Three Dimensional Computation of Fluid and Kinetic Drift Alfven Turbulence in Tokamak Geometry

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A two-fluid moment model for drift Alfvén turbulence is constructed, in which the electron temperature is allowed to be anisotropic. For weakly collisional edge plasmas, the fact that only the parallel temperature is coupled through the Alfvenic dynamics to the ExB flows is quantitatively significant. Electron dominated edge turbulence throughout typical edge operation space is drift Alfven turbulence, with weak dependence on either collisionality or ballooning effects. ExB flow generation is significant if the collisionality is low enough or the plasma beta is high enough, suggesting an important role for magnetic flutter, or magnetic Reynolds stress, in the L-H transition.

In the construction of fluid drift turbulence models which are valid in hot tokamak regimes, the fact that the electron collisional relaxation frequency is slow compared to the turbulence has to be faced in some reasonable way, even if a fully rigorous fluid model may not be possible to construct [1]. For the purposes of electron pressure gradient driven turbulence it is sufficient to carry the temperature and parallel heat flux as self-consistent, time-dependent variables in order to capture the correct time scales of the processes involved in the coupling of ExB flows to the things they transport [2]. The type of "mode" most prominent in this process is the drift Alfven wave system, in whose most basic form with $\widetilde{n}$, $\widetilde{\phi}$, and $\widetilde{A}_{\|}$ as dependent variables the three eigenmodes are an almost electrostatic drift mode and two almost ideal shear Alfven waves — at finite $k_{\perp}\rho_s$ each eigenmode has a small amount of admixture from the other two [3]. In turbulence all these are coupled, and while there are no well-defined eigenmodes *per se*, the turbulence is still well-described as an almost electrostatic ExB turbulence doing the transport coupled through thermal/kinetic Alfvenic dynamics to two electromagnetic transients which do all the dissipation and control the amplitude of the turbulence. Additional effects such as magnetic flutter or curvature (*i. e.*, ballooning) may be important in deciding the level of the turbulence but have no qualitative effect on its mode structure [2].

In this work the drift Alfven turbulence model is further extended to weak collisionality by allowing the parallel and perpendicular temperatures to differ. Each is coupled to dissipation through the corresponding parallel flux of parallel or perpendicular energy, respectively, in what can be called a six moment closure [4]. The only important differences to [4] is that the equation system is not a guiding center system, so that it must reduce to the Braginskii equations [5] in the collisional limit, and that since the field lines can be perturbed, $k_\parallel$ may not be used as a parameter in the model for Landau damping. For the present, Landau damping is modelled via direct damping of the two heat flux moments.

Under drift ordering, with the usual normalisations of drift-wave, drift-Alfvén, or universal-mode turbulence (t to $L_\perp/c_s$ and perpendicular coordinates to ρ_s), and under the cold-ion restriction (in which the polarisation drift divergence reduces to ExB advection of the ExB vorticity and the parallel ion velocity is forced by the electron pressure), this version of the fluid drift system becomes

$$\frac{d}{dt}\nabla_\perp^2\phi = \nabla_\parallel J_\parallel - \mathcal{K}\left(T_\perp + n\right) \tag{1}$$

$$\frac{d}{dt}n = -\omega_n\frac{\partial\phi}{\partial y} + \nabla_\parallel\left(J_\parallel - u_\parallel\right) - \mathcal{K}\left(T_\perp + n - \phi\right) \tag{2}$$

$$\frac{1}{2}\frac{d}{dt}T_\parallel = -\frac{1}{2}\omega_t\frac{\partial\phi}{\partial y} + \nabla_\parallel\left(J_\parallel - u_\parallel - q_{\parallel\parallel}\right) - \mathcal{K}\left(0.5T_\parallel\right) - \nu\left(T_\parallel - T_\perp\right) \tag{3}$$

$$\frac{d}{dt}T_\perp = -\omega_t\frac{\partial\phi}{\partial y} - \nabla_\parallel q_{\perp\parallel} - \mathcal{K}\left(3T_\perp + n - \phi\right) + \nu\left(T_\parallel - T_\perp\right) \tag{4}$$

$$\hat\beta\frac{\partial}{\partial t}A_\parallel + \hat\mu\frac{d}{dt}J_\parallel = \nabla_\parallel\left(T_\parallel + n - \phi\right) - \hat\mu\nu\left[J_\parallel + \frac{0.71}{1.6}\left(q_{\parallel\parallel} + q_{\perp\parallel} + 0.71J_\parallel\right)\right] \tag{5}$$

$$\hat\mu\frac{d}{dt}q_{\parallel\parallel} = -\frac{3}{2}\nabla_\parallel T_\parallel - \hat\mu a_L q_{\parallel\parallel} - \hat\mu\nu\frac{5/2}{1.6}\left[q_{\parallel\parallel} + \frac{3}{5}0.71J_\parallel + 1.28\left(q_{\parallel\parallel} - 1.5q_{\perp\parallel}\right)\right] \tag{6}$$

$$\hat\mu\frac{d}{dt}q_{\perp\parallel} = -\nabla_\parallel T_\perp - \hat\mu a_L q_{\perp\parallel} - \hat\mu\nu\frac{5/2}{1.6}\left[q_{\perp\parallel} + \frac{2}{5}0.71J_\parallel - 1.28\left(q_{\parallel\parallel} - 1.5q_{\perp\parallel}\right)\right] \tag{7}$$

$$\hat\epsilon\frac{d}{dt}u_\parallel = -\nabla_\parallel\left(n + T_\parallel\right) - \mu_\parallel\nabla_\parallel^2 u_\parallel \tag{8}$$

where d/dt includes advection by the ExB velocity, $\nabla_\parallel$ includes the contribution from magnetic fluctuations, $J_\parallel = -\nabla_\perp^2 A_\parallel$, and $\mathcal{K}$ is the magnetic curvature operator.

This set is similar to that for transcollisional drift Alfvén turbulence with an isotropic temperature [2], and the parameters in the equations are the same. The main ones are those controlling the ratios of important frequencies:

$$\hat\beta = \frac{4\pi nT}{B^2}\left(\frac{qR}{L_\perp}\right)^2 \qquad \hat\mu = \frac{m_e}{M_i}\left(\frac{qR}{L_\perp}\right)^2 \qquad \nu = \frac{\nu_e}{(c_s/L_\perp)} \tag{9}$$

giving the relative speeds of the Alfvén, thermal transit, and collision frequencies, respectively. The ion mass in these units, $\hat{\epsilon}$, is also normalised by the same two factors of $qR/L_\perp$, reflecting the field line connection length as $2\pi qR$ and baseline parallel wavenumber of $k_\parallel qR \approx 1$. The other parameters are secondary; ω_n and ω_t give the relative strengths of the density and temperature gradients and $\mu_\parallel$ is an artificial parallel viscosity, all of order unity. The Landau damping parameter is set to $\hat{\mu}^{-1}$.

In these equations it is important to note the different roles for $T_\parallel$ and $T_\perp$ — $T_\parallel$ participates in the parallel Alfvenic dynamics (through $J_\parallel$) while $T_\perp$ is directly coupled to $\widetilde{\phi}$ through the magnetic curvature. With finite collisionality, however, the distinction between $T_\parallel$ and $T_\perp$ or $q_{\parallel\parallel}$ and $q_{\perp\parallel}$ disappears and we revert to the isotropic model of [2].

The computations are done in a flux-tube model tokamak geometry, with the coordinates aligned to the magnetic field; the treatment is as in [6], but the x-dependence of the geometric coefficients is dropped as in The parallel boundary condition is maintained as in [6], due to the importance of having the correct discrete spectrum for $k_\parallel$ in the parallel electron dynamics. This self-consistent dynamics is responsible for the basic mechanism of the nonlinear drift wave [7,8], or drift Alfvén [2], instability.

Computations in the slab geometry of [2] show the effects of the anisotropy in the temperature and heat flux. The comparison was done for the nominal case of [2], with $\hat{\beta} = \hat{\mu} = 10$, $\nu = 0.5$, and $2L_\perp/R = 0.03$ corresponding to the pre-transition parameters in ASDEX Upgrade, and also for a "low-density" case with $\nu = 0.1$. The same drift Alfvén mode structure emerges, and the transport difference at $\nu = 0.5$ is only about 6 to 5, with the anisotropic case lower. At $\nu = 0.1$ the difference is greater, 4 to 3, so the dependence on ν is slightly greater with the anisotropic temperature. This is consistent with the departure of $T_\perp$ from $T_\parallel$, and the thermal wave in the $T_\parallel, q_{\parallel\parallel}$ pair is faster than that in the $T, q_\parallel$ pair by a factor of $\sqrt{9/5}$, which slightly reduces the total drive due to the temperature gradient. This is not a sharp contrast, but it is enough of one to affect comparisons between theory and experiment in the serious first-principles L-H computations of the near future. A tokamak like ASDEX Upgrade should be more affected by this in edge turbulence than one like Alcator C-Mod.

Another process affected by the anisotropy is the generation of sheared ExB flow by magnetic flutter, again because this is mediated by $\nabla_\parallel J_\parallel$ (the magnetic Reynolds stress) as well as $\mathbf{v}_E \cdot \nabla\nabla_\perp^2\phi$ (the ExB Reynolds stress), and only $T_\parallel$ affects $J_\parallel$. The comparative studies show that a well-formed shear layer — a vorticity profile nearly flat and a ϕ profile nearly parabolic — forms most likely for $\nu < 0.2$ and $\hat{\beta} > \hat{\mu} = 10$.

A parabolic $\phi(x)$ is significant since the effects of the boundaries in the x-domain can be judged less important than if $\phi(x)$ has many local extrema (one of the undesirable effects of periodic boundary conditions in too small a domain). Although these runs have not yet produced a self-consistent L-H transition, this effect may have an important role in one.

We are also developing a drift-kinetic electron model to more solidly ground the fluid model in the collisionless regime. This is done by replacing Eqs. (2–7) with the drift kinetic equation:

$$\frac{df_e}{dt} = -\omega_T \frac{\partial \phi}{\partial y} - \alpha_e w_\parallel \nabla_\parallel f_e - \alpha_e E_\parallel \left(w_\parallel f^M - \frac{\delta}{2} \frac{\partial f_e}{\partial w_\parallel} \right) \tag{10}$$

where $\alpha_e = \sqrt{2/\hat{\mu}}$, $\omega_T = \omega_n + \omega_t(w^2 - 3/2)$, and $\delta = \rho_s/L_\perp$. So far, this has been done with $\hat{\beta} \to 0$ and $\nu \to 0$ and no curvature; extension to tokamak geometry is in progress. We have found that the velocity space nonlinearity in Eq. (10) has a strong effect in the absence of magnetic shear, because it damps the higher $k_\parallel$ modes which populate the $k_\parallel = 0$ portion of ϕ required for significant transport [9]. With moderate shear ($\hat{s} = 1$), however, there are no purely $k_\parallel = 0$ modes and therefore ϕ is coupled to f_e at al degrees of freedom. The portion with $|k_\parallel qR| < 1/2$ then does the transport more or less by itself and the damping of higher $k_\parallel$ modes has little effect. In this case, the Landau resonance is smeared out by ExB turbulence [10]. Since this effect is captured in the fluid model by the presence of ExB advection in Eqs. (6,7), there is genuine hope that a relatively simple fluid model can capture the important processes in fluid drift turbulence in the weakly collisional regime, even quantitatively.

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