NONLOCAL THREE-DIMENSIONAL SIMULATIONS OF PLASMA EDGE TURBULENCE

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1. Introduction

The anomalous transport in the plasma edge has a great influence on overall tokamak confinement, since the central temperature to a great deal depends on the temperature reached at the edge. Most fluid simulations of plasma edge turbulence assume the plasma parameters and their gradients to be constant throughout the computational domain (local approximation). This assumption is not completely valid in the edge since the density and temperature gradient scale lengths there approach the turbulence scale lengths. In this work, we take the radial variation of the plasma parameters into account, so that different turbulence regimes are contained in the simulation volume and the different types of turbulence can interact with each other. The turbulence in turn self-consistently determines the evolution of the temperature and density profiles.

2. Equations

At the present stage we use the electrostatic drift-Braginskii equations with complete ion-dynamics [1] in a flux-tube domain [2]. The equations are normalized to the typical space and time scales of the resistive ballooning mode [2,3].

$$L_\perp = 2\pi q \sqrt{\frac{n_0 e^2 \eta_\parallel \rho_s R}{m_i \omega_{ci}}} \sqrt{\frac{2R}{L_n}}, \qquad L_\parallel = 2\pi q_a R, \qquad t_0 = \sqrt{RL_n/2}/c_s,$$

$$c_s = \sqrt{(T_{e0}+T_{i0})/m_i}, \qquad \rho_s = c_s/\omega_{ci}, \qquad T_i = \lambda_n T_i^P/T_{i0}, \qquad T_e = \lambda_n T_e^P/T_{e0},$$

$$n = \lambda_n n^P/n_0, \qquad v_\parallel = \lambda_n v_\parallel^P/c_s, \qquad \phi = \lambda_n \alpha e\phi^P/T_{e0},$$

$$\alpha = \frac{\rho_s c_s t_0}{(1+\tau)L_n L_\perp}, \qquad \epsilon_n = 2L_n/R, \qquad \epsilon_v = c_s t_0/L_\parallel, \qquad \lambda_n = L_n/L_\perp,$$

where T_{e0}, T_{i0}, n_0 are typical values of the simulated plasma, L_n is the density e-folding length, ω_{ci} is the ion gyro frequency. Dimensional quantities are indicated by a "P". The parameter λ_n is a measure for the "locality" of the simulation. In these units, the typical density fluctuation amplitude (for the ballooning mode) is $(\lambda_n/n_0)(n_0 L_\perp/L_n) = 1$. Similar estimates hold for ϕ and T_i if $L_{T_i} \sim L_n$. The background profiles $\langle n\rangle$ and $\langle T_i\rangle$ are by definition of order λ_n. Hence the parameter λ_n is also the ratio of the background quantities to the fluctuations.

In particular, the following equations are advanced in time:

$$\nabla_\perp \cdot \left(\frac{\langle n \rangle}{\lambda_n} D_t \left(\nabla_\perp \phi + \frac{\alpha\tau}{\langle n \rangle} \nabla_\perp \left(\langle T_i \rangle n + \langle n \rangle T_i - \langle n \rangle \langle T_i \rangle \right) \right) \right) +$$

$$+ \frac{(\tau \langle T_i \rangle + \langle T_e \rangle)}{\lambda_n (1+\tau)} \hat{\mathrm{C}}(n - \langle n \rangle) + \frac{\tau \langle n \rangle}{\lambda_n (1+\tau)} \hat{\mathrm{C}}(T_i - \langle T_i \rangle) + \frac{1}{\eta_{\langle T_e \rangle}} \partial_\parallel^2 h = 0, \qquad (1)$$

$$D_t n - \frac{\langle n \rangle}{\lambda_n} \left(\epsilon_n \hat{\mathrm{C}}(h - \langle h \rangle) - \epsilon_v \partial_\parallel v_\parallel \right) + \epsilon_n \alpha (1+\tau) \frac{1}{\eta_{\langle T_e \rangle}} \partial_\parallel^2 h = 0, \qquad (2)$$

$$D_t T_i - \frac{2\langle T_i \rangle}{3\lambda_n} \left(\epsilon_n \hat{\mathrm{C}}(h - \langle h \rangle) + \frac{5}{2} \alpha\tau\epsilon_n \hat{\mathrm{C}}(T_i - \langle T_i \rangle) - \epsilon_v \partial_\parallel v_\parallel \right) +$$

$$+ \epsilon_n \alpha (1+\tau) \frac{2\langle T_i \rangle}{3\langle n \rangle \eta_{\langle T_e \rangle}} \partial_\parallel^2 h = 0, \qquad (3)$$

$$D_t v_\parallel + \epsilon_v \left(\frac{\tau \langle T_i \rangle + \langle T_e \rangle}{\langle n \rangle (1+\tau)} \partial_\parallel n + \tau \partial_\parallel T_i \right) = 0, \qquad (4)$$

where $\langle . \rangle$ means the flux surface average, $\langle T_e \rangle$ is assumed to be equal to $\langle T_i \rangle$, and

$$h = \phi - \alpha \langle T_e \rangle n / \langle n \rangle, \qquad D_t = \partial_t + \mathbf{v} \cdot \nabla,$$

$$\mathbf{v} = \hat{\mathbf{z}} \times \nabla \phi, \qquad \hat{\mathrm{C}} = (\cos(2\pi z) - \epsilon)\, \partial_y + \sin(2\pi z) \partial_x, \qquad \partial_\parallel = \partial_z + 2\pi \hat{s} x \partial_y.$$

The flux surface averages in the vorticity equation (1) and the parallel derivatives are applied in a way to maintain as many nonlinear terms as possible while still being able to solve the equations with a pseudospectral solver in the y-direction.

The curvature operators are applied on the fluctuating quantities only and flux surface averages are allowed to commute past the curvature operators since for now we are not interested in an exact reproduction of the plasma equilibrium in the presence of neoclassical effects.

With the above equations, the local limit can be readily obtained by letting $\lambda_n \to \infty$, keeping the other parameters fixed and noting the following proportionalities:

$$\langle n \rangle \propto \langle T_i \rangle \propto \lambda_n \qquad n - \langle n \rangle \propto T_i - \langle T_i \rangle \propto \phi \propto \text{const.} \qquad \nabla_\perp \langle \xi \rangle \propto D_t \langle \xi \rangle \propto 1/\lambda_n \langle \xi \rangle.$$

Some additional perpendicular and parallel diffusion and viscosity terms with low amplitude have been introduced to damp the turbulence at the grid scale.

3. Numerical scheme

To allow the use of a large time step, the time integration of the resistive terms is carried out semi-implicitly with the trapezoidal rule, while all other terms are integrated with the mid-point rule. The resulting equations are Fourier transformed in the y-direction, and the equations (1), (2) and (3) are combined into a single equation for h at the next time step, which is then solved by direct matrix inversion in the x–z-plane. To limit the matrix band width, the computational domain is divided in the z direction into 8-16 boxes, of which the boundarys are treated explicitly by a Du-Fort-Frankel scheme. This is possible because the $\partial_\parallel^2$ contributions in the equation carry

a small factor Δt, the numerical time step. The resulting value of h is substituted back into equations (2) and (3). Equation (4) is solved explicitly since it does not contain large diffusive terms.

The convection terms are calculated in configuration space while the rest of the computation takes place with y in Fourier space. The flux surface averages and the inverse matrix used in the solver are recalculated only every ~ 1000 time steps, since they change very slowly. The numerical scheme has been implemented on a Cray T3E and runs on 256 processors with about 20% overhead due to communication.

4. Simulations

The results of a simulation with an initial profile covering the range of local $\alpha = 0.1 - 1.1$ and nearly constant local $\epsilon_n = .05$ and $\eta_i = L_{T_i}/L_n = 3$ and other paramaters $\lambda_n = 25, \tau = 1, \epsilon_v = 0.01, \epsilon = 0.2, \hat{s} = 1$ are presented.

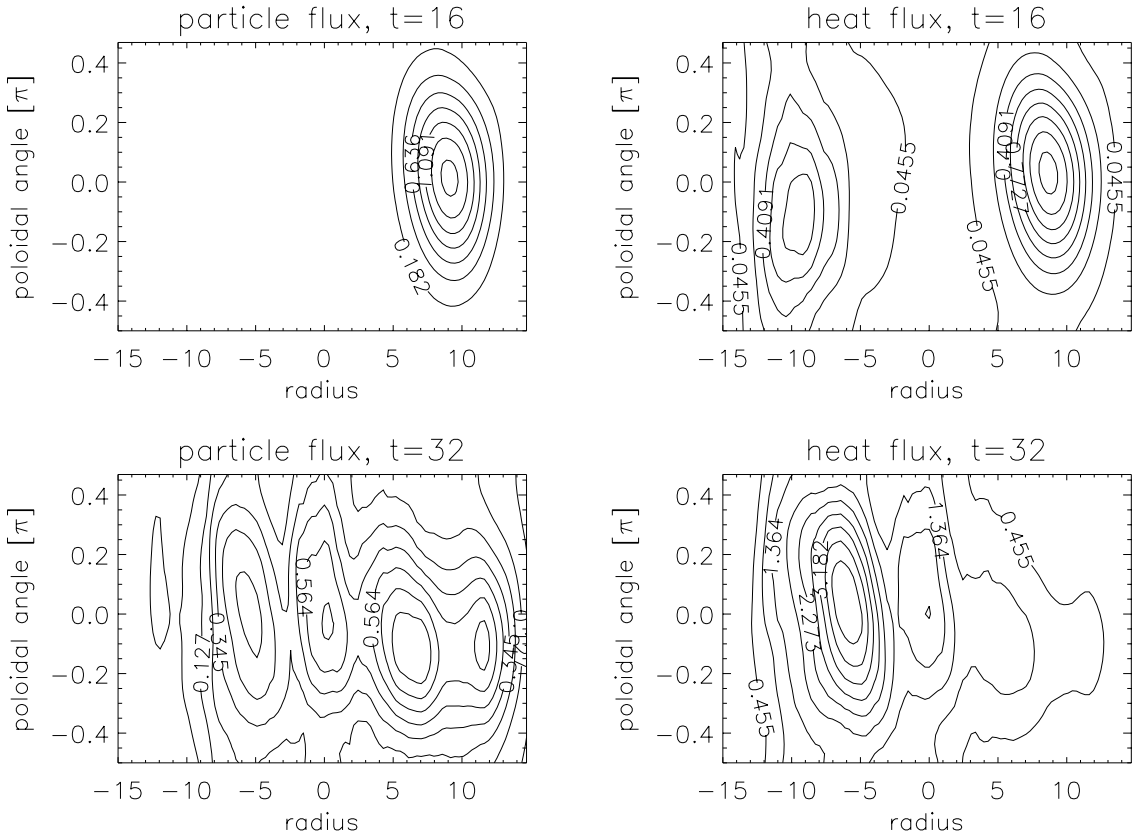

Figure 1. Particle and heat flux at $t = 16$ and $t = 32$

Figure shows the smoothed turbulence-driven particle (nv_r) and heat fluxes ($T_i v_r$) at $t = 16$, where the turbulence is not yet saturated, and at $t = 32$, where it is saturated, but the temperature and density profiles are not yet relaxed. At the outside of the tokamak the ballooning mode is observed, with a ratio of heat to density flux of order of $L_n T_i/(L_{T_i} n)$, while at the inside

the η_i-mode, with practically no particle transport, prevails. In the course of saturation of the turbulence the two regimes mix, but the ratio of heat transport to particle transport at the inside is still much larger than in the outer region. The occurrence of both modes is consistent [3] with the initial local α and η_i shown in figure (dashed).

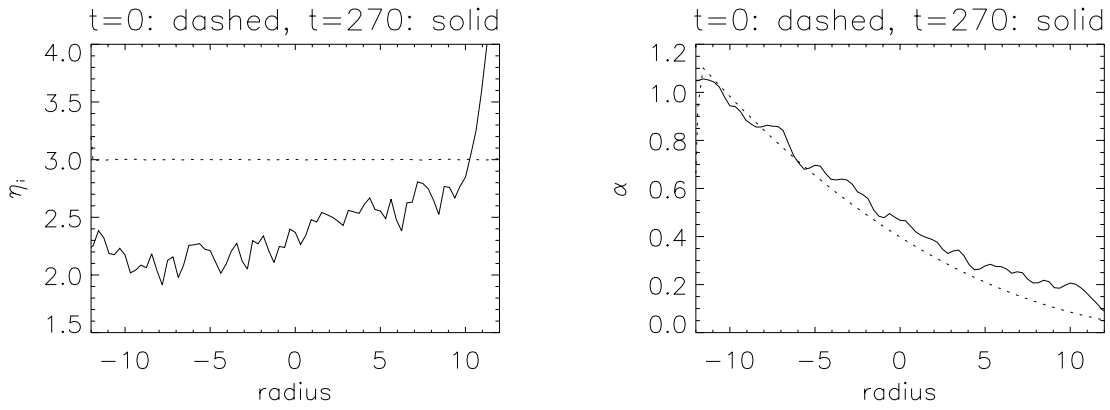

Figure 2. η_i and α profiles at $t = 0$ and $t = 270$

After a while, the unequal ratios of particle transport to heat transport for the ballooning and η_i modes leads to an increase in η_i at the outside and a decrease at the inside as shown in figure . This leads to only marginally unstable long-wavelength η_i-modes in the hot region of the computational domain [3].

5. Summary and Conclusions

A simulation code for the electrostatic drift Braginskii equations with ion temperature has been developed, which takes into account non-local effects and the self-consistent relaxation of the profiles. The code runs in parallel on up to 256 processors on a T3E. First results show a transition from η_i to ballooning-like turbulence from the inside to the outside of the simulation volume in the expected parameter range. Due to the relatively low particle transport rate of the η_i turbulence, the profiles relax to a state where η_i is very high at the outermost edge and is at the threshold for long-wavelength η_i-modes at the inner boundary of the computational domain. To model the physical situation more closely, it is planned to include the ionization of neutrals in the edge in the simulation.

References

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