MOMENTUM CONSERVATION AND FRICTION EFFECTS ON NEOCLASSICAL FLOWS IN STELLARATORS

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The monoenergetic drift kinetic equation and the adjoint approach

With the usual notation, described in [1, 2], the monoenergetic linearized drift kinetic equation for each one of n species a is:

$$(V - C_a^P) f_{a1} = \left[(\vec{v}_{da}.\nabla\rho)(A_1 + KA_2) + Bv_\parallel A_3 + \Delta C_a + S_{fa}\right] f_{a0}$$

$$V = (v_\parallel \vec{b} + \vec{v}_E).\vec{\nabla} + \dot{\alpha}\,\partial/\partial\alpha; \quad C_a^P = \sum\nolimits_b \nu_{ab}(v) L(f_{a1}) \tag{1}$$

$$A_{1a} = n_a'/n_a - 1.5T_a'/T_a - e_a E_\rho/T_a,\ A_{2a} = T_a'/T_a,\ A_{3a} = -(e_a/T_a)\langle \vec{E}.\vec{B}\rangle/\langle B^2\rangle$$

Here, f_{a1} is the first order perturbed distribution function, $\vec{B} = -\chi'\nabla\rho\times\nabla\zeta + \psi'\nabla\rho\times\nabla\theta$ is the magnetic field, $\vec{b} = \vec{B}/B$, $v_\parallel = \vec{b}.\vec{v}$, $\cos\alpha = v_\parallel / v = \xi$ gives the pitch angle, $L(f_{a1})$ is the pitch angle scattering collision operator, ν_{ab} is the collision frequency, $\vec{v}_E = \Phi' c(\vec{B}x\vec{\nabla}\rho)/\langle B^2\rangle$, K= $m_a v^2/2$ is the kinetic energy. Because C^P, the pitch angle collision operator, only acts on the pitch angle variable, and $\dot{v} = 0$, v is only a parameter in the equation. The ΔC_a term is a momentum conserving correction to the pitch angle collision operator, described in section 2. $S_{fa}(v)$ is the momentum input due to friction with fast particles, also described later. Both are represented as proportional to $v_\parallel$ through a function of v.

Defining the adjoint equations

$$\left(V + C_a^P\right)\begin{bmatrix} g_a^{(1)} \\ g_a^{(2)} \end{bmatrix} = \begin{bmatrix} Bv_\parallel \\ \vec{v}_{da}.\nabla\rho \end{bmatrix} f_{a0} \tag{2}$$

the plasma flows and heat flows are calculated as

$$\begin{bmatrix} \langle n_a u_{a\parallel} B\rangle \\ \langle q_{a\parallel} B\rangle \end{bmatrix} = -\left\langle \int d^3v\, \frac{g_a^{(1)}}{f_{a0}} \begin{bmatrix} 1 \\ K - 5T_a/2 \end{bmatrix} \left[(\vec{v}_{da}.\nabla\rho)(A_1 + KA_2) + (Bv_\parallel)A_3 + \Delta C_a + S_{fa}\right]\right\rangle \tag{3}$$

Taking into account the symmetry properties of the V and C^P operators,

$$\left\langle \int d^3v\, \frac{f_{a1}}{f_{a0}} \begin{bmatrix} V \\ L \end{bmatrix} g_a^{(n)} \right\rangle = \left\langle \int d^3v\, \frac{g_a^{(n)}}{f_{a0}} \begin{bmatrix} -V \\ L \end{bmatrix} f_{a1} \right\rangle \tag{4}$$

the particle and heat flows are given by:

$$\begin{bmatrix} \langle n_a u_{a\|} B \rangle \\ \langle q_{a\|} B \rangle \end{bmatrix} = -\left\langle \int d^3v \, \frac{g_a^{(1)}}{f_{a0}} \begin{bmatrix} 1 \\ K - 5T_a/2 \end{bmatrix} \left[(\vec{v}_{da}.\nabla\rho)(A_1 + KA_2) + (Bv_\|)A_3 + \Delta C_a + S_{fa} \right] \right\rangle \qquad (5)$$

which are simply a convolution of the solution of the adjoint equation (2a), with the source terms in equation (1), with some added weight functions. Because the momentum corrections to the collision operator are proportional to the particle and heat flows themselves, equation (5) actually becomes a 2n system of equations in the unknowns $\langle n_a u_{a\|} B \rangle$ and $\langle q_{a\|} B \rangle$. Inverting the system, the flows can be calculated, and then the total parallel current is $\langle j_\| B \rangle = \sum_a e_a \langle n_a u_{a\|} B \rangle$.

Similarly, it can be shown that the plasma particle and heat fluxes are given by

$$\begin{bmatrix} \langle \Gamma_a \rangle \\ \langle Q_a \rangle \end{bmatrix} = -\left\langle \int d^3v \, \frac{g_a^{(2)}}{f_{a0}} \begin{bmatrix} 1 \\ K - 5T_a/2 \end{bmatrix} \left[(\vec{v}_{da}.\nabla\rho)(A_1 + KA_2) + (Bv_\|)A_3 + \Delta C_a + S_{fa} \right] \right\rangle \qquad (6)$$

The diffusive transport coefficients are not affected by the momentum correction term or the momentum input, since these are proportional to the parallel velocity. The Ware pinch terms, now generalised to include other momentum sinks and sources, change.

The momentum conserving correction to the collision operator is obtained by expanding the l=0 spherical harmonic of f_{a1} in a series of orthogonal generalized Laguerre polynomials, as described in [3], and keeping the first two terms in the expansion (particle flow and heat flow). The result, described in [2], is proportional to $v_\|$:

$$\Delta C_{ab}(f_{a1}, f_{b1}) = 2\frac{v\xi}{v_a^2}\left[a_0 + \frac{2}{5} a_1 \left(\frac{5}{2} - \frac{v^2}{v_a^2} \right) \right] + \nu_{ab}^D 2\frac{v\xi}{v_a^2}\left[u_{a\|} - \frac{2}{5}\left(\frac{5}{2} - \frac{v^2}{v_a^2} \right)\frac{q_{a\|}}{p_a} \right] f_{a0}, \qquad (7)$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{4\nu_{ab}}{3\sqrt{\pi}} \left((u_{a\|} - u_{b\|}) \begin{bmatrix} M_{ab}^{00} \\ M_{ab}^{10} \end{bmatrix} - \frac{2}{5}\frac{q_{a\|}}{p_a} \begin{bmatrix} M_{ab}^{01} \\ M_{ab}^{11} \end{bmatrix} - \frac{2}{5}\frac{q_{b\|}}{p_b} \begin{bmatrix} N_{ab}^{01} \\ N_{ab}^{11} \end{bmatrix} \right). \qquad (8)$$

M_{ab}^{ij}, N_{ab}^{ij} are the classical friction coefficients, given in [4].

First case study: e-i plasma, $m_e/m_i << 1$

In the case of a pure electron-ion plasma, the ion momentum can be considered to be unaffected by the friction with the electrons, while the electron fluid velocity is altered by the very massive ions. Because of their lower thermal velocities, there is an E_ρ resonance of the ion flow. Shown in Fig. 1a and 1b is the E_ρ dependence of the electron and ion parallel currents, calculated with (Taguchi) and without (DKES) e-i friction, at a fixed radial position (r=12 cm) and plasma collisionality. The plasma represented is from a neutral beam heated discharge of W7AS, with n_e= 6 . 10^{20} m^{-3}, T_e= 1.7 keV, T_i= 1.2 keV, and a strong ion root. Note that the electron flow responds to the ion E_ρ resonance because of the friction with the ions.

Profiles of total current density, calculated with and without electron friction, are shown in Fig. 1c. In this case the friction reduces the total plasma current by about 20% (this result is sensitive to the actual E_ρ profile considered). This calculation was done ignoring the effect of friction with the fast particles on the electron and ion rotation, and the ion rotation was assumed to be determined by the local gradients. Clearly, if the ions rotate in the direction of the total current, the e-i friction reduces the current locally. If the ions rotate in the opposite direction, the electron-ion friction would increase the total current.

The effect of fast particles (neutral beam injection) on electron current.

As derived in [5], the Green's function response representing the momentum input to the electron distribution function from a monoenergetic source of fast ions with velocity v_b and parallel velocity $u_{\|b}$ is:

$$S_b^{(1)}(v, v_\|) = -\left(\frac{2v_\| u_{\|b}}{v_e^2}\right)\left(\frac{n_b e_b^2}{n_e e^2}\right)\nu_{e0}\ s(x)\ f_{e0},, \quad s(x) = \begin{cases} x^{-3}\left[1.2(v_b/v_e)^2 + 1\right] & (v_b/v_e) < x \\ (v_b/v_e)^{-3}\left[1.2x^2 - 2\right] & (v_b/v_e) > x \end{cases}$$

The velocity distribution function of the fast ions produced by the ionisation and slowing down of the beam is calculated in the usual way. Then the total contribution of the beam to S_{fe} is given by convolution of the beam energy distribution function with $S_b^{(1)}$.
The Ohkawa current can also be computed with a low collisionality model [6].

Shown in Fig. 2a is the profile of current density for the same W7AS shot as before, now including the neutral beam current drive (with both, the beam current and the electron response to it), but ignoring main ion rotation. The local current balance is still affected by the E_ρ profile, shown in 2b. Note that in this case the low collisionality model produces a larger beam current drive, as expected. With a model of S_{fi} a more complete solution of the problem could be given, but this has not been done yet.

References

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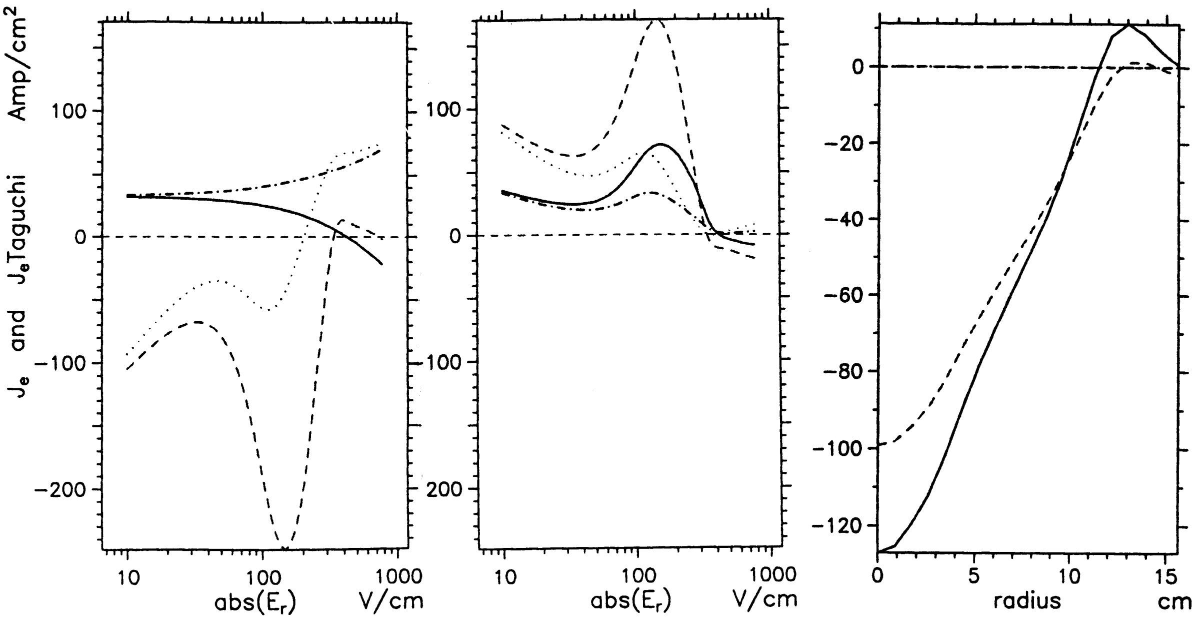

Fig. 1. Parallel currents as a function of E_ρ value, at r=12 cm.
a) Electrons: solid (dash-dot) are calculated without e-i friction for positive (negative) E_ρ, Dashed (dotted) with ei friction, for positive (negative) E_ρ. b) Ions (same convention) c) Total current profiles, same convention. In this shot, E_ρ is positive.

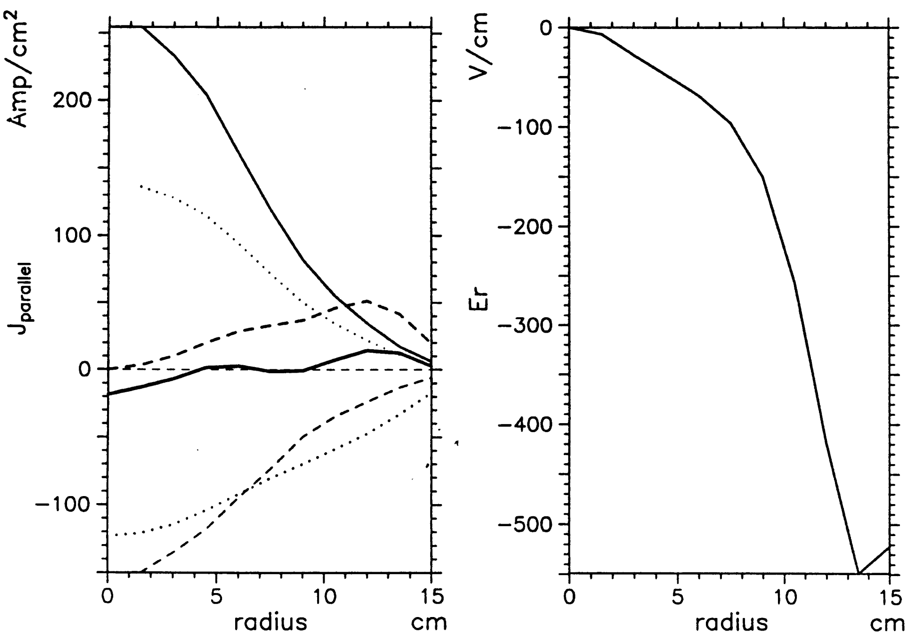

Fig. 2. a): Parallel currents: dashed lines are from electron bootstrap (13.4 kA); dotted from loop voltage (-22. kA); solid and positive from fast ions (27.9 kA); dashed from electron response to beam (-17.2 kA); dotted and negative from the fast ion and electron response, assuming a collisionless plasma (24. kA); thick solid is the total (2. kA). The error in the current balance is of the order of the main ion rotation minus its electron response. b) Radial electric field profile.