

# FLUID AND KINETIC COMPUTATION OF DRIFT ALFVÉN TURBULENCE

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Recent three dimensional computations of electromagnetic drift wave dynamics are extended to cover the effects of warm ions, so that the model now covers both branches of low frequency fluid edge turbulence. In addition, the companion model with drift kinetic electrons has been extended to the electromagnetic regime. For cold ions, the two models are quantitatively consistent, more successfully in comparison than are their counterparts with gyrokinetic ions and adiabatic electrons. Finite ion temperature does not change the mode structure below the ballooning boundary. Sensitivity of transport to collisionality depends strongly on beta.

## 1. Introduction

Previous work in this series has emphasized the importance of addressing the fact that tokamak edge turbulence of the ExB-vortex type is not collisional enough to be treated with conventional fluid equations [1,2]. Transport due to electrostatic collisional drift wave turbulence has a dramatic dependence on the collision frequency,  $\nu_e$  [3], since the parallel electron dynamics in such a model results solely from a finite resistivity [4]. If electromagnetic effects are considered for Alfvén transit frequency  $v_A/qR$  comparable to drift frequencies  $c_s/L_\perp$ , where  $L_\perp$  is the profile scale length and  $2\pi qR$  is the field line connection length, the dependence on  $\nu_e$  is somewhat reduced [1]. Ion temperature dynamics was recently introduced into this drift Alfvén model, adding the  $\nabla T_i$  (ITG) modes to the mix but still feeling the Alfvén dynamics through the rising tendency of the transport with the ratio of drift to Alfvén transit frequencies [5]. Below, we find a weakened dependence of transport on  $\nu_e L_\perp/c_s$  with the ions present, for values in the edges of present day tokamak plasmas (see also early studies by Waltz [6]).

An important experimental consideration is that when the anomalous fluxes of particles or thermal energy are expressed as a diffusivity,  $\chi$ , the  $\chi$  rises sharply with minor radius,  $r$ , irrespective of the parameter regime [7]. Traditionally this rise has been attributed to some sort of underlying turbulence process thought to be similar to resistive MHD [8,9], but tokamak edge turbulence is not within this regime. The rise of  $\chi$  with  $r$  can on the other hand be explained by a strongly nonlinear dependence

of  $\chi$  on the profile gradients. Below, we find  $\chi \propto L_{\perp}^{-3}$  within the regime of interest, in absolute units (see also [10]).

When treating collisionless dynamics with “extended” fluid models (e. g., [11]), one needs careful checking of the models against more complete kinetic ones [12]. We have also developed a drift kinetic electron model, which substitutes the electron distribution function  $f_e(w_{\parallel}, w^2)$ , in terms of the velocity space variables  $w_{\parallel}$  and  $w^2$ , for the fluid model’s equations for the electron moments  $n_e, v_{\parallel}, T_e$ , and  $q_{e\parallel}$ , providing direct checks of the results within the same computational geometry. Although incorporation of finite- $T_i$  into this model is still in progress, we report a level of quantitative agreement between the two models in the cold ion limit which surpasses that shown by their counterparts in the “ion world” which allows arbitrary  $T_i$  but neglects parallel electron dynamics.

## 2. Fluid and Kinetic Models

The fluid model is presented in [5]. It is a generalization of the one detailed in [1] to finite  $T_i$  subject to the same considerations. The geometry is described by a globally consistent flux tube, [13] with the curvature operator reducing to the large aspect ratio model [14]. Drift ordering is applied to the fluid equations, which are extended to treat Landau damping by incorporating the parallel heat fluxes as dynamical variables. Normalization is in gyro Bohm units.

Under drift ordering, with the usual normalizations of drift turbulence ( $t$  to  $L_{\perp}/c_s$  and perpendicular coordinates to  $\rho_s$ ), the collisionless drift kinetic model appears as:

$$\frac{d}{dt} \nabla_{\perp}^2 \phi = \nabla_{\parallel} \left( u_{\parallel} - \int d^3 w w_{\parallel} f_e \right) \quad \hat{e} \frac{d}{dt} u_{\parallel} = E_{\parallel} \quad (1)$$

$$\frac{df_e}{dt} = -\omega_T f^M \frac{\partial \phi}{\partial y} - \alpha_e w_{\parallel} \left( \nabla_{\parallel} f_e - \omega_T f^M \hat{\beta} \frac{\partial A_{\parallel}}{\partial y} \right) - \alpha_e E_{\parallel} \left( w_{\parallel} f^M - \frac{\delta_0}{2} \frac{\partial f_e}{\partial w_{\parallel}} \right) \quad (2)$$

subject to

$$E_{\parallel} = -\hat{\beta} \frac{\partial A_{\parallel}}{\partial t} - \nabla_{\parallel} \phi \quad \nabla_{\perp}^2 A_{\parallel} = -J_{\parallel} = \int d^3 w w_{\parallel} f_e - u_{\parallel} \quad (3)$$

where  $d/dt$  includes advection by the ExB velocity,  $\nabla_{\parallel}$  is the gradient along the perturbed magnetic field, and  $f^M = \pi^{-3/2} e^{-w^2}$  is the background Maxwellian.

The main parameters are those controlling the ratios of important frequencies:

$$\hat{\beta} = \frac{4\pi n T}{B^2} \left( \frac{qR}{L_{\perp}} \right)^2 \quad \hat{\mu} = \frac{m_e}{M_i} \left( \frac{qR}{L_{\perp}} \right)^2 \quad \nu = \frac{\nu_e}{(c_s/L_{\perp})} \quad (4)$$

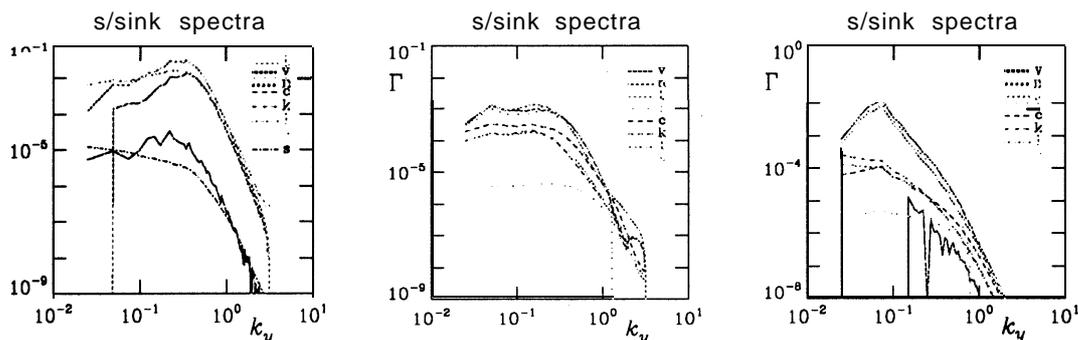
giving the relative speeds of the Alfvén thermal transit, and collision frequencies, respectively. The normalized ion mass,  $\hat{e}$ , gives the parallel sound time. The other

parameters are secondary;  $\omega_n, \omega_t$ , and  $\omega_i$  give the relative strengths of  $\nabla n$ ,  $\nabla T_e$ , and  $\nabla T_i$ ;  $\tau_i = T_i/T_e$  in the background; the strength of the curvature operator is controlled by  $\omega_B$ , nominally equal to  $2L_\perp/R$ ; sound waves are controlled by 2, nominally equal to  $(qR/L_\perp)^2$ ;  $\alpha_e = \sqrt{2/\hat{\mu}}$ ;  $\omega_T = \omega_n + \omega_t(w^2 - 3/2)$ ; and  $\delta_0 = \rho_s/L_\perp$ . In the kinetic model the curvature operator and  $T_i$  are neglected; they will be added in the future.

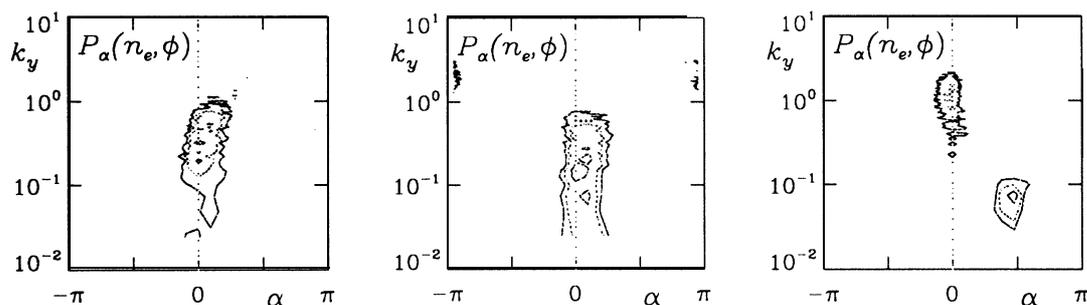
### 3. Results

We briefly describe three of the most important sets of new results. Unless stated otherwise, parameters were  $\hat{\beta} = 2$ ,  $\hat{\mu} = 10$ ,  $\nu = 0.5$ ,  $\delta_0 = 0.02$ ,  $\omega_n = \omega_t = \omega_i = \tau_i = 1$ ,  $\omega_B = 0.03$ , and  $\hat{\epsilon} = 3600$  (speeding up the sound waves), and the runs were arranged and diagnosed as described in [1].

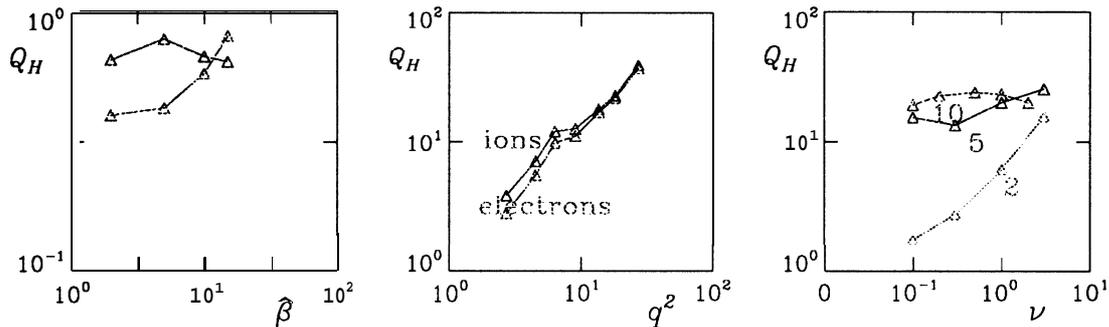
Following from left to right are the spectra of the free energy source and sink processes, for the kinetic model, the fluid model, and the fluid model with  $\hat{\beta} = 10$ , i.e., ideal ballooning with  $\alpha_M = 2\hat{\beta}\omega_B(1 + \tau_i) = 1.2$ . Ballooning mode turbulence is controlled by the largest scales available in the  $\nabla p$  direction.



Following from left to right are the phase shift probability distributions, for  $n$  phased ahead of  $\phi$  by angle  $\alpha$  for each wave  $k_y$  normalized to  $\rho_s$ , for the above three cases. Again, the warm ion system has drift Alfvén structure for  $\hat{\beta} = 2$  (middle), contrasted to a ballooning mode structure for  $\hat{\beta} = 10$  (right). For ballooning modes the parallel dynamical coupling between  $p_e$  and  $\phi$  is de-emphasized and so the dominant modes have phase shifts near  $\pi/2$  for maximum free energy access.



Following are a series of total thermal energy flux scalings. (left) Comparison of the scalings with  $\hat{\beta}$  between the kinetic and fluid models for  $\nu = \tau_i = \omega_B = 0$ ; transport results are always within 50 per cent of each other. (middle) The scaling with  $qR/L_{\perp}$  for the fluid model, equivalently with  $\nabla \log p$  or  $q$  (for  $\hat{\beta} = 10$ ). The  $\hat{\beta}$  scaling is similar below  $\alpha_M = 1$ . Resulting from the strength ratio of perpendicular to parallel dynamics, it is consistent with the rising tendency of  $\chi$  with minor radius. (right) The scaling with  $\nu$  for  $\hat{\beta} = 2$  and 5, drift Alfvén regime, and  $\hat{\beta} = 10$  with  $\alpha_M = 1.2$ . The values of  $\nu$  for tokamak edges are in the range 0.3 to 2.0.



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