COILS FOR 3D MHD EQUILIBRIA

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1. Introduction

In 3D toroidal MHD configuration research it has proved advantageous to separate the optimization of the plasma equilibrium from its realization by external currents. That is suggested by the fact that the properties of a toroidal magnetic field are essentially (except for profile effects) determined by the shape of the outermost closed flux surface. Advanced configurations as the W7-X stellarator [1] or the quasi-axisymmetric tokamak [2],[3] have been optimized by considering the plasma domain only.

The magnetic field $\mathbf{B}_{\mathrm{p}}$ of an optimized fixed boundary equilibrium is the sum of a vacuum field $\mathbf{B}_{\mathrm{vac}}$ and the field $\mathbf{B}_{\mathrm{j}}$ produced by the plasma current: $\mathbf{B}_{\mathrm{p}} = \mathbf{B}_{\mathrm{vac}} + \mathbf{B}_{\mathrm{j}}$ with $\mathbf{B}_{\mathrm{p}} \cdot \mathbf{n} = \mathbf{0}$ on the boundary ($\mathbf{n}$ = exterior normal). Given the equilibrium one has to determine a set of coils producing the vacuum part $\mathbf{B}_{\mathrm{vac}}$ of the equilibrium field.

Originally, the NESCOIL coil code [4],[5] was developed to compute external currents for the Helias class of optimized stellarator configurations on which the W7-X device is based. For these equilibria the effect of the plasma current on the magnetic field structure is small because of the strongly reduced parallel plasma current density. Therefore the coils were designed to produce the vacuum solution of the optimized configuration. However, to treat cases with large plasma currents as quasi-axisymmetric tokamak configurations the effect of the plasma current cannot be neglected anymore.

This paper describes the method which takes into account the effect of the plasma current and presents an application of the extended version of NESCOIL to a quasi-axisymmetric tokamak configuration [6].

2. Virtual casing principle

The magnetic field $\mathbf{B}_{\mathrm{p}}$ of a fixed boundary equilibrium is the sum $\mathbf{B}_{\mathrm{p}} = \mathbf{B}_{\mathrm{vac}} + \mathbf{B}_{\mathrm{j}}$. In order to determine coils producing the vacuum field in the plasma domain one needs to know $\mathbf{B}_{\mathrm{vac}}$ separately. One possible way of calculating $\mathbf{B}_{\mathrm{vac}}$ is to compute the field generated by the plasma current and to subtract it from the total field. Here, a more attractive method - sometimes called the virtual casing principle [7] - is preferred: The sources of a field $\mathbf{B}$ defined by

$$\mathbf{B} = \begin{cases} \mathbf{B}_{\mathrm{vac}} + \mathbf{B}_{\mathrm{j}} & \text{in the plasma domain,} \\ 0 & \text{otherwise,} \end{cases} \qquad (1)$$

are the plasma current $\mathbf{j}_\mathrm{p}$ and a surface current

$$\mathbf{j}_\mathrm{s} = \mathbf{B}_\mathrm{p} \times \mathbf{n} \tag{2}$$

on the boundary where the normal component of $\mathbf{B}$ vanishes and the tangential component is discontinuous. Hence, $\mathbf{B}_\mathrm{vac}$ is generated by the surface current and can easily be computed with Biot-Savart's formula,

$$\mathbf{B}_\mathrm{vac} = \frac{1}{4\pi} \int df' \, \frac{\mathbf{j}_{\mathrm{s}'} \times (\mathbf{x} - \mathbf{x}')}{\mid \mathbf{x} - \mathbf{x}' \mid^3} \tag{3}$$

It is sufficient to calculate the normal component of $\mathbf{B}_\mathrm{vac}$ on the boundary only because $\mathbf{B}_\mathrm{vac}$ is uniquely determined by $\mathbf{B}_\mathrm{vac} \cdot \mathbf{n}$ on the boundary and a toroidal normalization integral $I_\mathrm{pol} = \int \mathbf{B}_\mathrm{vac} \cdot d\mathbf{s}$, where I_pol is the poloidal current producing $\mathbf{B}_\mathrm{vac}$.

With $\mathbf{B}_\mathrm{vac}$ known external currents are determined by minimizing

$$\int df [(\mathbf{B}_\mathrm{coil} - \mathbf{B}_\mathrm{vac}) \cdot \mathbf{n}]^\mathbf{2} = \mathrm{min!} \tag{4}$$

on the plasma boundary where $\mathbf{B}_\mathrm{coil}$ is the field produced by the external currents.

In the NESCOIL code two procedures for determining coils are provided. Coils can be designed either by determining a current distribution on a toroidal surface surrounding the plasma domain and discretizing it into a set of filaments or by a nonlinear optimization procedure determining the position of current filaments between two constraining tori. Geometrical properties of the filaments (distances between filaments, curvatures) are guaranteed by prescibed constraints [8].

3. Application

The extended version of the NESCOIL code has been applied to a quasi-axisymmetric tokamak [6]. These configurations combine the small aspect-ratio of a tokamak with stellarator properties by creating part of the rotational transform by external currents. The case presented here is determined by the shape of the boundary, two field periods, a value of $< \beta > \approx .03$ and a toroidal current $I_\mathrm{tor} = -.015 I_\mathrm{pol}$, which results by fixing the ι profile to be $.51(\mathrm{axis}) < \iota < .56(\mathrm{edge})$. Flux surfaces of the fixed-boundary equilibrium computed with the VMEC code [9] are shown in Fig.1. Using the equilibrium field a modular coil system with $N_\mathrm{c} = 12$ filaments per period is determined producing the appropriate vacuum field with the finite-β NESCOIL version described above. Flux surfaces of the free-boundary equilibrium computed with the NEMEC (=VMEC+NESTOR) code [9],[10],[11] are shown in Fig.2. With an additional auxiliary set of modular coils (see Fig.3) the position of the plasma domain can be kept fixed for different values of plasma pressure and toroidal current. For $I_\mathrm{aux} = 0$ one gets the equilibrium shown in Fig.2. For $I_\mathrm{aux} = -0.16 \cdot I_\mathrm{pol}$ the field produced by the combined coil system approximates the vacuum field of the quasi-axisymmetric configuration [$I_\mathrm{pol}, (I_\mathrm{aux})$ = total current of main (auxiliary) set of coils]. Poincaré plots of the vacuum field are shown in Fig.4.

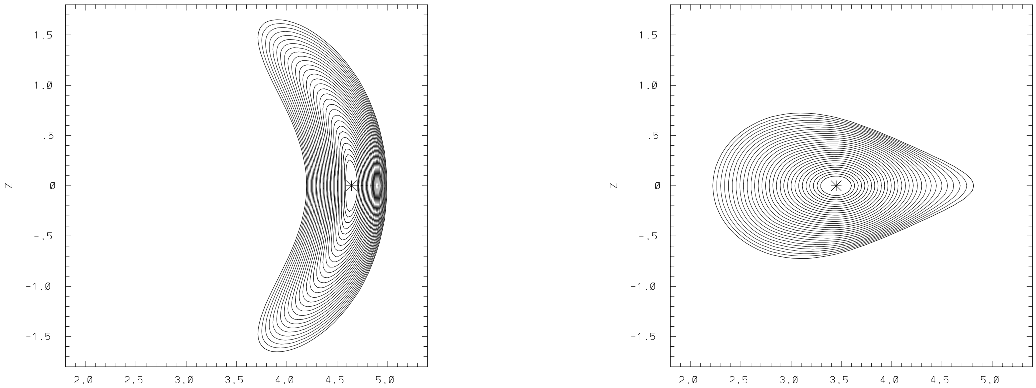

Fig. 1. Fixed-boundary equilibrium of a quasi-axisymetric tokamak configuration: Aspect ratio $A \approx 4$, $< \beta > \approx .03$, rotational transform $\iota \approx 0.51(\mathrm{axis}),\ \ 0.56(\mathrm{edge})$, toroidal plasma current $I_{\mathrm{tor}} = -0.015 \cdot I_{\mathrm{pol}}$, $I_{\mathrm{pol}} =$ coil current

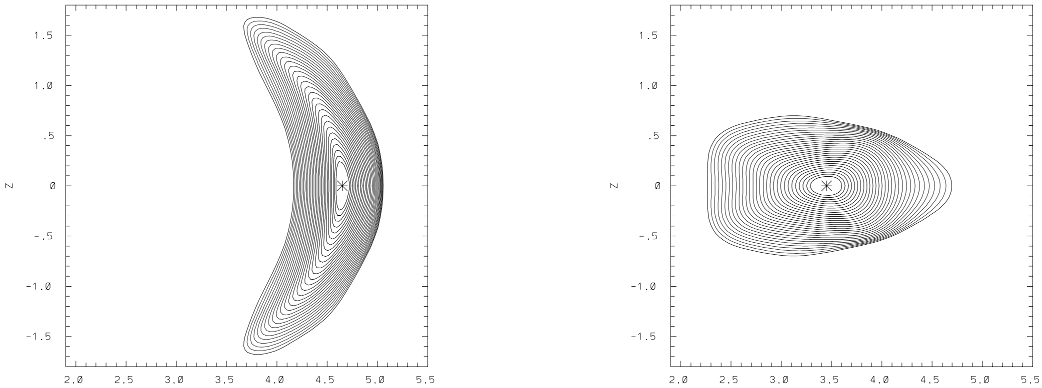

Fig. 2. Free-boundary equilibrium of the above case obtained in the external field of the main coil set. The coils (Fig.3) are determined such that the plasma configuration, defined in Fig.1, is reproduced.

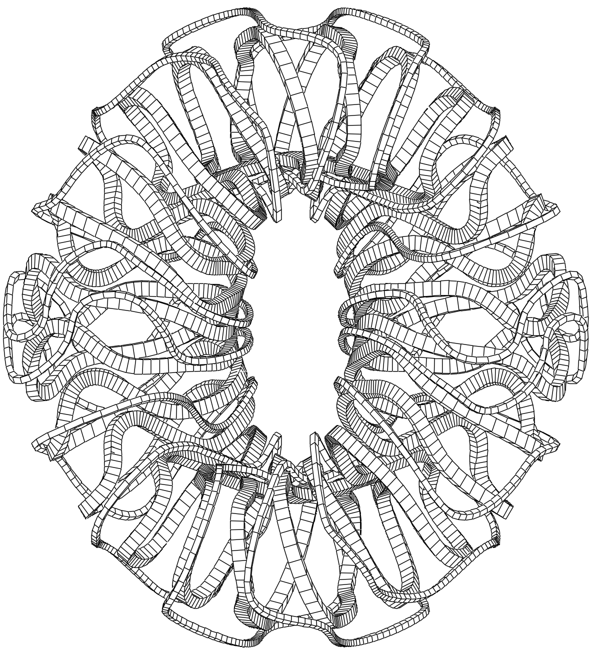

Fig. 3. Modular coil system for a quasi-axisymmetric tokamak [6]: set of main coils with $N_{\mathrm{c}} = 12$ coils per period, set of auxiliary coils with $N_{\mathrm{aux}} = 8$ coils per period. Number of periods $N_{\mathrm{p}} = 2$

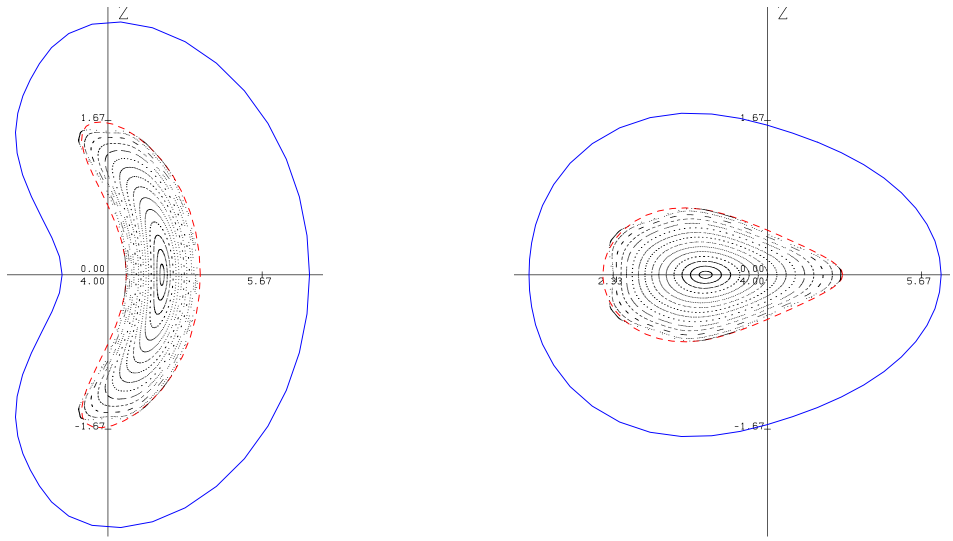

Fig. 4. Poincaré plots of the vacuum field produced by the coil system with $I_{\rm aux} = -0.16 \cdot I_{\rm pol}$. Boundary (red) of the quasi-axisymmetric tokamak, filament carrying surface (blue)

4. Conclusions

Coil sets for a quasi-axisymmetric tokamak configuration have been determined by the extended NESCOIL code. The fixed- and free-boundary equilibria are computed with codes (VMEC,NEMEC) assuming the existence of nested flux surfaces in the plasma domain. The quasi-axisymmetric configurations are three-dimensional in real space. Therefore, it is necessary and essential that the magnetic field properties of these equilibria be investigated by codes allowing for islands and stochastic regions.

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