Extending HELENA to equilibria with incompressible parallel plasma rotation

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INTRODUCTION

Plasma rotation is believed to affect the confinement properties of a tokamak device regarding the equilibrium, stability and transport. The appearance of highly peaked density, pressure and temperature profiles, the possible suppression of some instabilities and the creation of transport barriers, either in the edge region (H-mode) or inside the plasma core (Internal Transport Barriers), are associated with plasma flow (see for example [1], [2]). Therefore plasma rotation is one of the elements present in most -if not all- of the Advanced Tokamak Scenarios. Understanding the way plasma rotation affects the equilibrium properties of a tokamak is of importance, especially in preparation towards ITER operation. The EFDA Task Force for Integrated Tokamak Modeling (ITM-TF), included in the workprogram of IMP12 project the development of independent modules that will be able to deal with the stationary equilibrium problem. Within the framework of the aforementioned project we extended the well known and widely used for the ITM-TF purposes, code HELENA[3] in order to solve the problem of plasma equilibria with parallel plasma rotation.

THE EQUATIONS OF STATIONARY EQUILIBRIUM

The MHD equilibrium states of an axisymmetric magnetized plasma with incompressible flows are determined by the following equation written in convenient units by setting $\mu_0 = 1$ [4], [5]

$$(1-M_p^2)\Delta^\star\psi - \frac{1}{2}(M_p^2)'|\nabla\psi|^2 + \frac{1}{2}\left(\frac{X^2}{1-M_p^2}\right)' + R^2P_s' + \frac{R^4}{2}\left(\frac{\rho(\Phi')^2}{1-M_p^2}\right)' = 0 \qquad (1)$$

along with the Bernoulli relation for the pressure,

$$P = P_s(\psi) - \rho\left[\frac{\upsilon^2}{2} - \frac{R^2(\Phi')^2}{1-M_p^2}\right] \qquad (2)$$

Here, the poloidal magnetic flux function $\psi(R,z)$ labels the magnetic surfaces; $M_p(\psi)$ is the Mach function of the poloidal velocity with respect to the poloidal Alvén velocity; $\rho(\psi)$ and $\Phi(\psi)$ are the density and the electrostatic potential; $X(\psi)$ relates to the toroidal magnetic field;

for vanishing flow the surface function $P_s(\psi)$ coincides with the pressure; υ is the velocity modulus which can be expressed in terms of surface functions and R; $\Delta^\star = R^2\nabla\cdot(\nabla/R^2)$; and the prime denotes a derivative with respect to ψ. In the absence of flow (1) reduces to the usual Grad-Shafranov equation. Derivation of (1) and (2) is provided in [4]. The surface quantities $M_p(\psi)$, $\Phi(\psi)$, $X(\psi)$, $\rho(\psi)$ and $P_s(\psi)$ are free functions for each choice of which (1) is fully determined and can be solved whence the boundary condition for ψ is given. For a typical thermonuclear plasma and magnetic field of the order of 1 Tesla, υ is of the order of $10-10^2$ Km/sec.

Equation (1) can be simplified by the transformation

$$u(\psi) = \int_0^\psi \left[1 - M^2(f)\right]^{1/2} df \tag{3}$$

which reduces (1) to

$$\Delta^\star u + \frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-M_p^2}\right) + R^2\frac{dP_s}{du} + \frac{R^4}{2}\frac{d}{du}\left[\rho\left(\frac{d\Phi}{du}\right)^2\right] = 0 \tag{4}$$

Also, (2) is put in the form

$$P = P_s(\psi) - \rho\left[\frac{\upsilon^2}{2} - R^2\left(\frac{d\Phi}{du}\right)^2\right] \tag{5}$$

Note that no quadratic term as $|\nabla u|^2$ appears anymore in (4). Transformation (3) does not affect the magnetic surfaces, it just relabels them.

Considering now rotation parallel to the magnetic field,

$$\vec{\upsilon} = \frac{M}{\sqrt{\rho}}\vec{B}$$

where M is the Afvénic Mach number of the (total) parallel velocity which is exactly equal to the poloidal Mach number M_p, the electric field vanishes; therefore (4) becomes:

$$\Delta^* u + \frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-M^2}\right) + R^2\frac{dP_s}{du} = 0 \tag{6}$$

which is identical in form with the Grad-Shafranov equation describing static equilibria, while (5) takes the form:

$$P = P_s - \rho\left(\frac{\upsilon^2}{2}\right) \tag{7}$$

STATIONARY FIXED BOUNDARY EQUILIBRIUM CODE HELENA

The code HELENA, is a fixed boundary equilibrium solver [6] available on the EFDA ITM Gateway and used for ITM-TF purposes. The static Grad-Shafranov equation used in the code is written as:

$$\Delta^*\psi = -F\frac{dF}{d\psi} - \mu_0 R^2\frac{dP}{d\psi} = -\mu_0 R j_{tor} \tag{8}$$

By observing that there is a correlation between the quantities in (6) and (8):

$$\psi \longleftrightarrow u \tag{9}$$

$$F\frac{dF}{d\psi} \longleftrightarrow \frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-M^2}\right) \tag{10}$$

$$P(\psi) \longleftrightarrow P_s(u) \tag{11}$$

the solver of the static code HELENA can be used to calculate the stationary equilibrium for plasma rotation parallel to the magnetic field, though the output will no longer correspond to the "natural" quantities in the ψ-space. In order to preserve compatibility with the EFDA ITM-TF conventions the calculated by the solver quantities (now in the u-space) must be mapped to the "natural" ψ-space. For the mapping one must consider the following basic correspondence:

$$P_{\text{HELENA}} \longleftrightarrow P_s \tag{12}$$

$$F_{\text{HELENA}} \longleftrightarrow \frac{X}{\sqrt{1-M^2}} \tag{13}$$

$$\psi_{\text{HELENA}} \longleftrightarrow u \tag{14}$$

By applying the inverse of transformation (3) and taking into account the relations (12)-(14), we get the following expressions for the magnetic field, current density and pressure.

$$\vec{B} = \frac{F_{\text{HELENA}}}{\sqrt{1-M^2}}\vec{\nabla}\phi - \frac{1}{\sqrt{1-M^2}}\vec{\nabla}\phi \times \vec{\nabla}u \tag{15}$$

$$\vec{J} = \left[\frac{-1}{\sqrt{1-M^2}}\Delta^* u + \frac{1}{2}\frac{1}{(1-M^2)^{3/2}}\frac{dM^2}{du}|\vec{\nabla}u|^2\right]\vec{\nabla}\phi + \frac{d}{du}\left(\frac{F_{\text{HELENA}}}{\sqrt{1-M^2}}\right)\vec{\nabla}\phi \times \vec{\nabla}u \tag{16}$$

$$P = P_{\text{HELENA}} - \frac{1}{2R^2}\frac{M^2}{1-M^2}\left(F^2_{\text{HELENA}} + |\vec{\nabla}u|^2\right) \tag{17}$$

where the subscript HELENA refers to the computed by the code quantities.

The profile of the Mach number is chosen peaked on-axis, vanishes at the boundary and there are three independent parameters (M_0, a, b) that control its shape:

$$M^2 = M_0^2(\psi^a - \psi_0^a)^b$$

where M_0 is the maximum value on axis, ψ_0 is the value of the poloidal magnetic flux on the boundary and a, b parameters controlling the shear of the Mach number profile.

CALCULATED EQUILIBRIUM WITH INCOMPRESSIBLE PLASMA ROTATION PARALLEL TO THE MAGNETIC FIELD

In the following we will present some preliminary results from the code. The boundary as well as the input profiles P' and FF', obtained from a scenario of a 15MA ITER equilibrium based on Ref. [7].

In (Fig. 1) the pressure profile for a stationary case with possible ITER relevant plasma rotation ($M_0 = 0.03$, $a = 2$, $b = 3$) is plotted against the static one. It is shown that the flow decreases the values at the core as well as the shear of the profile. This result is expected since in Eq. 17 the flow term is subtracted from the static one. This decrease indicates that the poloidal rotation alters the effect of plasma rotation since in [8] the purely toroidal direction of the flow and the density variation on the magnetic surfaces, results in an increase of the pressure. In (Fig. 2) the safety factor is plotted for the same cases as in (Fig. 1). The impact of the flow appears to be much weaker for q than that in the pressure profile. The flow increases the values of q at the plasma core though by a small fraction, a result different than that in [9], where equilibria with flow of arbitrary direction is calculated.

These results indicate that the flow affects some equilibrium quantities, while others are almost insensitive to it. The fact that some equilibrium quantities are affected by the plasma rotation suggests that the latter may change the stability and transport properties of magnetically confined plasmas in connection with the formation of transport barriers. Further extension of the code to non-parallel plasma rotation is underway.

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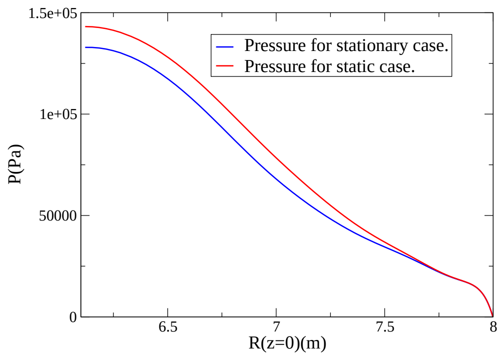

Figure 1: *Pressure profiles for* $M_0 = 0.03$, $a = 2$, $b = 3$ *and static.*

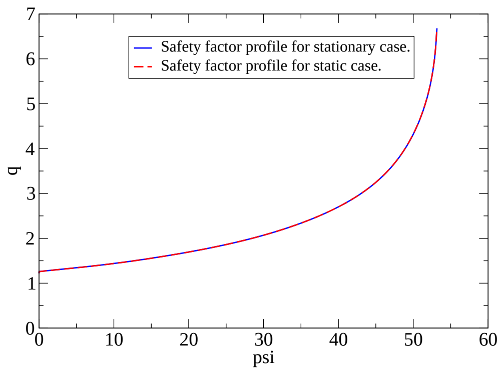

Figure 2: *The small difference of the q values in the two configurations is the cause of the overlapping profiles.*