SIMULATION OF ION-TEMPERATURE-GRADIENT-DRIVEN (ITG) MODES FOR THE BUMPY PINCH

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1. Introduction

ITG-instabilities are now commonly held responsible for turbulence giving rise to anomalous ion heat transport in the core of tokamaks. Therefore, the focus of the theoretical analysis has been on axisymmetric configurations of the tokamak type. In contrast, only very little has been done for stellarators although it is very likely that ITG-turbulence could become the dominant transport mechanism as collisional transport has been optimized in modern stellarators such as W7-X. Recently Villard et al. [1] have presented the first simulations for ITG-modes in straight helical configurations. Having in mind that the toroidal curvature is very low in the equilibrium of W7-X, i. e. smaller than that of a tokamak of aspect ratio 20, we investigate a 2D approximation to the equilibrium of W7-X, namely a straight so-called bumpy pinch configuration. Approaching the problem in this way can be seen as a main step in the long-term goal of modeling microinstabilities in three-dimensional W7-X type configurations.

2. The bumpy pinch configuration

We limit the plasma to a straight bumpy pinch configuration, i. e. an axisymmetric plasma in which the axis of symmetry and the magnetic axis coincide. Using cylindrical coordinates (r, θ, z), we can describe the divergence-free magnetic field $\mathbf{B}$ and the magnetic flux ψ in the form:

$$\mathbf{B}(r,z) = \nabla\psi(r,z) \times \nabla\theta, \qquad \psi(r,z) = \frac{B_0(r)}{2}\, r^2 \left(1 + c_0 \cos\left(\frac{N}{R_0} z\right)\right) \tag{1}$$

where $B_0(r)$ represents a magnetic well, c_0 gives the variation of B_z along the magnetic axis and N gives the number of periods of the cylinder of length $2\pi R_0$ ("topological torus”). A flux coordinate system (s, z) is given by $s = \sqrt{\psi/\psi_{\rm a}}$ where $\psi_{\rm a} = B(r_{\rm a})/2\, r_{\rm a}^2$ is the value of ψ at the plasma edge $(s = 1)$.

In this paper we will consider four cases: a) the θ-pinch with an ad hoc magnetic field of $B_0(r) = B_0$, $c_0 = 0$ and $N = 0$, b) the θ-pinch where the magnetic field $B_0(r)$ is given by the MHD equilibrium conditions consistently with the density and pressure profiles considered, c) the bumpy pinch approximating W7-X with a bumpiness of $c_0 = 0.1$ and a periodicity of $N = 5$ and d) such as case c) with an additional magnetic well $B_0(r)$ varying from the inside to the outside by 10%.

3. The gyrokinetic model

We adapted the linear particle-in-cell (PIC) code GYGLES (GYrokinetic Global Linear Electrostatic Solver) [2] to handle a plasma in the aforementioned straight bumpy pinch configuration which causes essentially a new orientation of the field solver from tokamak to θ-pinch geometry. For the case of periodicity $(N \neq 0)$ an analytic phase factor transformation in the z-direction reduces the simulation to only one period and a certain mode family which significantly cuts back the computational effort. In order to run the code on the T3E it was necessary to rewrite the GYGLES code from Cray Research Adaptive Fortran (CRAFT) to the Shared Memory Library. Optimal scaling with the number of Processor Elements (PE) is achieved with a 60 Mflop rate per PE.

The corresponding gyrokinetic equations describe the evolution of the perturbed ion guiding center distribution function $\tilde{f}(\mathbf{R}, v_\perp, v_\parallel)$: The unperturbed guiding center trajectories in phase space $(\mathbf{R}, v_\perp, v_\parallel)$ for the considered case are

$$\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}t} = v_\parallel \mathbf{e}_\mathrm{B} + \frac{v_\parallel^2 + v_\perp^2/2}{\Omega_\mathrm{i}} \mathbf{e}_\mathrm{B} \times \frac{\nabla B}{B} - \frac{v_\parallel^2}{\Omega_\mathrm{i}} \mathbf{e}_\mathrm{B} \times \left(\mathbf{e}_\mathrm{B} \times \frac{\nabla \times \mathbf{B}}{B} \right) \tag{2}$$

$$\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = \frac{1}{2} v_\perp^2 \nabla \cdot \mathbf{e}_\mathrm{B}, \quad \frac{\mathrm{d}v_\perp}{\mathrm{d}t} = -\frac{1}{2} v_\perp v_\parallel \nabla \cdot \mathbf{e}_\mathrm{B} \tag{3}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{e}_\mathrm{B} = \mathbf{B}/B$ its unit vector, $\Omega_\mathrm{i} = q_\mathrm{i}/m_\mathrm{i}B$ the ion cyclotron frequency, q_i and m_i the ion charge and mass. The evolution of the perturbed distribution function $\tilde{f}(\mathbf{R}, v_\perp, v_\parallel)$ along the unperturbed guiding center trajectories is

$$\begin{aligned} \frac{\mathrm{d}\tilde{f}}{\mathrm{d}t} = -\frac{\tilde{\mathbf{E}}}{B} \cdot \Bigg((\mathbf{e}_\mathrm{B} \times \nabla\psi) \frac{\partial f_0}{\partial \psi} - \Omega_\mathrm{i}\, \mathbf{e}_\mathrm{B} \frac{\partial f_0}{\partial v_\parallel} + \left(\mathbf{e}_\mathrm{B} \times \frac{\nabla B}{B} \right) \left(v_\parallel \frac{\partial f_0}{\partial v_\parallel} + \frac{v_\perp}{2} \frac{\partial f_0}{\partial v_\perp} \right) \\ - \left(\mathbf{e}_\mathrm{B} \times \left[\mathbf{e}_\mathrm{B} \times \frac{\nabla \times \mathbf{B}}{B} \right] \right) v_\parallel \frac{\partial f_0}{\partial v_\parallel} \Bigg) \end{aligned} \tag{4}$$

with f_0 the equilibrium distribution function assumed as an isotropic Maxwellian with density $n_0(s)$ and temperature $T_\mathrm{i}(s)$ constant on magnetic surfaces. The self-consistent electrostatic potential $\tilde{\phi}$ and therefore the gyro-averaged electric field $\tilde{\mathbf{E}}$ are derived by assuming Boltzmann electrons and quasi-neutrality which closes the system of equations:

$$\frac{n_0 e}{T_\mathrm{e}} \tilde{\phi}(\mathbf{x}) - \nabla_\perp \cdot \frac{n_0}{B\Omega_\mathrm{i}} \nabla_\perp \tilde{\phi}(\mathbf{x}) = \int \tilde{f}(\mathbf{R}, v_\parallel, v_\perp)\, \delta\left(\mathbf{R} - \mathbf{x} + \boldsymbol{\rho}\right) \mathrm{d}\mathbf{R}\mathrm{d}\mathbf{v} \tag{5}$$

$$\tilde{\mathbf{E}}(\mathrm{R}, v_\perp) = -\frac{1}{2\pi} \int \nabla\tilde{\phi}(\mathbf{x})\, \delta\left(\mathbf{R} - \mathbf{x} + \boldsymbol{\rho}\right) \mathrm{d}\mathbf{x}\mathrm{d}\alpha \tag{6}$$

where $\boldsymbol{\rho}$ is the gyroradius and $T_\mathrm{e}(s)$ the electron temperature. The quasi-neutrality Eq. (5) is valid up to $(k_\perp \rho)^2$ because the polarization density [3] (second term) is approximated by a differential expression to avoid solving a numerically time consuming integral equation. It is inverted using a quadratic spline finite element method so that charge assignment is naturally coupled to the basis functions.

The simulation is done over the whole cross-section of the cylinder with perturbations calculated separately for each poloidal Fourier harmonic m. This reduces the problem to a time-evolution of the distribution function $\tilde{f}$ (see Eq. (4)) in a 4-D reduced phase space. A small electrostatic perturbation is evolved over a finite time and, if a given configuration is unstable, the system is dominated by its most unstable eigenmode. For symmetry reasons no difference between the time-evolution of e. g. the toroidal Fourier modes $n = -1$ and $n = 1$ is tolerable, therefore appropriate changes in the "quiet starts" initial condition had been done. By analyzing the electrostatic potential $\tilde{\phi}$ and its time-evolution, we can compute the frequency ω, growth rate γ and spatial structure of the most unstable mode.

4. Results

We consider the following parameters: $q_\mathrm{i}/m_\mathrm{i} = 4.79 \cdot 10^7$ C/kg (Deuterium), $B_0 = 2.5$ T, aspect ratio $A = R_0/r_\mathrm{a} = 10$ with $R_0 = 5.5$ m and $r_\mathrm{a} = 0.55$ m, flat density profile, $T_\mathrm{e} = T_\mathrm{i}$, $\mathrm{d}\ln T_\mathrm{i}/\mathrm{d}s$ profile peaking at $s = s_0 = 0.5$ with $L_\mathrm{T}^{-1} = |\nabla \ln T_\mathrm{i}| = 1/3$ m, $T_\mathrm{i}(s_0) = 5$ keV.

As stated before we will consider four different axisymmetric configurations: The first and the simplest is case a) of a θ-pinch with a constant ad hoc magnetic field B_0 and an infinite $\eta = L_\mathrm{n}/L_\mathrm{T}$ over the whole cross-section where $L_\mathrm{n}^{-1} = |\nabla \ln n_0|$. The Fourier harmonics are the eigenmodes of the θ-pinch which makes it possible, if an appropriate Fourier filter in n is used, to trace the time evolution of not only the most unstable mode. In Fig. a) the growth rate γ for the toroidal mode numbers $n = 1, 2, 3, 4$ is plotted as a function of the poloidal mode number m. All four curves are tending to zero for low and high poloidal mode numbers: For low m's it is the Landau damping of the ions, for high m's the finite gyroradius which damps the growth rate to zero. Modes with $|n| > 4$ seem to be stable. The spatial structure of e. g. the $n = 1, m = 24$ mode is given in Fig. b) where the real part of the potential $\tilde{\phi}$ over the s-z-plane is plotted. The mode is centered around $s = 0.433$ (dashed line) and has one node in the s-direction.

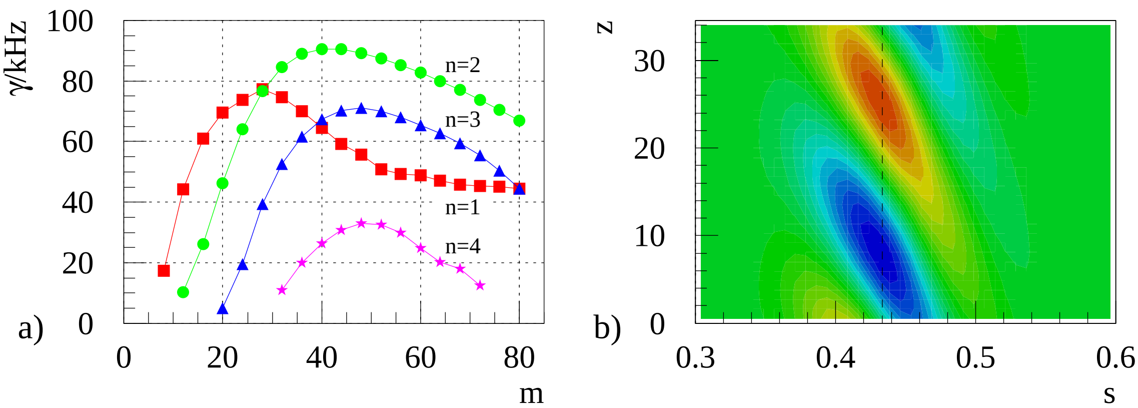

Figure 1. a) Growth rate γ for the toroidal mode numbers $n = 1$ (squares), $n = 2$ (circles), $n = 3$ (triangles) and $n = 4$ (stars) as a function of the poloidal mode number m at infinite $\eta = L_\mathrm{n}/L_\mathrm{T}$. b) Contour plot in the s-z-plane of $\Re(\tilde{\phi})$ for the $n = 1, m = 24$ mode.

For case b) we change the density profile so that it implements the boundary condition $n_0(s = 1) = 0$ but still with a very large η. The considered MHD equilibrium condition gives rise to a magnetic well and hence a ∇B drift opposite to the diamagnetic drift. By way of

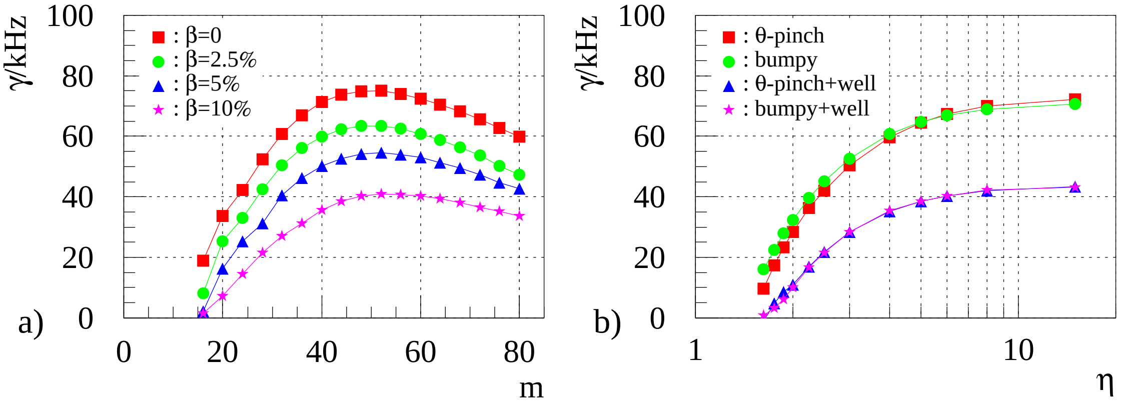

Figure 2. a) Growth rate γ for different β-values with $\beta = 2\mu_0 p(s_0)/B_0^2$ at $s_0 = 0.5$ but $n = 1$ const. as a function of the poloidal mode number m at infinite η. b) Growth rate γ of the $n = 1$ and $m = 24$ Fourier mode for different configurations as a function of η.

example, Fig. a) shows for $n = 1$ that this has a stabilizing effect on the slab-ITG modes as the growth rate γ for fixed m is decreasing with increasing β and magnetic well.

Figure b) shows the growth rate of the $n = 1, m = 24$ Fourier mode for all the cases a)–d) as a function of η. The main difference between the resulting growth rates come form the presence of a 10 % deep magnetic well, while the effect of 10 % bumpiness is of small influence although it causes 30 % trapped particles. The position of the marginal point at $\eta \approx 1.6$ is not changed in any of the configurations.

5. Conclusions

These first simulations of ITG modes in a bumpy pinch configuration show that the influence of trapped ions which occur due to the field variation along the magnetic axis of W7-X is an effect of minor importance as compared to the effect of the magnetic well caused by the MHD equilibrium. But even this effect reduces the growth rate by less than a factor of two so that ITG instabilities remain as a candidate for the ion heat transport for stellarators like W7-X.

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References

[1] Villard L., Fivaz M., Vaclavik J.: *Proc. of the 7th European Fusion Theory Conference*, Jülich, Germany, p. 139–142 (1997).

[2] Fivaz M., Brunner S., De Ridder G., Sauter O., Tran T.M., Vaclavik J., Villard L. and Appert K.: Comp. Phys. Comm., *submitted (1998).*

[3] Hahm T.S.: Phys. Fluids **31**, p. 2670–2673 (1988).