape without experiencing a collision, roughly for $\nu_{ii}\tau_{cr} < 1$, with ν_{ii} the velocity-dependent ion collision rate and τ_{cr} the pedestal ion stored energy over the ion heat flux. For thermal pedestal-top ions, $\nu_{ii}\tau_{cr}$ takes values around 1 for typical H-modes in AUG, JET, and DIII-D, so superthermal pedestal-top ions tend to exit the plasma collisionlessly while subthermal ones do not. Boundary conditions are $f_i(-\infty, y) \rightarrow f_{i0}(v)$, $f_i(\infty, y) \rightarrow 0$, $f_i(x < 0, y_0) = f_i(x < 0, y_0 + 2\pi)$, $f_i(x > 0, y_0, v > 0) = 0$, and $f_i(x > 0, y_0 + 2\pi, v < 0) = 0$, with y_0 the poloidal X-point angle. Eq. (1) is invariant to a rigid toroidal rotation v_{rig} , normalized to $v_{ti}|_{pt}B_\phi/B_0$ and positive for co-current motion, and trivially conserves a simplified toroidal angular momentum $\int (v + v_{rig})f_i dv$, density $\int f_i dv$ and energy $\int (1 + v^2/2)f_i dv$.

Eq. (1) can be approximately analytically solved v -by- v in steady state for both large and small effective diffusivity D_{eff} , results agreeing for $D_{eff} \approx 1$. The solution procedure is briefly described here, details given in [28]. Since v appears only as a parameter, v -dependent variable transforms can greatly simplify Eq. (1). First, use new spatial variables

$$\bar{x} \doteq x - \delta v (\cos y - \cos y_0), \quad (2a)$$

$$\bar{y} \doteq D_{y0}^{-1}(v) \int_{y_0}^y D(y') e^{-\delta v (\cos y' - \cos y_0)} dy', \quad (2b)$$

with $D_{y0}(v) \doteq \int_{y_0}^{y_0+2\pi} D(y') \exp(-\delta v (\cos y' - \cos y_0)) dy'$. Physically, \bar{x} is a drift-surface label and D_{y0} an orbit-averaged diffusivity. Next, for $v < 0$, take $\bar{y} \rightarrow 1 - \bar{y}$. Finally, transform from \bar{x} to $u \doteq e^{\bar{x}/2}$, obtaining

$$\partial_{\bar{y}} f_i = (D_{eff}/4) (\partial_u^2 f_i - u^{-1} \partial_u f_i) \quad (3)$$

for $f_i(u, \bar{y}, v)$, in which $D_{eff}(v) \doteq D_{y0}(v)/|v|$. Boundary conditions are now $f_i(0, \bar{y}) = f_{i0}(v)$, $f_i(\infty, \bar{y}) \rightarrow 0$, $f_i(u < 1, 0) = f_i(u < 1, 1)$, and $f_i(u > 1, 0) = 0$. The normalized flux of particles with velocity v through any closed poloidal contour, $\Gamma(v)$, takes the simple form

$$\Gamma(v) = -\frac{1}{2} D_{y0}(v) u^{-1} \int_0^1 \partial_u f_i d\bar{y}, \quad (4)$$

evaluated at any constant $u \leq 1$.

Eq. (3) shows the original problem to reduce to a one-parameter family of otherwise-identical differential equations. Remarkably, the spatially-constant effective diffusivity D_{eff} depends not only on the magnitude of v , but also on its sign! Physically, as shown in Fig. 1, this symmetry breaking results from the fact that co- (counter-) current ions' drift orbits are displaced major-radially outwards (inwards). For the typical case of turbulent diffusivities larger at the outboard, counter-current ions experience larger orbit-averaged diffusivities. Preferentially

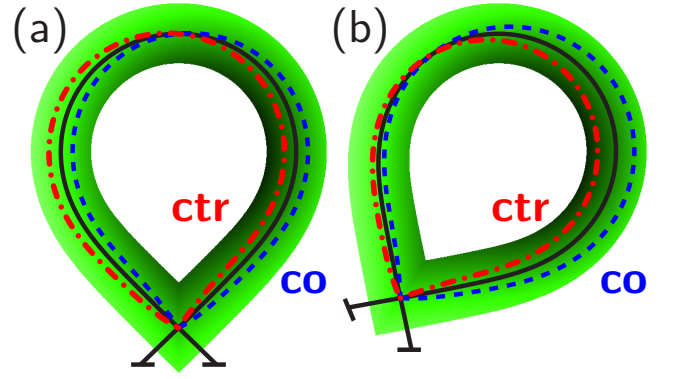


Figure 1. (color online). Co- and counter-current passing ion drift orbits over turbulence, plotted for a straight-down (a) and inboard (b) X-point. Darker shading indicates stronger diffusivity. Co- (ctr-) orbits are displaced major-radially outward (inward) as shown, regardless of the sign of B_ϕ or I_p .

exhausting counter-current ions represents a co-current residual stress, although momentum transport at any given spatial point is purely diffusive.

In solving Eq. (3), a Laplace transform approach similar to [30] yielded the exact Green's function,

$$G(u, \xi, \tau) = \frac{2u}{D_{eff}\tau} \exp\left(-\frac{\xi^2 + u^2}{D_{eff}\tau}\right) I_1\left(\frac{2u\xi}{D_{eff}\tau}\right), \quad (5)$$

in terms of which the solution may be written as

$$f_i(u, \bar{y}) = f_{i0} e^{-u^2/D_{eff}\bar{y}} + \int_0^1 f_i(\xi, 0) G(u, \xi, \bar{y}) d\xi. \quad (6)$$

A first-order iterative approximation $f_i^{(1)}$, obtained using $f_i(\xi, 0) \rightarrow f_{i0} \exp(-\xi^2/D_{eff})$ in Eq. (6), was demonstrated to yield an approximate normalized flux $\Gamma/f_{i0}|v|$ with absolute error strictly less than $\min(0.58D_{eff}^{1/2}(1 - e^{-1/D_{eff}}), 0.75/D_{eff}^{3/2})$, tight bounds for large D_{eff} . For small D_{eff} , a two-region solution may be used, representing f_i with a Fourier series for $u < 1$ (edge) and Laplace transforming for $u > 1$ (SOL), requiring continuity in f_i and $\partial_u f_i$ at $u = 1$, except possibly at the single point $u = 1, \bar{y} = 0$. The resulting edge and SOL ODEs possess explicit solutions in terms of modified Bessel functions. Slightly generalizing [31], continuity at $u = 1$ then requires the edge solution to satisfy

$$f_i \approx -\frac{1}{2} D_{eff}^{1/2} \frac{1}{\sqrt{\pi}} \int_0^{\bar{y}} \frac{\partial_u f_i dy'}{\sqrt{\bar{y} - y'}} - \frac{1}{8} D_{eff} \int_0^{\bar{y}} \partial_u f_i dy' \quad (7)$$

at $u = 1$. The resulting dense matrix for the Fourier coefficients has been solved numerically at various D_{eff} , retaining 10000 modes in \bar{y} .

The two approximate solutions,

$$\frac{\Gamma}{f_{i0}|v|} \approx \begin{cases} D_{eff}/(1 + a_1 D_{eff}^{1/2} + a_2 D_{eff}), & D_{eff} \lesssim 1 \\ -\frac{1}{2} D_{eff} \int_0^1 \partial_u|_{u=1} (f_i^{(1)}/f_{i0}) d\bar{y}, & D_{eff} \gtrsim 1 \end{cases}, \quad (8)$$

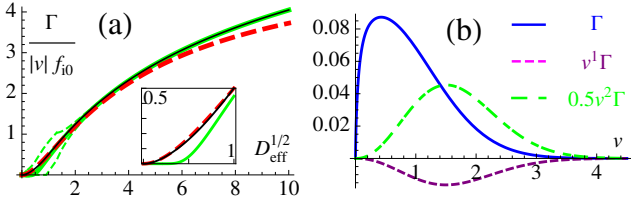


Figure 2. (color online). (a) normalized flux as a function of the v -dependent effective diffusivity, with uniform approximation (thin solid black), small- D_{eff} approximation (thick dashed red), large- D_{eff} approximation (thick solid green), and large- D_{eff} error bounds (thin dashed green). (b) speed distribution of particle, momentum and parallel heat fluxes, assuming a Maxwellian at the inner boundary.

are well-approximated for all D_{eff} by

$$\frac{\Gamma}{f_{i0}|v|} \approx \frac{1}{4} \ln \left(1 + \sum_{j=2}^8 c_j D_{\text{eff}}^{j/2} \right) \approx \ln \left(1 + \frac{D_{\text{eff}}}{e^\gamma} \right), \quad (9)$$

with $a_1 = 0.8224$, $a_2 = 0.1763$, $c_2 = 4$, $c_3 = -4a_1$, $c_4 = 4a_1 + e^{4/(1+a_1+a_2)} - 5 - 9e^{-4\gamma}$, $c_5 = c_7 = 0$, $c_6 = 8e^{-4\gamma}$, $c_8 = e^{-4\gamma}$, and Euler's constant $\gamma \approx 0.5772$. The second approximation is used for the simplified explicit forms in Eqs. (11)–(12) and corresponding plots, the first for all other plots. The results of Eqs. (8) and (9) are plotted in Fig. 2(a), along with the large- D_{eff} error bounds.

Explicit forms for normalized particle, momentum, and parallel heat fluxes may now be obtained for any specified $f_{i0}(v)$ and $D(y)$. Assuming a Maxwellian $f_{i0}(v) = e^{-v^2/2}/(2\pi)^{1/2}$ and simple ballooning form $D(y) = D_0(1 + d_c \cos y)$, thus

$$D_{\text{eff}}(v) = 2\pi D_0 e^{\delta v \cos y_0} [I_0(\delta v) - d_c I_1(\delta v)] / |v|, \quad (10)$$

the relevant flux moments may be reasonably approximated for small δ as

$$\Gamma^p \doteq \int_{-\infty}^{\infty} \Gamma dv \approx \sqrt{\frac{2}{\pi}} g_1, \quad (11a)$$

$$\Pi \doteq \int_{-\infty}^{\infty} v \Gamma dv \approx 8\delta \sqrt{\frac{2}{\pi}} \left(\cos y_0 - \frac{d_c}{2} \right) (g_3 - g_5), \quad (11b)$$

$$Q_{\parallel} \doteq \int_{-\infty}^{\infty} \frac{v^2}{2} \Gamma dv \approx \sqrt{\frac{2}{\pi}} g_3, \quad (11c)$$

in which $g_p(D_0) \doteq \ln(1 + 2\pi D_0 / e^\gamma p^{1/2})$. The integrands are plotted in Fig. 2(b), summed over the sign of v . (All plots use representative AUG H-mode values $D_0 = 0.033$, $d_c = 0.8$, $y_0 = -5\pi/8$, and $\delta = 0.28$.)

The total momentum flux, incorporating v_{rig} , is just $v_{\text{rig}}\Gamma^p + \Pi$. Since toroidal rotation damping is very weak [32], a vanishing momentum input implies that momentum flux must also vanish, resulting in the pedestal-top

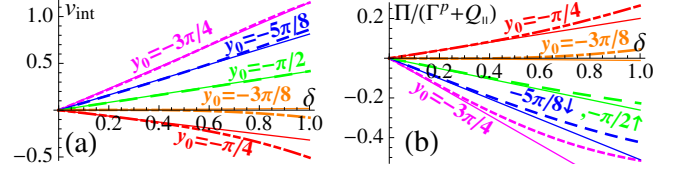


Figure 3. (color online). Normalized intrinsic rotation velocity v_{int} (a) and unbalanced NBI fraction $\Pi/(\Gamma^p + Q_{\parallel})$ (b), plotted as functions of drift orbit width δ for several values of poloidal X-point angle y_0 , with numerical integrals (thick dashed) and analytical approximations (thin solid).

intrinsic rotation rate

$$v_{\text{int}} = -\frac{\Pi}{\Gamma^p} \approx 8\delta (d_c/2 - \cos y_0) \frac{g_3 - g_5}{g_1}, \quad (12)$$

plotted in Fig. 3(a). Alternatively, one may balance an applied NBI torque with the outward momentum flux resulting from the NBI-driven ion heat flux. For zero pedestal-top toroidal rotation, $v_{\text{rig}} = 0$, one must set the unbalanced NBI fraction $f_{\text{unb}} \doteq (P_{\text{NBI}}^{\text{co}} - P_{\text{NBI}}^{\text{ctr}})/P_{\text{NBI}}$ to

$$f_{\text{unb}} = \frac{f_c}{2f_{\text{NBI}}} \frac{B_\phi}{B_0} \frac{v_{\text{NBI}}}{v_{ti}|_{\text{pt}}} \frac{\Pi}{\Gamma^p + Q_{\parallel}}, \quad (13)$$

in which f_{NBI} is the fraction of heating by NBI, f_c is the fraction of heat transported by ions, and v_{NBI} is the beam ion velocity. The ratio $\Pi/(\Gamma^p + Q_{\parallel})$ is plotted in Fig. 3(b). Since $v_{\text{NBI}}/v_{ti}|_{\text{pt}}$ is typically large, f_{unb} may be a significant fraction of unity, as observed by [33].

Several features of this solution deserve comment. First, the steady-state results given in Eqs. (12)–(13) are due to a balance between large momentum transport terms [Fig. 2(b)], thus are robust. Relaxation time to the edge intrinsic rotation profile should occur roughly on an ion transport time through the pedestal, $\sim \tau_{\text{cr}}$. The v -asymmetric diffusion Π is independent of the toroidal velocity and its radial gradient, thus $-\Pi$ represents a residual stress. For typical experimental parameters, it acts in the co-current direction with experimentally-relevant magnitude. The dimensional intrinsic rotation prediction, given here for small D_0 , is

$$v_{\text{int}}^{\text{dim}} \approx 1.04 \frac{B_\phi}{B_0} \left(\frac{d_c}{2} - \cos y_0 \right) \frac{q\rho_i|_{\text{pt}}}{L_\phi} v_{ti}|_{\text{pt}} \propto \frac{T_i|_{\text{pt}}}{B_\theta}. \quad (14)$$

The $1/B_\theta$ dependence corresponds to experimentally-observed $1/I_p$ scalings [2, 3], while proportionality to ion temperature $T_i|_{\text{pt}}$ provides an alternative explanation for recent observations [25, 34]. Co-current spin-up at the L-H transition [2, 3] is expected due to the increase in $T_i|_{\text{pt}}$ and probable decrease in L_ϕ . The predicted dependence on X-point poloidal angle has yet to be experimentally tested. The physics presented here may also have implications for internal transport barrier (ITB) rotation:

for outboard-ballooning and radially-increasing diffusivity (as outside an ITB), the asymmetric diffusivity causes a counter-current core rotation increment, as seen by [35].

Simplifications used in this model must be kept in mind. The presented calculations omitted both the ∇B drift and the radial electric field E_r , outside the latter's contribution to v_{rig} . While the ∇B drift has modest qualitative effect, a uniform uncanceled poloidal $\mathbf{E} \times \mathbf{B}$ drift of magnitude approaching $v_{ti}|_{\text{pt}} B_\theta / B_0$ may contribute a nonnegligible residual stress, a transport effect due to a shifted relation between D_{y0} and D_{eff} [28]. Treatment of sheared E_r effects, $\mathbf{E} \times \mathbf{B}$ divergence, electrostatic confinement [36], or collisions would require nontrivial extensions to the theory. Direct collisional effects on the rotation-driving flux Π may often be small, since Π results mainly from ions that are somewhat superthermal at the pedestal top [Fig. 2(b)], thus very superthermal at the separatrix, with an accordingly low collision rate. However, lower-energy ions may affect both E_r and the rotation saturation $v_{\text{rig}} \Gamma^p$. Finally, recall that the turbulence parameters are taken as an input to the present model, not calculated self-consistently.

In summary, radial displacements of passing-ion orbits and typical tokamak-edge turbulence inhomogeneity are shown to result in orbit-averaged diffusivities that depend on the sign of v_{\parallel} . Even in the absence of nondiffusive effects, this results in residual stress and corresponding pedestal-top intrinsic rotation at experimentally-relevant levels. The rotation is co-current for typical H-mode parameters and scales with $T_i|_{\text{pt}} / B_\theta$, in agreement with experimental observations.

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