The Conformal Standard Model

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We present an extended version of the Conformal Standard Model (characterized by the absence of any new intermediate scales between the electroweak scale and the Planck scale) with an enlarged scalar sector coupling to right-chiral neutrinos in such a way that the scalar potential and the Yukawa couplings involving only right-chiral neutrinos are invariant under a new global symmetry $\mathrm{SU}(3)_N$ which is broken explicitly only by the Yukawa interaction coupling right-chiral neutrinos and the electroweak lepton doublets. We point out four main advantages of such an enlargement, namely: (1) the economy of the (non-supersymmetric) Standard Model, and thus its observational success, is preserved; (2) thanks to the enlarged scalar sector the RG improved one-loop effective potential is everywhere positive with a stable global minimum, thereby avoiding the notorious instability of the Standard Model vacuum; (3) the pseudo-Goldstone bosons resulting from spontaneous breaking of the $\mathrm{SU}(3)_N$ symmetry are natural Dark Matter candidates with calculable small masses and couplings; and (4) the Majorana Yukawa coupling matrix acquires a form naturally adapted to leptogenesis. The model is made perturbatively consistent up to the Planck scale by imposing the vanishing of quadratic divergences at the Planck scale. Observable consequences of the model occur mainly via the mixing of the new scalars and the standard model Higgs boson.

1 Introduction

Experimental searches at LHC have so far not revealed any evidence of 'new physics' beyond the Standard Model (SM), and in particular no signs of low energy supersymmetry, technicolor or large extra dimensions [1]. Of course, this state of affairs may change in the near future with new data, but the possibility that there is in fact not much new structure beyond the SM is by no means excluded. There thus remains the distinct possibility that – apart from 'small' modifications of the type suggested by the present work – the SM may survive essentially *as is* all the way to the Planck scale. This prospect is further strengthened by the excellent quantitative agreement between the SM predictions and several precision experiments that has emerged over the past decades, and which so far has not shown any deviation from SM predictions. In our view all this indicates that any 'beyond the standard model' (BSM) scenario must stay as close as possible to the SM as presently understood.

The present work takes up this point of view, in an attempt to formulate a more comprehensive and coherent scheme beyond the SM, within the general framework proposed in [2]. More specifically, this is to be done in such a way that, on the one hand, the economy of the SM is maintained as much as possible, by extending it only in a very minimal way, but on the other hand, such that – besides explaining the observed structure – the extension solves all outstanding problems that belong to particle physics proper. The latter comprise in particular the explanation of the neutrino sector (with light and heavy neutrinos), the explanation of the origin of Dark Matter with suitable dark matter candidates, and finally leptogenesis. Whereas the solution of these problems is usually assumed to involve large intermediate scales and new heavy degrees of freedom (GUT-scale Majorana masses, new heavy quarks to generate axion gluon couplings, and the like) that will be difficult, if not impossible, to observe, the important point here is that we try to make do without such large scales between the electroweak and the Planck scale. This postulate entails strong restrictions that we will analyze in this work and that may be falsified by observation. By contrast, we do *not* consider to belong to the realm of particle physics the problems of the cosmological constant, the origin of Dark Energy and the ultimate explanation of inflation. Beyond their effective description in terms of scalar fields, these are here assumed to involve quantum gravity in an essential way, whence their solution must await the advent of a proper theory of quantum gravity.

A crucial assumption of the present work, and the defining property of the term 'Conformal Standard Model' (CSM)[1], is thus the absence of any new scales intermediate between the electroweak scale and the Planck scale. This assumption is motivated on the one hand by the absence of any direct evidence in this direction (such as proton decay), and on the other hand by the 'near conformality' of the SM, that is, the fact that the SM *is* classically conformally invariant, *except* for the the explicit mass term in the scalar potential introduced to trigger spontaneous symmetry breaking. In previous work we have formulated a scenario which attempts to exploit this fact, and thus to explain the stability of the electroweak scale as well as the supposed absence of large intermediate scales, by imposing classical conformal symmetry as a basic symmetry. Importantly, we thus do not rely on low energy supersymmetry to explain the stability of the electroweak scale. In [2] the Coleman-Weinberg mechanism was invoked to provide a quantum mechanical source of conformal symmetry breaking, but more recently we have adopted a variant of this scheme, by allowing for explicit mass terms, but with the extra restriction of vanishing quadratic divergences in terms of bare parameters at the Planck scale, in a realization of what we call 'softly broken conformal symmetry' [3]. With either realization there is then only one scale other than the Planck scale in the game; this scale, which should be tiny in comparison with the Planck scale, is here assumed to be $\mathcal{O}(1)$ TeV. The challenge, then, is to accommodate within such a scenario all observed SM phenomena and, in particular, the considerable differences in scales observed in the SM. To these requirements we add the triple conditions of *perturbative consistency* (absence of Landau poles up to the Planck scale), of *lower boundedness of the RG improved one-loop effective potential*, and finally, of *vacuum stability* (the global minimum of the potential should stay the same all the way up to the Planck scale, so it cannot decay). It is a non-trivial check on our assumptions that there do exist parameter values satisfying all of these constraints.

Accordingly, we present in this paper a slightly modified version of the model proposed in [2, 3], with the aim of working towards a more comprehensive scenario of BSM physics. The modification consists in an enlargement of the scalar sector that couples to the right-chiral neutrinos, and the introduction of a new global $SU(3)_N$ symmetry acting only on the right-chiral neutrinos and the new scalar fields. This symmetry is assumed to be spon-

[1]In order to avoid an unnecessary proliferation of names, we have decided to use this name for the whole class of models satisfying the stated requirements.

taneously broken, giving rise to several Goldstone bosons. The latter are converted to pseudo-Goldstone bosons by the one-loop corrections induced by the Yukawa interaction coupling right-chiral neutrinos and the electroweak lepton doublets, which is the only term in the Lagrangian that breaks $SU(3)_N$ explicitly. Besides preserving the economy of the (non-supersymmetric) SM, this version of the CSM comes in particular with the following advantages: (1) the pseudo-Goldstone bosons resulting from spontaneous symmetry breaking can in principle serve as Dark Matter candidates with calculable small masses and couplings, and (2) the Majorana Yukawa coupling matrix dynamically acquires a form naturally adapted to leptogenesis via the mechanism proposed and investigated in [4]. Furthermore, there remains the possibility that a certain linear combination of the pseudo-Goldstone bosons may be identified with the axion required for the solution of the strong CP problem [2].

We should note that there is a substantial body of work along similar lines as proposed here. The idea of exploiting the possible or postulated absence of intermediate scales in order to arrive at predictions for the Higgs and top quark masses was already considered in [7]. However, it appears that the actual values of the SM parameters with only the standard scalar doublet cannot be reconciled with the stability of the electroweak vacuum over the whole range of energies up to the Planck mass (see [8] for a more recent re-assessment of this scenario). The possible importance of conformal symmetry in explaining the electroweak hierarchy was already emphasized in [9]. More recently, there have been a number of approaches proceeding from the assumption of conformal symmetry, in part based on the Coleman-Weinberg mechanism, as in [10], and [11, 12] which discusses aspects of neutrino physics in conformal theories; see also [13] for a discussion of the phenomenology of such models. Conformal models with gauged $(B-L)$ symmetry have been investigated in [14, 15]. RG improved effective potentials and their applications in the conformal context were have been considered in [16, 17, 18]. It has also been pointed out in [19] that the vanishing of the SM scalar self-coupling and the associated β-function at the Planck scale could be interpreted as evidence for a hidden conformal symmetry at that scale (and also for asymptotic safety); this is in some sense the opposite of the present

[2] In our previous work [5] it was wrongly claimed that the Majoron can become a pseudo-Goldstone boson. The error in that argument, which was based on a rather involved three-loop calculation, was uncovered thanks to the new technology developed in [6], which shows that only fields orthogonal to the identity in the matrix of Goldstone fields can become pseudo-Goldstone bosons, cf. (47) and section 2.4 for details.

scenario, where conformal symmetry is assumed to be relevant *below* the Planck scale. Among the non-supersymmetric attempts at a comprehensive approach to BSM physics the so-called νMSM model of [20] has been widely discussed; this model is somewhat related to the present work in that it is also based on a minimal extension of the SM, but differs in other aspects (for instance, in trying to incorporate inflation, with the Higgs boson as the inflaton). Here we will have nothing to say about supersymmetric models, which are characterized by more than just minimal additions to the SM, where there is a vast literature, see e.g. [21] for a recent overview and bibliography.

The main message of this paper is therefore that there may exist a (potentially rich) sector of 'sterile' scalar particles not far above the electroweak scale that would manifest itself only through the mixing with the SM Higgs boson and the appearance of *narrow* resonances in the TeV range or below. This would be the main observable consequence of the present work. In addition we will show that the masses and couplings of the pseudo-Goldstone bosons can make them viable Dark Matter candidates, at least in principle. To illustrate the different possibilities we briefly discuss (in section 2.2) a further extension of our main model. A representative list of possible mass values compatible with all consistency requirements is given in Table 1. Finally, we argue that *if* $\mathrm{U}(1)_{B-L}$ is gauged, the mass of the associated Z' vector boson would be expected to lie not too far above the TeV range with our assumptions (whereas it is expected to be much heavier in the context of GUT-type scenarios, as e.g. in [21]).

2 Basic features of the model

To present our point of view in as clear a manner as possible this paper is structured in line with our basic assumptions, which concern in particular

- Scalar sector
- Fermionic sector
- Pseudo-Goldstone bosons and their couplings

and which we will discuss in this section. In the following section we will discuss the constraints that self-consistency and compatibility with the SM and other data impose on the model and its parameters. Possible checks (that could in principle falsify our approach) are also discussed there, as well as possible signatures that may discriminate the present proposal from other proposals.

2.1 Scalar Sector

Although full confirmation is still pending, there is good evidence that the SM Higgs boson does not distinguish between different families (generations) [1]. Consequently, its different couplings to the SM fermions are entirely due to the different Yukawa coupling matrices, implying for instance that the Higgs couplings to quarks and leptons are directly proportional to their masses. It would therefore seem natural to assume that possible extensions of the scalar sector to include Majorana-like couplings to the right-chiral neutrinos should also proceed through a 'family blind' electroweak singlet scalar ϕ whose vacuum expectation value generates the usual Majorana mass term required for the seesaw mechanism, with an appropriate Majorana-type Yukawa coupling matrix Y^M_{ij}, and this path has been followed mostly in past work. By contrast, we here wish to explore an alternative scenario relaxing this assumption, and to point out several advantages that come with making the extended scalar sector sensitive to the family structure of right-chiral neutrinos. These concern in particular the appearance of pseudo-Goldstone bosons that are natural Dark Matter candidates, with calculable small masses and couplings. Furthermore, thanks to the new scalar fields, the much advertised instability of the Higgs coupling and the one-loop effective potential in the (un-extended) SM (see e.g. [22, 23, 24] for a recent discussion) can be avoided without great effort.

Accordingly, the main new feature of our model in comparison with the SM is its enlarged scalar sector, while there is no corresponding enlargement in the fermionic sector, other than the *ab initio* incorporation of right-chiral neutrinos (see below). The scalar sector is assumed, on the one hand, to allow for a Majorana mass matrix for the right-chiral neutrinos to be generated by spontaneous symmetry breaking, and with a breaking pattern adapted to leptogenesis, and on the other to allow for the existence of very light pseudo-Goldstone bosons that can serve as natural axion and dark matter candidates. The appearance of extra scalar degrees of freedom is a common feature of many proposed extensions of the SM, and in particular, of supergravity and superstring scenarios. A distinctive feature of the present scheme is that the new scalars, while carrying family indices, are otherwise 'sterile', except for those scalars that mix with the standard Higgs boson; as we will explain below this can lead to new experimental signatures, different from low energy supersymmetry and other scenarios where extra scalars carry electroweak or strong charges. The assumed sterility safeguards principal successes of the SM, in particular the absence of FCNC. While it might appear desirable to also extend the family structure of the scalars to the quark and lepton sector, our assumption of 'near conformality' seems difficult to reconcile with the existence of scalars relating different generations of quarks and leptons: by softly broken conformal invariance these would have to have relatively low masses, and thus conflicts would with SM data would be inevitable. In this respect, the situation is different in GUT-type scenarios, where such extra scalars can in principle be made sufficiently heavy so as to avoid any direct conflict with observation. However, even in that context, fully consistent models with family sensitive scalars seem hard to come by, and we are not aware of a single example of a model of this type that works all the way (see, however, [25] and references therein for a recent attempt to explain the observed hierarchy of quark masses in terms of discrete subgroups of a family symmetry SU(3)).

A new feature in comparison with [2] is thus that the scalars coupling to the right-chiral neutrinos are assumed to admit a family-type symmetry $U(3)_N$, that extends the standard $U(1)_{B-L}$ symmetry. This enlarged symmetry is broken explicitly by the Dirac-Yukawa couplings (to be discussed later) to the $U(1)_{B-L}$ subgroup. More specifically, in addition to the standard Higgs doublet H, we introduce a complex scalar sextet $\phi_{ij} = \phi_{ji}$ (with family indices $i, j, ...$), which are 'blind' to the SM gauge symmetry, hence sterile. This sextet replaces the standard Majorana mass term triggered by a family singlet scalar ϕ according to

$$\langle\phi\rangle\, Y^M_{ij} \quad \longrightarrow \quad y_M\, \langle\phi_{ij}\rangle \tag{1}$$

and similarly for the associated Majorana-type Yukawa couplings. The scalar field Lagrangian is

$$\mathfrak{L}_{scalar} = -(D_\mu H)^\dagger (D^\mu H) - \mathrm{Tr}(\partial_\mu \phi^* \partial^\mu \phi) - \mathcal{V}(H, \phi) \tag{2}$$

with the potential

$$\begin{aligned} \mathcal{V}(H,\phi) \;\; &= \;\; m_1^2\, H^\dagger H + m_2^2\, \mathrm{Tr}(\phi\phi^*) + \; \lambda_1\, (H^\dagger H)^2 \\ &+ 2\lambda_3\, (H^\dagger H)\mathrm{Tr}(\phi\phi^*) \; + \; \lambda_2\, [\mathrm{Tr}(\phi\phi^*)]^2 \; + \; \lambda_4\, \mathrm{Tr}(\phi\phi^*\phi\phi^*) \end{aligned} \tag{3}$$

where all coefficient are real (traces are over family indices). This potential is manifestly invariant under

$$\phi(x) \quad \rightarrow \quad U\phi(x)U^T \;, \qquad\qquad U \in \mathrm{U}(3)_N \tag{4}$$

The scalar fields ϕ_{ij} are inert under the usual SM symmetries, unlike the Higgs doublet H.

There are three different cases that ensure positive definiteness of the quartic part of the potential

- $\lambda_1, \lambda_2, \lambda_4 > 0, \quad \lambda_3 > -\sqrt{\lambda_1(\lambda_2 + \lambda_4/3)}$;
- $\lambda_4 < 0, \quad \lambda_1 > 0, \quad \lambda_2 > -\lambda_4, \quad \lambda_3 > -\sqrt{\lambda_1(\lambda_2 + \lambda_4)}$;
- $\lambda_2 < 0, \quad \lambda_1 > 0, \quad \lambda_4 > -3\lambda_2, \quad \lambda_3 > -\sqrt{\lambda_1(\lambda_2 + \lambda_4/3)}$.

One of these conditions has to hold *for all scales* between the electroweak and the Planck scales to avoid the problem of vacuum instability, and thus to overcome one of the main open problems of the SM in its current form. More concretely, we will require them to hold for the running couplings $\lambda_i(\mu)$ over this whole range when these are evolved with the β-functions (56).

Assuming the following values of the mass parameters

$$m_1^2 = -2\lambda_1 v_H^2 - 6\lambda_3 v_\phi^2, \qquad m_2^2 = -2\lambda_3 v_H^2 - (6\lambda_2 + 2\lambda_4)\, v_\phi^2, \tag{5}$$

(and thus parametrising them directly in terms of the positive parameters v_H and v_ϕ) it is straightforward to show that the global minimum of the potential takes the form[3]

$$\langle H\rangle = \begin{pmatrix} 0 \\ v_H \end{pmatrix}, \qquad \langle\phi\rangle = \mathcal{U}_0 \begin{pmatrix} v_\phi & 0 & 0 \\ 0 & v_\phi & 0 \\ 0 & 0 & v_\phi \end{pmatrix} \mathcal{U}_0^T, \qquad \mathcal{U}_0 \in \mathrm{U}(3), \tag{6}$$

provided that (in addition to the above positivity conditions) the following inequalities are also satisfied

$$\lambda_1 \left\{ \lambda_2 + \frac{\lambda_4}{3} \right\} - \lambda_3^2 > 0, \qquad \lambda_4 > 0.$$

[3]Thus, v_H and v_ϕ are the expectation values of the complex fields (we here drop the customary factor $1/\sqrt{2}$).

At the classical level the U(3) matrix $\mathcal{U}_0$ remains undetermined. The explicit breaking of $SU(3)_N$ symmetry to be discussed below will, however, lift this degeneracy and produce a 'vacuum alignment' with $\mathcal{U}_0 \neq \mathbb{1}$ according to [26], and also introduce small corrections that will lift the degeneracy of eigenvalues in $\langle\phi_{ij}\rangle$.

A second motivation for the replacement of a single complex scalar by a sextet is the following. Because the $U(3)_N$ invariance is assumed to be broken both spontaneously and explicitly (by the Yukawa interaction coupling right-chiral neutrinos to the lepton doublets via the matrix Y^{ν}_{ij}, see (27) below) there exist various light particles, i.e. (pseudo-)Goldstone bosons. It is a general result that the manifold of Goldstone bosons $\mathcal{M}$ is the quotient of the symmetry group by the symmetry of the vacuum. For (6) the residual symmetry is SO(3), and therefore

$$\mathcal{M} = U(3)_N/SO(3) \equiv U(1)_{B-L} \times SU(3)_N/SO(3) \tag{7}$$

whence there are altogether six (pseudo-)Goldstone bosons in our model. One of them is the *genuine* Goldstone boson associated with the exact $U(1)_{B-L}$ symmetry (so we can take out the U(1) factor).

After the symmetry breaking we have as usual the real Higgs field H_0

$$H(x) = \begin{pmatrix} 0 \\ v_H + \frac{1}{\sqrt{2}} H_0(x) \end{pmatrix}, \tag{8}$$

(in the unitary gauge) while a convenient parametrisation of the coset space $\mathcal{M}$ is given by

$$\phi(x) = \mathcal{U}_0 \, e^{\mathrm{i}\tilde{A}(x)} \, (v_\phi + \tilde{R}(x)) \, e^{\mathrm{i}\tilde{A}(x)} \, \mathcal{U}_0^{\mathrm{T}} \tag{9}$$

where $\tilde{A}_{ij}$ and $\tilde{R}_{ij}$ are real symmetric matrices. The trace part of

$$\mathcal{G}(x) \equiv \mathcal{U}_0 \tilde{A}(x) \mathcal{U}_0^{\dagger}, \tag{10}$$

is the $(B-L)$ Goldstone boson $\mathfrak{a}(x)$ that remains a Goldstone boson even when the $U(3)_N$ symmetry is broken, while the traceless part of $\mathcal{G}(x)$ yields the five Goldstone bosons that will be converted to pseudo-Goldstone bosons. In accordance with the decomposition $\mathbf{6} \to \mathbf{1} \oplus \mathbf{5}$ under the residual SO(3) symmetry we can thus write

$$\tilde{A}_{ij}(x) = \frac{1}{2\sqrt{6} v_\phi} \mathfrak{a}(x) \delta_{ij} \, + \, A_{ij}(x) \ , \qquad \mathrm{Tr}\, A(x) = 0 \tag{11}$$

with

$$A_{ij} \equiv \frac{1}{v_\phi} G(x) \equiv \frac{1}{4v_\phi} {\sum_a}' G_a \lambda^a_{ij} \tag{12}$$

where the restricted sum is only over the five *symmetric* Gell-Mann matrices (with the standard normalization $\mathrm{Tr}\,(\lambda^a \lambda^b) = 2\delta^{ab}$) and where the real fields $a(x)$ and $G_a(x)$ are canonically normalized. The matrix $\tilde{R}_{ij}(x)$ likewise can be split into a trace and a traceless part, *viz.*

$$\tilde{R}_{ij}(x) = \frac{1}{\sqrt{6}} r(x) \delta_{ij} \, + \frac{1}{2} {\sum_a}' R_a \lambda^a_{ij}(x) \tag{13}$$

Because the new scalars are thus only very weakly coupled to the remaining SM fields, the main observable effects are due to the mixing between the standard Higgs boson and the new scalars. In fact, the five modes R_a are already the proper mass eigenstates with eigenvalues

$$M_R^2 = 4\lambda_4 v_\phi^2, \tag{14}$$

The fields r can mix with H_0 and the combined mass matrix for the fields (H_0, r) reads

$$\mathcal{M}^2 = \begin{pmatrix} 4\lambda_1 v_H^2 & 4\sqrt{3}\lambda_3 v_H v_\phi \\ 4\sqrt{3}\lambda_3 v_H v_\phi & 4(3\lambda_2 + \lambda_4) v_\phi^2 \end{pmatrix}, \tag{15}$$

and determines the mass eigenstates h_0 and h'

$$h_0 \; = \; \cos\beta \, H_0 \; + \; \sin\beta \, r \quad , \quad h' \; = \; -\sin\beta \, H_0 \; + \; \cos\beta \, r, \tag{16}$$

with the mixing angle β. We identify the lighter of the two mass eigenstates h_0 with the observed Higgs boson, with $M_{h_0} \approx 125$GeV. The mixing will lead to a second resonance associated with h', which is one of the main predictions of the present model. This resonance should be rather narrow because of the factor $\sin^2 \beta$ [27]. It will have the same decay channels to the SM particles as the standard Higgs boson (hence look like a 'shadow Higgs'), but depending on the actual mass of h', there may also be other decay channels which could broaden the resonance. We will return to this point below.

2.2 More sterile scalars?

Given the fact that many approaches to unification and quantum gravity come with an abundance of scalar fields it is entirely conceivable that there exists an even larger sector of scalar fields, and in this sense our model is just the simplest example. As one further example, we briefly discuss in this section an extension of the model obtained by introducing a complex scalar triplet ξ_i transforming as a **3** under $\mathrm{SU}(3)_N$, and how the presence of such an extra field would modify the vacuum structure and other aspects of the model. One new feature here is that ξ_i is even 'more sterile' than ϕ_{ij} in that not only it does not directly couple to SM particles (like ϕ_{ij}), but cannot even couple to right-chiral neutrinos if we insist on renormalizability and classical conformal invariance. As a consequence the associated new pseudo-Goldstone excitations are even more weakly coupled to SM matter than those coming from ϕ_{ij}.

With the extra triplet ξ_i, the most general renormalizable and $\mathrm{U}(3)_N$ symmetric scalar field potential reads

$$\begin{aligned}\mathcal{V}(H,\phi,\xi) &= m_1^2\, H^\dagger H + m_2^2\,\mathrm{Tr}(\phi\phi^*) + m_3^2\xi^\dagger\xi + (m_4\xi^\dagger\phi\xi^* + \mathrm{h.c}) \\ &\quad +\lambda_1\,(H^\dagger H)^2 + 2\lambda_3\,(H^\dagger H)\mathrm{Tr}(\phi\phi^*) + \lambda_2\,[\mathrm{Tr}(\phi\phi^*)]^2 + \lambda_4\,\mathrm{Tr}(\phi\phi^*\phi\phi^*) \\ &\quad +\lambda_5\,\xi^\dagger\phi\phi^*\xi + 2\lambda_6\,H^\dagger H\,\xi^\dagger\xi + 2\lambda_7\,\xi^\dagger\xi\,\mathrm{Tr}(\phi\phi^*) + \lambda_8\,(\xi^\dagger\xi)^2 \end{aligned} \tag{17}$$

where all coefficient are real except for m_4 (traces are over family indices). This potential is manifestly invariant under

$$\phi(x) \;\to\; U\phi(x)U^T\,, \quad \xi(x) \;\to\; U\xi(x) \qquad U\in\mathrm{U}(3)_N \tag{18}$$

One point to note is that with ξ_i one can easily arrange for 'anisotropic' expectation values $\langle\phi_{ij}\rangle$ not proportional to the unit matrix. As before one shows that there exists a range of parameters for which the global minimum of the potential takes the form

$$\langle\xi\rangle = \mathcal{U}_0\begin{pmatrix}0\\0\\e^{i\alpha}v_\xi\end{pmatrix}, \quad \langle H\rangle = \begin{pmatrix}0\\v_H\end{pmatrix}, \quad \langle\phi\rangle = \mathcal{U}_0\begin{pmatrix}v_1&0&0\\0&v_1&0\\0&0&v_2\end{pmatrix}\mathcal{U}_0^T \tag{19}$$

with positive parameters $v_\xi, v_H, v_1,\, v_2\,(\neq v_1)$, the phase α fixed by $\arg(m_4)$ and the vacuum alignment matrix $\mathcal{U}_0$ is of the same origin as before. The important new feature due to the presence of ξ_i is the special form of the matrix $\langle\phi_{ij}\rangle$, with the equality of the first two diagonal entries being due to the fact that the expectation value $\langle\xi_i\rangle$ singles out one particular direction in family space, thus also lifting the degeneracy in the heavy neutrino mass matrix obtained from (6).

Because the residual symmetry of (19) is SO(2), and the manifold of Goldstone bosons is the coset

$$\mathcal{M} = \mathrm{U}(3)_N \big/ \mathrm{SO}(2) \tag{20}$$

whence there are now altogether eight (pseudo-)Goldstone bosons. These can be parametrized as

$$\begin{aligned} \phi(x) &= \mathcal{U}_0\, e^{\mathrm{i}A(x)}\, \tilde{\phi}(x)\, e^{\mathrm{i}A(x)^T}\, \mathcal{U}_0^T \\ \xi(x) &= \mathcal{U}_0\, e^{\mathrm{i}A(x)}\, \tilde{\xi}(x) \end{aligned} \tag{21}$$

with

$$A(x) \equiv {\sum_a}' A_a(x)\lambda^a, \tag{22}$$

and where the sum runs over those generators λ^a (now including $\lambda^9 \equiv \mathbb{1}$) that are spontaneously broken by vacuum (19).

The analysis of the vacuum structure is now more cumbersome than before. Expanding $\tilde{\phi}(x)$ and $\tilde{\xi}(x)$ about the vacuum expectation values (19)

$$\tilde{\phi}_{ij}(x) = \langle \phi_{ij} \rangle + \phi'_{ij}(x), \qquad \tilde{\xi}_i(x) = \langle \xi_i \rangle + \xi'_i(x), \tag{23}$$

we have to ensure that the quantum fluctuations $\phi'_{ij}(x)$ and $\xi'_i(x)$ do not contain Goldstone bosons, as the latter are to be absorbed into $\mathcal{U}(x)$. In other words, the fields ϕ'_{ij} and ξ'_i should only contain the ten heavy non-Goldstone modes. This is ensured by imposing the condition (see [28], chapter 19)

$$\mathrm{Im}\Big\{ \xi'(x)^\dagger \lambda^a \langle \xi \rangle + \mathrm{Tr}\big[\phi'(x)^\dagger \left\{ (\lambda^a \otimes \mathbb{1} + \mathbb{1} \otimes \lambda^a) \langle \phi \rangle \right\} \big] \Big\} = 0, \quad \forall a \in \{1, \ldots, 9\}. \tag{24}$$

As before the main observable effects are due to the mixing between the standard Higgs boson and the new scalars, but there may now appear more narrow resonances above the standard Higgs resonance.

2.3 Fermionic sector

With right-chiral neutrinos, the SM comprises altogether 48 fundamental spin-$\frac{1}{2}$ degrees of freedom, in three generations (families) of 16 fermions each. It is one of our basic assumptions that there are no other spin-$\frac{1}{2}$ degrees of freedom.[4] This assumption is mainly motivated by observation, that is, the complete lack of evidence so far of such new fermionic degrees of freedom at LHC. In fact, already the LEP experiment had produced strong evidence that there exist only three generations, so any extra spin-$\frac{1}{2}$ fermions beyond the known quarks and leptons would have to be either sterile, or otherwise appear as heavy fermionic superpartners of the known SM bosons (thus not implying the existence of new families of fermions).

We here concentrate on the Yukawa part of the extended CSM Lagrangian, referring to [29, 30] for the complete SM Lagrangian and its properties. With the above assumptions concerning the fermionic sector and the new scalar sextet introduced in the foregoing section, we can write down right away the most general Yukawa couplings: the Higgs doublet couples in the usual way, while ϕ_{ij} couples only to the right-chiral neutrinos. Accordingly, the complete Yukawa part of the Lagrangian is [5]

$$\begin{aligned} \mathfrak{L}_{\mathrm{Y}} \;\; &= \;\; \big\{ - Y_{ij}^{E} H^{\dagger} L^{i\alpha} E_{\alpha}^{j} - Y_{ij}^{D} H^{\dagger} Q^{i\alpha} D_{\alpha}^{j} - Y_{ij}^{U} H^{T} \varepsilon Q^{i\alpha} U_{\alpha}^{j} \\ & \qquad - Y_{ij}^{\nu} H^{T} \varepsilon L^{i\alpha} N_{\alpha}^{j} - \frac{1}{2} y_{M} \phi_{ij} N^{i\alpha} N_{\alpha}^{j} \big\} \; + \; \text{h.c.} \end{aligned} \tag{25}$$

Here Q_{α}^{i} and L_{α}^{i} are the left-chiral quark and lepton doublets, $\bar{U}^{i\dot{\alpha}}$ and $\bar{D}^{i\dot{\alpha}}$ are the right-chiral up- and down-like quarks, while $\bar{E}^{i\dot{\alpha}}$ are the right-chiral electron-like leptons, and $\bar{N}^{i\dot{\alpha}}$ the right-chiral neutrinos; the family indices $i, j = 1, 2, 3$ as well as $SL(2, \mathbb{C})$ indices are written out explicitly. Classically, the full SM Lagrangian is invariant under lepton number symmetry $\mathrm{U}(1)_{L}$ as well as under the usual baryon number symmetry $\mathrm{U}(1)_{B}$; these two U(1) symmetries combine to the anomaly free $\mathrm{U}(1)_{B-L}$ symmetry which is hence preserved to all orders.

The main new feature of our model is that the right-chiral neutrinos transform under the previously introduced symmetry $\mathrm{SU}(3)_{N}$ according to

$$N^{i}(x) \quad \rightarrow \quad (U^{*})^{i}{}_{j} N^{j}(x) \tag{26}$$

[4]The occurrence of 16 fermions in one generation is often interpreted as strong evidence for an underlying SO(10) GUT symmetry. However, apart from the fact that SO(10) cannot explain the origin of the family reduplication, there may be alternative explanations. In particular, $48 = 3 \times 16$ is also the number of physical spin-$\frac{1}{2}$ fermions in maximally extended ($N = 8$) supergravity remaining after complete breaking of supersymmetry. See [31] for a fresh look at this coincidence.

[5]We will make consistent use of Weyl (two-component) spinors throughout, see e.g. [5] for our conventions, as we have found them much more convenient than 4-spinors in dealing with the intricacies of the neutrino sector.

whereas all other SM fermions are inert under this symmetry.[6] This reflects the essential difference in our model between the quarks and leptons on the one hand, where the Yukawa couplings are given by numerical matrices, and the right-chiral neutrinos on the other, where the *effective* couplings are to be determined as vacuum expectation values of sterile scalar fields. The $SU(3)_N$ symmetry is thus broken explicitly only by one term in (25), namely the interaction

$$\mathfrak{L}'_{\mathrm{Y}} = -Y^{\nu}_{ij} H^T \varepsilon L^{i\alpha} N^j_\alpha \,+\, \mathrm{h.c.} \tag{27}$$

coupling the lepton doublet and the right-chiral neutrinos. Consequently, (27) is the only term in the SM Lagrangian by which right-chiral neutrinos communicate with the rest of the SM, and hence will play a key role in the remainder. We repeat that this interaction *does* preserve $U(1)_{B-L}$. The numerical matrix Y^{ν}_{ij} here must be assumed very small [with entries of order $\mathcal{O}(10^{-6})$], in order to explain the smallness of light neutrino masses via the see-saw mechanism [32, 33, 34, 35] with TeV scale heavy neutrinos.

The neutrino masses emerge upon spontaneous symmetry breaking in the usual way, and thus depend on the matrices m_{ij} and M_{ij} defined by the vacuum expectation values of the corresponding scalar fields, *viz.*

$$M_{ij} = y_M \langle \phi_{ij} \rangle \tag{28}$$

and

$$m_{ij} \,=\, Y^{\nu}_{ij}\, v_H \tag{29}$$

Given these matrices, the (squared) masses of the light neutrinos are then determined as the eigenvalues of the following matrices (see e.g. [5] for a derivation), namely

$$\mathbf{m}^2_\nu = \boldsymbol{m}^\dagger \boldsymbol{m}, \qquad \text{with} \;\; \boldsymbol{m} \equiv m M^{-1} m^T + \ldots \tag{30}$$

for the light neutrinos, and

$$\mathbf{M}^2_N = \boldsymbol{\mathcal{M}}^\dagger \boldsymbol{\mathcal{M}}, \qquad \text{with} \;\; \boldsymbol{\mathcal{M}} \equiv M + \tfrac{1}{2} m^T m^* M^{-1} + \tfrac{1}{2} M^{-1} m^\dagger m + \ldots \tag{31}$$

for the heavy neutrinos. These formulas generalize the well-known seesaw mass formulas [32, 33, 34, 35]. Assuming $m \sim 100$ keV and $M \sim 1$ TeV we

[6]Strictly speaking, we should therefore use two different kinds of family indices, one for the usual quarks and leptons, the other for the right-chiral neutrinos. We will refrain from doing so in order to keep the notation simple.

get light neutrinos with masses of order 0.01 eV, and heavy neutrinos with masses of order 1 TeV. The mass eigenvalues are furthermore constrained by the known mass differences δm_ν^2.

We conclude this section by giving the neutrino propagators derived in [5] for the case when M_{ij} is given by (28) with (6). With a proper change of basis in the space of right-chiral neutrinos we can assume, that $M_{ij} = M\delta_{ij}$ with a positive parameter M (this change will also modify m, see below). Moreover, because the effects we are looking for depend on the small matrix m_{ij} we can simplify the expressions further by expanding in powers of m_{ij}. Up to and including terms $\mathcal{O}(m^3)$ this gives (suppressing family indices)

$$\begin{aligned}
\langle \nu_\alpha \bar{\nu}_{\dot\beta} \rangle &= -\mathrm{i}\frac{\not{p}_{\alpha\dot\beta}}{p^2}\big(1 - m^*\mathcal{D}(p)^*(p^2 + m^\dagger m)m^T\big) = \\
&= -\mathrm{i}\ \not{p}_{\alpha\dot\beta}\Big(\frac{1}{p^2} - \frac{m^* m^T}{p^2(p^2+M^2)} + \ldots\Big) \\
\langle N_\alpha \bar{N}_{\dot\beta} \rangle &= -\mathrm{i}\ \not{p}_{\alpha\dot\beta}\mathcal{D}(p)(p^2 + m^T m^*) = \\
&= -\mathrm{i}\ \not{p}_{\alpha\dot\beta}\Big(\frac{1}{p^2+M^2} - \frac{m^\dagger m}{(p^2+M^2)^2} + \frac{M^2 m^T m^*}{p^2(p^2+M^2)^2} + \ldots\Big) \\
\langle \nu_\alpha \nu_\beta \rangle &= \mathrm{i}\epsilon_{\alpha\beta} M m^* \mathcal{D}(p)^* m^\dagger = \\
&= \mathrm{i}\epsilon_{\alpha\beta}\Big(\frac{M m^* m^\dagger}{p^2(p^2+M^2)} + \ldots\Big) \\
\langle N_\alpha N_\beta \rangle &= -\mathrm{i}\epsilon_{\alpha\beta} M p^2 \mathcal{D}(p) = \\
&= -\mathrm{i}\epsilon_{\alpha\beta}\Big(\frac{M}{p^2+M^2} - \frac{M(m^\dagger m + m^T m^*)}{(p^2+M^2)^2} + \ldots\Big) \\
\langle \nu_\alpha \bar{N}_{\dot\beta} \rangle &= \mathrm{i}\ \not{p}_{\alpha\dot\beta} M m^* \mathcal{D}(p)^* = \\
&= \mathrm{i}\ \not{p}_{\alpha\dot\beta}\Big(\frac{M m^*}{p^2(p^2+M^2)} - \frac{M m^*(m^T m^* + m^\dagger m)}{p^2(p^2+M^2)^2} + \ldots\Big) \\
\langle \nu_\alpha N_\beta \rangle &= -\mathrm{i}\epsilon_{\alpha\beta} m^* \mathcal{D}(p)^*(p^2 + m^\dagger m) = \\
&= -\mathrm{i}\epsilon_{\alpha\beta}\Big(\frac{m^*}{p^2+M^2} - \frac{m^* m^T m^*}{(p^2+M^2)^2} + \frac{M^2 m^* m^\dagger m}{p^2(p^2+M^2)^2} + \ldots\Big) \quad (32)
\end{aligned}$$

where

$$\not{p}_{\alpha\dot\beta} = p_\mu \sigma^\mu_{\alpha\dot\beta}, \qquad \sigma^\mu = (\mathbb{1}, \sigma^i)$$
$$\mathcal{D}(p) \equiv \left[(p^2 + m^T m^*)(p^2 + m^\dagger m) + p^2 M^2\right]^{-1} = \mathcal{D}(p)^T \tag{33}$$

In evaluating the Feynman integrals we should keep in mind that expressions containing the matrix m_{ij} can originate both from this expansion as well as from the interaction vertex (35) below.

2.4 Pseudo-Goldstone bosons

As we already pointed out, besides the Majoron, there appear five Goldstone bosons. The latter can be converted to pseudo-Goldstone bosons via radiative corrections that originate from the Yukawa term (27). To make all this more explicit we need to parametrize the Goldstone manifold $\mathcal{M}$ in (7). To this aim, we first separate off the (pseudo-)Goldstone modes by means of the formula (9). According to (11) we can then split $\tilde{A}_{ij}(x)$ into a trace part and the rest. As we will see below, because of the explicit breaking of $SU(3)_N$, and hence also its SO(3) subgroup, induced by the Yukawa couplings Y^ν_{ij}, the five Goldstone fields contained in $A_{ij}(x)$ will actually acquire very small masses, and thus metamorphose into pseudo-Goldstone bosons.

To proceed it is convenient to eliminate the pseudo-Goldstone boson fields from the Majorana Yukawa coupling $\propto \phi NN$ by absorbing them into the right-chiral neutrino spinors

$$N^i_\alpha(x) = (\mathcal{U}_0^* e^{-\mathrm{i}\tilde{A}(x)} \mathcal{U}_0^T)^i{}_j \tilde{N}^j_\alpha(x), \tag{34}$$

where we have included the (constant) 'vacuum realignment matrix' $\mathcal{U}_0$ that is implicitly determined by requiring absence of tadpoles (or equivalently, $\langle A \rangle = 0$ for the vacuum of the one-loop corrected effective potential). For the remaining SM fermions there is a similar redefinition only involving the Majoron $\mathfrak{a}(x)$. After this redefinition the Goldstone modes only appear in the Dirac-Yukawa coupling (27) and via derivative couplings of the type $\partial_\mu A \bar{f} \gamma^\mu \frac{1+\gamma_5}{2} f$. The only *non-derivative* couplings of the pseudo-Goldstone fields are thus given by

$$\begin{aligned}\mathfrak{L}'_{\mathrm{Y}} &= -(Y^\nu \mathcal{U}_0^* e^{-\mathrm{i}A(x)} \mathcal{U}_0^T)_{ij} H^T \varepsilon \tilde{L}^{i\alpha} \tilde{N}^j_\alpha + \text{h.c.} \\ &= -v_H (Y^\nu \mathcal{U}_0^* e^{-\mathrm{i}A(x)} \mathcal{U}_0^T)_{ij} \tilde{\nu}^{i\alpha} \tilde{N}^j_\alpha + \text{h.c.} + \cdots \end{aligned} \tag{35}$$

The Majoron $\boldsymbol{a}(x)$ has disappeared from the above interaction term because of the accompanying redefinitions of the left-chiral leptons, in accordance with exact $\mathrm{U}(1)_{B-L}$ symmetry (thus, $\boldsymbol{a}(x)$ has *only* derivative couplings). Even though the interaction (35) now looks non-renormalizable, it is, of course, not. However, in order to recover renormalizability in this 'picture' one must expand the exponential as appropriate. For instance, at one loop we will have to take into account both linear and quadratic terms in $\tilde{A}(x)$ when computing mass corrections, see below.

At this point we can also absorb the vacuum realignment matrix $\mathcal{U}_0$ into a redefinition of the Yukawa couplings. For this purpose we redefine the right-chiral neutrino fields once again

$$\tilde{N}^j_\alpha(x) \equiv \left(\frac{y_M^*}{|y_M|}\right)^{1/2} (\mathcal{U}_0^*)^j{}_i \widehat{N}^i_\alpha(x)\,, \tag{36}$$

so that, in terms of new fermion fields, the vertex (35) reads

$$\mathfrak{L}'_{\mathrm{Y}} \;=\; -v_H(\hat{Y}^\nu\, e^{-\mathrm{i}A(x)})_{ij}\tilde{\nu}^{i\alpha}\hat{N}^j_\alpha \,+\, \text{h.c.} \,+\, \cdots \tag{37}$$

with the redefined Yukawa coupling matrix

$$\hat{Y}^\nu = \left(\frac{y_M^*}{|y_M|}\right)^{1/2} Y^\nu \mathcal{U}_0^*. \tag{38}$$

The presence of a non-trivial vacuum realignment matrix $\mathcal{U}_0$ entails the following redefinition of the Dirac and Majorana mass matrices

$$\hat{m} = \hat{Y}^\nu\, v_H, \qquad \hat{M} = |y_M|v_\phi \mathbb{1}. \tag{39}$$

so the redefined Majorana mass matrix is diagonal.

For the calculation of the radiative correction we employ the neutrino matrix propagators listed in (32), with $M \equiv |y_M|v_\phi$ and the replacement $m \mapsto \hat{m}$. While the original potential did not depend on $A(x)$ at all, this vacuum degeneracy is lifted at one loop due to the interactions induced by the term (35) in the effective potential. The result can be expanded in powers of A, but we are interested here only in the linear and quadratic terms. There is a finite contribution to the term linear in A_{ij}, which is proportional to

$$\int \frac{d^4p}{(2\pi)^4}\frac{M^2}{p^2(p^2+M^2)^2}\,\mathrm{Tr}\left(\left[\hat{m}^\dagger\hat{m}\,,\,\hat{m}^T\hat{m}^*\right] A\right) \tag{40}$$

and comes from the tadpole diagram, using the $\langle \nu_\alpha N_\beta \rangle$ and $\langle \bar{\nu}_{\dot{\alpha}} \bar{N}_{\dot{\beta}} \rangle$ propagators from (32), with one factor m from the vertex and the other factors containing m from the propagators. *Importantly, there are neither quadratic nor logarithmic divergences.* To switch to the true vacuum we now require absence of tadpoles [26], or equivalently, the vanishing of the linear term above. This amounts to choosing the vacuum realignment matrix $\mathcal{U}_0$ in such a way that $\hat{m}^\dagger \hat{m} = \mathcal{U}_0^T (m^\dagger m) \mathcal{U}_0^*$ is real (such a matrix always exists because $m^\dagger m$ is hermitean). Consequently,

$$\left[\hat{m}^\dagger \hat{m} \,,\, \hat{m}^T \hat{m}^*\right] \;=\; 0 \tag{41}$$

We emphasize that (41) does not restrict the parameters of the Lagrangian in any way, but simply tells us that the matrix $\mathcal{U}_0$ is determined from Y^ν in order to reach the true vacuum of the one-loop effective potential.

Remarkably, the explicit form of the matrix $\mathcal{U}_0$ is thus not needed, it is enough to simply impose the condition (41). For instance, in the so-called Casas-Ibarra parametrization [36] the redefined $\hat{Y}^\nu$ matrix has the form

$$\hat{Y}^\nu = \frac{1}{v_H} U_\nu^* \sqrt{\bar{m}_\nu} R_{CI} \sqrt{\hat{M}}, \tag{42}$$

with a complex orthogonal R_{CI} matrix and a unitary matrix U_ν (sometimes called PMNS matrix, being the neutrino analog of the CKM matrix); furthermore, the diagonal matrix $\bar{m}_\nu$ of eigenmasses of light neutrinos

$$\bar{m}_\nu = \mathrm{diag}\big(\bar{m}_{\nu,1}, \bar{m}_{\nu,2}, \bar{m}_{\nu,3}\big). \tag{43}$$

The general solution to (41) then requires (assuming $\det \bar{m}_\nu \neq 0$)

$$R_{CI}^* = R_{CI}. \tag{44}$$

Thus all phases of $\hat{Y}^\nu$ are contained in the PMNS matrix. *To simplify the notation we will from now on assume that the couplings have been appropriately redefined and drop the hats in all formulas.*

At quadratic order in A there are eight contributions from the usual loop diagrams and two contributions from the tadpole diagrams with two external A legs which endow the erstwhile Goldstone bosons with a (small) mass. At one loop the relevant contributions come from the diagrams depicted in Fig. 1 below.

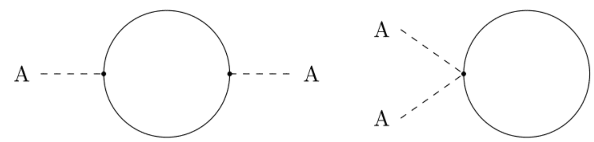

Fig. 1. Two types of diagrams that contribute to the quadratic terms in the potential for A. Every vertex couples to either $\nu^{i\alpha} N^j_\alpha$ or $\bar{\nu}^i_{\dot{\alpha}} \bar{N}^{j\dot{\alpha}}$; solid lines represent neutrino propagators from (32).

Up to and including $\mathcal{O}(m^4)$ terms they are given by

$$
\begin{aligned}
(1) \quad = \quad & \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \Bigg\{ \frac{-2}{p^2+M^2} m^\dagger m A^2 + \frac{2}{(p^2+M^2)^2} (m^\dagger m)^2 A^2 \\
& + \frac{-2M^2}{p^2(p^2+M^2)^2} m^\dagger m A m^T m^\star A \\
& + \frac{-M^2}{p^2(p^2+M^2)^2} \Big(\big[m^\dagger m, A\big] \big[m^T m^\star, A\big] \Big) \Bigg\}
\end{aligned}
\tag{45}
$$

and

$$
\begin{aligned}
(2) \quad = \quad & \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \Bigg\{ \frac{2}{p^2+M^2} m^\dagger m A^2 + \frac{-2}{(p^2+M^2)^2} (m^\dagger m)^2 A^2 \\
& + \frac{2M^2}{p^2(p^2+M^2)^2} m^\dagger m A m^T m^\star A \\
& + \frac{-M^2}{p^2(p^2+M^2)^2} \Big(\big[m^\dagger m, A\big] \big[m^T m^\star, A\big] \Big) \Bigg\} \ .
\end{aligned}
\tag{46}
$$

Adding the two contributions we see that all the divergent terms cancel, so we are left with a finite integral. Integrating over the momentum we arrive at the very simple and suggestive formula (now in terms of the dimensionful fields $G(x)$ introduced in (12))

$$
\mathcal{L}_{\mathrm{eff}}(A) \ni \frac{1}{8\pi^2 v_\phi^2} \mathrm{Tr} \Big(\big[m^\dagger m, G\big] \big[m^T m^*, G\big] \Big)
\tag{47}
$$

Since the terms of order $\mathcal{O}(m^2)$ cancel at one loop, and only terms $\mathcal{O}(m^4)$ remain, one can worry that higher loop corrections can be more important than the contribution calculated above. However, there is a very simple argument showing that the terms of order $\mathcal{O}(m^2)$ will always cancel. Namely, if we focus on terms that do not contain derivatives of A, the only way A can appear in the formula is through the exponential factor in Yukawa coupling (37). That means that the potential for A can be calculated from the contributions to the vacuum energy by substituting $m \to me^{-\mathrm{i}A}$ in the formulae. Because the only structures of order $\mathcal{O}(m^2)$ that can appear with the breaking pattern (6)) in the vacuum diagram are $mm^\dagger$ and m^*m^T, which are invariant under this substitution, there will be no $\mathcal{O}(m^2)$ terms in the potential for A, at any loop order. Terms containing mm^T and $m^*m^\dagger$ that potentially could provide contributions, will not appear because lepton number is conserved in the SM. The terms of order $\mathcal{O}(m^4)$ can only appear because the allowed structure $mm^Tm^*m^\dagger$ is not invariant under this substitution. This also shows why commutator structures appear:

$$\begin{aligned} mm^Tm^*m^\dagger \to\ & me^{-2\mathrm{i}A}m^Tm^*e^{2\mathrm{i}A}m^\dagger = \\ & = mm^Tm^*m^\dagger - 2\mathrm{i}m[A, m^Tm^*]m^\dagger - 2m[A,[A,m^Tm^*]]m^\dagger + \ldots \end{aligned} \tag{48}$$

If we had additional scalar fields like in (17), or any other mechanism for which $M_{ij} \sim \langle\phi_{ij}\rangle$ is not proportional to the identity matrix, then the structures that can appear are more complicated. For example, instead of simple structure $mm^\dagger$, we could have $mf(M)m^\dagger$, with f being some function of matrix M. Substituting $m \to me^{-\mathrm{i}A}$ now produces the following terms:

$$\begin{aligned} mf(M)m^\dagger \to\ & me^{-\mathrm{i}A}f(M)e^{\mathrm{i}A}m^\dagger = \\ & = mf(M)m^\dagger - \mathrm{i}m[A, f(M)]m^\dagger + \ldots \end{aligned} \tag{49}$$

Ultimately, those of the fields A that do not commute with matrix M will obtain mass terms of order $\mathcal{O}(m^2) = \mathcal{O}((\sqrt{m_\nu M})^2)$. For those that do commute however, this terms will vanish, and the leading contribution to their mass will come from (47).

The finiteness of the result (47) is crucial, and this is the sense in which the approximate $\mathrm{SU}(3)_N$ symmetry 'protects' the pseudo-Goldstone bosons from acquiring large masses. If there were divergences the pseudo-Goldstone masses would have to be fixed by some renormalisation procedure, and we could no longer claim that they are 'naturally' small. We also note that (47) vanishes for diagonal $G(x)$, hence two of the Goldstone bosons remain massless at this order (but not beyond). As (47) shows, the mass values are slightly smaller than the (light) neutrino masses. Likewise, the part in A proportional to the unit matrix would drop out in this formula, and the associated Goldstone boson would thus remain massless (but we note that this formula is anyway not directly applicable to the Majoron $\boldsymbol{a}(x)$ as this field drops out from the vertex (35) after re-defining all SM fermions).

2.5 Pseudo-Goldstone couplings

The pseudo-Goldstone particles couple, via the Yukawa interaction (27), to the usual ('non-sterile') SM particles. Because these couplings receive non-vanishing contributions only at higher orders in the loop expansion they are naturally small, with calculable coefficients [5], and this fact makes them obvious Dark Matter candidates. In this subsection we briefly discuss some of the possible couplings, in particular the couplings to neutrinos and photons. These are not only relevant to the question which pseudo-Goldstone excitations can survive to the present epoch and hence serve as viable dark matter candidates, but also to the question whether their decays can be observed in principle. The decays of these pseudo-Goldstone bosons into other lighter pseudo-Goldstone bosons are strongly suppressed.

The first point to note is that our pseudo-Goldstone bosons cannot decay into light neutrinos because by (47) their masses are generically below the light neutrino mass values. This is crucial for them to be viable dark matter candidates, as otherwise they would have decayed long ago! However, they can decay into photons, with a calculable rate. This rate follows from an explicit calculation of the effective vertex

$$\mathcal{L} \sim \frac{1}{v_\phi M^4} \sum_i g_{A\gamma\gamma,i} \big(m^* \big[m^\dagger m, A\big] m^T\big)_{ii} F_{\mu\nu} F^{\mu\nu} \tag{50}$$

where $g_{A\gamma\gamma,i}$ is of the order of 10^{-4} for $M \sim 200$ GeV. Observe that the coefficient $g_{A\gamma\gamma\, i}$ depends on the family index i via the mass of the associated lepton $m_i \equiv (m_e, m_\mu, m_\tau)$, otherwise this term would vanish altogether with our minimization condition from (41). As a consequence the result depends on the (very small) differences between the contributions from different leptons. Even without taking this into account the effective decay rate is extremely small

$$\Gamma_{A\gamma\gamma} \sim \frac{g^2_{a\gamma\gamma} m^2_\nu}{8\pi M^2} \frac{m^3_A}{v^2_\phi} \ll 10^{-42}\text{eV} \tag{51}$$

This is many orders of magnitude less than the Hubble parameter ($H_0^{-1} \sim$

10^{-32} eV). Therefore we conclude that these pseudo-Goldstone bosons are stable.

The result (50) may also be important for axion searches (see [37]). However, for the present model with only a sterile scalar sextet, the effective coupling is of the order of

$$\frac{1}{f_\gamma} \sim \frac{m_\nu^2}{M^3} \sim 10^{-24} \text{ GeV}^{-1} \tag{52}$$

and thus far beyond the reach of current experiments. However, this situation may well change in the presence of more complicated scalar sectors, such as the one discussed in section 2.2: if the eigenvalues of the mass matrix M of the heavy neutrinos were fundamentally different from each other, the coupling would be of order of $\frac{m_\nu}{M^2} \sim 10^{-13}$ GeV^{-1}. This value would still pose a challenge, but could be much closer to experimental verification.

We would also like to emphasize that the present model in principle allows not only for couplings of the type (50), but also for axionic couplings $\propto AF\tilde{F}$, such that there can appear effective couplings

$$\mathfrak{L}_{\text{eff}} \ni \frac{1}{2} aA \left(\mathbf{E}^2 - \mathbf{B^2}\right) + bA\, \mathbf{E}\cdot\mathbf{B} \tag{53}$$

with computable small coefficients a and b, and $A \equiv \sum_{i,j} c_{ij} A_{ij}$ a certain linear combination of the pseudo-Goldstone bosons.

In principle the pseudo-Goldstone bosons also couple to gluons, again with computable coefficients. As before the coupling need not be purely axionic. Not unexpectedly, the coupling turns out to be extremely small: for the present model it is proportional to (see [6] for a derivation)

$$\mathfrak{L}_{\text{eff}} \ni \frac{\alpha_W^2 y_M}{8\pi^2 M_W^4 v_\phi} \text{Tr}\, \Big(m\big[M^\dagger M, A\big]m^\dagger\Big) \left[\frac{\alpha_s}{4\pi} \text{Tr}\left(G^{\mu\nu}\tilde{G}_{\mu\nu}\right)\right] \tag{54}$$

in lowest order (involving several three-loop diagrams as in [5]). This expression vanishes if the matrix $M^\dagger M$ is proportional to unity, in which case one would have to go to the next order to obtain a non-vanishing result. However, it is possible to obtain a non-vanishing result already at this order with a more complicated scalar sector.

3 Constraints and predictions

Any of the following observations would immediately falsify the model:

- Detection of new fundamental spin-$\frac{1}{2}$ degrees of freedom, as predicted by all supersymmetric models (gluinos, photinos, higgsinos, etc.).
- Detection of *non-sterile charged* scalar degrees of freedom, as predicted by two-doublet models or all models of low energy supersymmetry (squarks, sleptons, etc.).
- Discovery of WIMPs (as being indicative of new large scales)

If none of the above shows up in the near future the model presented in this paper (or some modified version thereof) can be considered as an alternative.

The first test of the proposed scenario is of course whether it is possible at all to arrange the parameters such that all the conditions and constraints imposed by observations can be simultaneously satisfied in such a way that no large intermediate scales are needed, and the subset of couplings already known from the SM agrees with the ones computed in our model. We now list the conditions that will have to be met for our scenario to be consistent and compatible with what has been observed so far.

3.1 Perturbative Consistency

Scalar fields are usually accompanied by quadratic divergences, which are generally viewed as posing a fine tuning challenge. With several new scalar fields beyond the SM scalar sector we have to address this issue. The desired cancellation of quadratic divergences is one of the main motivations for 'going supersymmetric', but we will here follow a different, and more economical strategy, by imposing the cancellation of quadratic divergences directly in terms of bare parameters at the Planck scale [3]. The underlying assumption here is that at this scale a proper and finite theory of quantum gravity (not necessarily a space-time based quantum field theory) 'takes over' that will explain the cancellation in terms of some as yet unknown symmetry (different from low energy $N = 1$ supersymmetry). The corresponding conditions were already evaluated for a simpler model in [3], where it was shown that a realistic window could be found for the couplings. This analysis can be generalized to the present case.

In addition we require that none of the couplings should exhibit Landau poles over the whole range of energies from the electroweak scale to the Planck scale. Likewise, there should be no instabilities (in the form of lower unboundedness) of the effective potential over this range. Realizing this assumption in the concrete model at hand shows that the putative instability of the Higgs potential in the (un-extended) SM (see e.g. [22, 23, 24]) can be avoided altogether. Obviously, these requirements lead to strong restrictions on the couplings, and it is one of the main challenges whether these can be met with our other assumptions.

As explained in [3] for each scalar we impose the vanishing of the quadratic divergence associated with this scalar at the Planck mass, and then evolve back to the electroweak scale, matching the couplings to the electroweak couplings as far as they are known. For the investigations of the scale dependence of the couplings at one loop we need the coefficients in front of the quadratic divergences; they read

$$\begin{aligned} f_H &= \frac{9}{4}g_w^2 + \frac{3}{4}g_y^2 + 6\lambda_1 + 12\lambda_3 - 6y_t^2, \\ f_\phi &= 14\lambda_2 + 4\lambda_3 + 8\lambda_4 - |y_M|^2. \end{aligned} \tag{55}$$

Non-zero values of Y^ν do not produce additional quadratic divergences at one loop (except for a negligible contribution to f_H). At one loop the β-functions do not depend on the renormalization scheme, and can be deduced from the general expressions given in [38]; they are ($\tilde{\beta} \equiv 16\pi^2\beta$)

$$\begin{aligned} \tilde{\beta}_{g_w} &= -\frac{19}{6}g_w^3\,, \qquad \tilde{\beta}_{g_y} = \frac{41}{6}g_y^3, \qquad \tilde{\beta}_{g_s} = -7g_s^3, \\ \tilde{\beta}_{y_t} &= y_t\left\{\frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g_w^2 - \frac{17}{12}g_y^2\right\}, \qquad \tilde{\beta}_{y_M} = \frac{5}{2}y_M|y_M|^2, \\ \tilde{\beta}_{\lambda_1} &= \frac{3}{8}\left(3g_w^4 + 2g_w^2g_y^2 + g_y^4\right) - 6y_t^4 - 3\left(3g_w^2 + g_y^2 - 4y_t^2\right)\lambda_1 + \\ &\quad +12\left(2\lambda_1^2 + 2\lambda_3^2\right), \\ \tilde{\beta}_{\lambda_2} &= 40\lambda_2^2 + 8\lambda_3^2 + 6\lambda_4^2 + 32\lambda_2\lambda_4 + 2\lambda_2|y_M|^2, \\ \tilde{\beta}_{\lambda_3} &= \lambda_3\left[|y_M|^2 + 6y_t^2 - \frac{9g_w^2}{2} - \frac{3g_y^2}{2} + 12\lambda_1 + 28\lambda_2 + 8\lambda_3 + 16\lambda_4\right], \\ \tilde{\beta}_{\lambda_4} &= 22\lambda_4^2 + \left(2|y_M|^2 + 24\lambda_2\right)\lambda_4 - |y_M|^4, \end{aligned} \tag{56}$$

and

$$\tilde{\beta}_{m_1^2} = m_1^2 \left(12\lambda_1 - \frac{3}{2}\left(3g_w^2 + g_y^2\right) + 6y_t^2\right) + 24\lambda_3 m_2^2,$$

$$\tilde{\beta}_{m_2^2} = 8\lambda_3 m_1^2 + m_2^2 \left(28\lambda_2 + 16\lambda_4 + |y_M|^2\right). \tag{57}$$

Anomalous dimensions (in the Landau gauge) can be derived from the above expressions and the effective potential given below

$$\gamma_\phi = \frac{1}{32\pi^2}|y_M|^2, \qquad \gamma_H = -\frac{3}{64\pi^2}\left(3g_w^2 + g_y^2 - 4y_t^2\right). \tag{58}$$

We also refer to [24] for an investigation of the scale dependence of f_H in the Standard Model.

3.2 Vacuum stability

One of the important open issues for the SM concerns the stability of the electroweak vacuum. There are strong indications that this vacuum develops an instability around $\sim 10^{11}$ GeV when radiative corrections are taken into account [22, 23, 24]. More specifically, the *RG improved* one-loop potential $V_{\rm eff}(H) \propto \lambda(\mu = H)H^4$ becomes negative when the running coupling $\lambda(\mu = H)$ dips below zero, as it does for large field values $H \sim 10^{11}$ GeV. Remarkably, however, the potential fails to be positive by very little, so one might hope that a 'small' modification of the theory might remedy the instability. Although the RG improved one-loop potential is much more difficult to obtain in the multifield case than with only one scalar field (as in the SM), we will now argue that this is indeed the case for the present model. [7]

To confirm that the point (6) is indeed the global minimum of the full effective potential we recall that we impose the conditions of positivity of the quartic potential (listed in section 2.1) for all values of the RG scale μ between the electroweak and the Planck scale. In order to investigate this issue more carefully we note that for $Y^\nu = 0$ the effective potential has an exact U(3) symmetry, and thus reaches all its values on a submanifold

[7] This stability requirement was already present in previous versions of the CSM [2, 3]. See also [40] for an alternative proposal how to stabilize the electroweak vacuum.

parametrized by

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_4 \end{pmatrix}, \qquad \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_3 \end{pmatrix}, \tag{59}$$

with nonnegative parameters φ_i. Its explicit form (in the Landau gauge and the $\overline{MS}$ scheme of dimensional regularization) reads

$$\mathcal{V}_{\text{eff}}(\varphi) \,=\, \mathcal{V}\big(H\big(\varphi),\phi(\varphi)\big) \,+\, \hbar\mathcal{V}^{(1)}(\varphi) \,+\, \mathcal{O}(\hbar^2), \tag{60}$$

with the tree-level potential given in (4) and (we follow the notation of [39])

$$\begin{aligned} 64\pi^2\,\mathcal{V}^{(1)}(\varphi) \;\; = \;\; & \sum_{i=1}^{13} S_i^2 \left\{ \ln\frac{S_i}{\mu^2} - \frac{3}{2} \right\} + 3\,\mathbb{G}^2 \left\{ \ln\frac{\mathbb{G}}{\mu^2} - \frac{3}{2} \right\} + \\ & -2\sum_{i=1}^{3} N_i^2 \left\{ \ln\frac{N_i}{\mu^2} - \frac{3}{2} \right\} - 12\,T^2 \left\{ \ln\frac{T}{\mu^2} - \frac{3}{2} \right\} + \\ & +3\,Z^2 \left\{ \ln\frac{Z}{\mu^2} - \frac{5}{6} \right\} + 6\,W^2 \left\{ \ln\frac{W}{\mu^2} - \frac{5}{6} \right\}, \end{aligned} \tag{61}$$

where

$$W = \frac{1}{4} g_w^2 \varphi_4^2, \quad Z = \frac{1}{4}(g_w^2 + g_y^2)\varphi_4^2, \quad T = \frac{1}{2} y_t^2 \varphi_4^2, \quad N_i = \frac{1}{2}|y_M|^2 \varphi_i^2, \tag{62}$$

$$\mathbb{G} = \lambda_1 \varphi_4^2 + \lambda_3 \left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) + m_1^2, \tag{63}$$

and

$$S_i = \lambda_4 \varphi_i^2 + \lambda_3 \varphi_4^2 + \lambda_2 \left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) + m_2^2, \qquad \text{for } i = 1, 2, 3, \tag{64}$$

$$S_{4-9} = -\lambda_4 \left(\varphi_k^2 \pm \varphi_l \varphi_n\right) + \lambda_3 \varphi_4^2 + (\lambda_2 + \lambda_4) \left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) + m_2^2, \tag{65}$$

with $k, l, n = 1, 2, 3$ and $k \neq l \neq n \neq k$. Finally S_{10}-S_{13} are eigenvalues of the following 4×4 matrix

$$\mathcal{S} = \begin{pmatrix} D_1 & 2\lambda_2\varphi_1\varphi_2 & 2\lambda_2\varphi_1\varphi_3 & 2\lambda_3\varphi_1\varphi_4 \\ 2\lambda_2\varphi_1\varphi_2 & D_2 & 2\lambda_2\varphi_2\varphi_3 & 2\lambda_3\varphi_2\varphi_4 \\ 2\lambda_2\varphi_1\varphi_3 & 2\lambda_2\varphi_2\varphi_3 & D_3 & 2\lambda_3\varphi_3\varphi_4 \\ 2\lambda_3\varphi_1\varphi_4 & 2\lambda_3\varphi_2\varphi_4 & 2\lambda_3\varphi_3\varphi_4 & E \end{pmatrix}, \tag{66}$$

where

$$\begin{aligned} D_i &= (2\lambda_2 + 3\lambda_4)\,\varphi_i^2 + \lambda_3\varphi_4^2 + \lambda_2\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) + m_2^2, \\ E &= 3\lambda_1\varphi_4^2 + \lambda_3\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) + m_1^2. \end{aligned} \tag{67}$$

Typically, the *unimproved* one-loop potential (60) with $\mu = M_t \approx 173\,\text{GeV}$ can exhibit an instability below the Planck scale. However, this effect is spurious, as its origin is entirely due to large logarithms. Although a closed formula for the RG improved one-loop potential in terms of running couplings is unavailable in the multifield case (unlike for effective potentials with only *one* scalar field), we can nevertheless formulate an RG improved version by taking the field dependent 'radial norm'

$$\mu^2(H,\phi) = 2\left\{H^\dagger H + \text{Tr}\,(\phi^*\phi)\right\} = \sum_{i=1}^{4}\varphi_i^2 \equiv \|\varphi\|^2, \tag{68}$$

as the scale parameter in field space. Then one checks numerically that (the RG improved version of) the potential (60) remains positive for large values of $\|\varphi\|$ in the range

$$10\text{TeV} \lesssim \|\varphi\| \lesssim M_{Pl},$$

(in particular this is true for all points in the Table). This is a strong indication that the electroweak vacuum (6) remains the global minimum over this whole range of energies. The apparent discrepancy between the unimproved and the improved effective potential is the same as for the SM, where the unimproved one-loop effective potential likewise reaches the instability already for much smaller field values than the RG improved one.

3.3 Dark Matter constraints

We have already pointed out that the pseudo-Goldstone bosons of our model are natural Dark Matter candidates. However, in order to verify that they are really viable we need to check (1) whether they can be non-relativistic, and (2) whether they can survive till the present epoch [41]. As for the second requirement, we have already checked that the pseudo-Goldstone cannot decay into light neutrinos. The decay rate into photons was found to be very small, and many orders of magnitude smaller that the present Hubble parameter. Hence the pseudo-Goldstone particles are indeed stable.

The first requirement can be satisfied if at the time of the electroweak phase transition, *i.e.* for temperatures around 100 GeV, the causally connected region is smaller than the inverse mass of the Dark Matter candidate. This requirement comes from the fact that the potential for the scalar fields started to be nonvanishing at the time of the electroweak transition. At that point, the phase fields start to oscillate coherently, and the fluctuations of smaller wavelength than the causal region are suppressed. To get a rough estimate, we note that the causally connected region at that time of the phase transition ($\sim 10^{-10}$ s) was about 0.01 m; expressed in mass units this corresponds to a mass bigger than about 10^{-4} eV. As we can see from the formula (47) the masses of the pseudo-Goldstone bosons are not too much below the mass of the light neutrinos, so this requirement can be satisfied and they are naturally in a (small) window between 10^{-4} eV and the light neutrinos masses.

An equally important point concerns the abundance with which the Dark Matter particles are produced, so as to arrive at the desired value $\Omega_{DM} \sim 0.3$. In order to derive a very rough estimate we note that this requires (amongst other things) not only a knowledge of the pseudo-Goldstone masses, but also of the effective potential $V_{\text{eff}}(G)$. All we know is that the latter must be a single-valued function on the Goldstone manifold $\mathrm{SU}(3)_N/\mathrm{SO}(3)$, cf. (7). It is also clear from our foregoing considerations this potential is in principle calculable via the determination of the effective higher point vertices of the pseudo-Goldstone fields. At one loop the effective potential in G derives from

$$V_{\text{eff}}(G) \propto \mathrm{Tr}\left(me^{-iG/v_\phi}m^T m^* e^{iG/v_\phi}m^\dagger\right) \tag{69}$$

which yields the estimate

$$V_{\text{eff}}^{max} \propto \mathrm{Tr}\left(mm^T m^* m^\dagger\right) \sim m_\nu^2 M^2 \tag{70}$$

for the height of the potential. The contribution to Ω then follows from scaling up the energy density of the pseudo-Goldstone particles to the present epoch by means of the factor $(R_*/R_0)^3 \sim (T_0/T_*)^3$ where R_0 (T_0) is the present radius (temperature) of the universe, and R_* (T_*) the radius (temperature) of the universe when the abundance is produced. To estimate the latter, we observe that for $T > V_{\text{eff}}^{max}$ we have thermal equilibrium, and only for $T < V_{\text{eff}}^{max}$ can the pseudo-Goldstone particles start to be produced nonthermally by coherent oscillations. Therefore setting $T_* = (V_{\text{eff}}^{max})^{1/4}$ seems a reasonable choice; this gives

$$\Omega \;\sim\; \rho_{crit}^{-1} V_{\rm eff}^{max} \left(\frac{T_0}{T_*}\right)^3 \;\sim\; \rho_{crit}^{-1}\,(V_{\rm eff}^{max})^{1/4}\,T_0^3 \;\sim\; \rho_{crit}^{-1}\sqrt{m_\nu M}\,T_0^3 \qquad (71)$$

This is indeed an estimate that also gives about the right order of magnitude for standard axions, with $V_{\rm eff}^{max} = \Lambda_{QCD}^4$. In our case, the result comes out too small by two or three orders of magnitude. However, the above estimate is fraught with several uncertainties, apart from the precise details of the production mechanism, which may give rise to all kinds of 'fudge factors'. In particular, since there is a 'collective' of scalar fields involved in this process it is not clear whether there cannot exist new enhancement effects, similar to the resonant effects giving rise to leptogenesis as in [4, 42]. Furthermore, a modification of the scalar sector along the lines of section 2.2 might change the value of $V_{\rm eff}^{max}$, for instance replacing $m_\nu^2 M^2$ by $m_\nu M^3$ in (70) which would give the desired number. So this issue clearly requires further and more detailed study.

3.4 Leptogenesis

An important feature of the present model is that it can account for the observed matter-antimatter asymmetry ($\sim 10^{-10}$) in a fairly natural manner, namely via the mechanism of leptogenesis in the form proposed and studied by [4, 42]. Since the masses of right-chiral neutrinos are smaller than the usually quoted bound ($\gg 10^5$ TeV) we have to assume that the source of the asymmetry is *resonant leptogenesis* [4, 42]. One of the necessary conditions for this mechanism to work is the approximate degeneracy of the masses of right-chiral neutrinos – exactly as obtained in our model. The shift δM induced by the Dirac-Yukawa term is naturally very small, and turns out to be exactly of the magnitude required by the condition given in [4]:

$$\delta M \;\sim\; \Gamma \qquad (72)$$

This is because, on the one hand, the decay rate of a massive neutrino in our model is $\Gamma \;\sim\; Y_\nu^2 M$. On the other hand, the mass splitting induced by the Dirac-Yukawa coupling is $\delta M \;\sim\; Y_\nu^2 M$; the latter is caused by two sources – the diagonalization of the neutrino mass matrix in the presence of the Dirac Yukawa term (35) and the RGE running of the Majorana-Yukawa couplings from M_{PL} down to TeV scale of heavy neutrinos. It is important to emphasize that the condition $\delta M \sim \Gamma$ is thus very natural in our model, whereas it usually requires a certain amount of fine tuning, especially in GUT type models.

If we use the formula to estimate the baryonic asymmetry given in [42] we get the correct asymmetry taking into account light neutrino data and assuming nonzero (but small) phases of the PMNS matrix in Eq. (42). In our case, as we have already said in (44), the Casas-Ibarra matrix has to be real, so that the PMNS phases are responsible for the leptogenesis. For example, the points shown in Tab. 1, give $\eta_B \approx 6 \times 10^{-10}$ with PMNS phases of order 10^{-3}. We leave the details of this and other leptogenesis related calculations to a future publication.

3.5 Gauging $(B-L)$

The cancellation of $(B-L)$ anomalies is widely viewed as an indication that this symmetry should be gauged (see e.g. [14, 15] for recent work in this direction). Because the decay rates of the pseudo-Goldstone particles into Majorons are exceedingly small such a gauging is not necessarily required for our scenario to work. Nevertheless, one can enquire under what conditions gauging $\mathrm{U}(1)_{B-L}$ would be consistent both with our assumptions and existing experimental bounds. The associated $\mathrm{U}(1)_{B-L}$ gauge boson (*alias* the Z'-boson) would then also appear in the scalar kinetic terms

$$\mathfrak{L}_{kin} = -\mathrm{Tr}\big[(\partial^\mu \phi^* - 2ig' Z'^\mu \phi^*)(\partial_\mu \phi + 2ig' Z'_\mu \phi)\big] \; + \; \cdots \tag{73}$$

From these and the expectation values (6) we immediately deduce the mass of the Z'-boson

$$m^2_{Z'} \;=\; 24 g'^2 v_\phi^2 \tag{74}$$

The present lower limit on $m_{Z'}$ is 2.5 TeV [1]; inspection of the representative values listed in Table 1 shows that we can in principle arrange for this mass bound to be comfortably satisfied with our assumptions (the mass value could be pushed up even further if there were more 'sterile' scalars coupling to $\mathrm{U}(1)_{B-L}$). However, we also see that the margin is not too large: whereas the Z'-mass is usually assumed to originate from GUT-scale symmetry breaking and hence should be expected to be far above the 1 TeV scale, it cannot be too large in the present context. Indeed, while the appearance of a Z' gauge boson in the 10 TeV range would seem difficult to reconcile with a GUT-type scenario, it would constitute clear evidence for the present scheme!

3.6 New scalar particles

Because much of the new structure of the model is sterile, not many dramatic new effects are expected to be observable beyond the SM. Nevertheless, there are distinctive signatures that are very specific to the present scenario, and that can be easily used to discriminate it from other BSM scenarios. These are mainly due to the mixing of the new scalars with (the H_0 component of) the Higgs doublet induced by the potential (4) with (6). From (16) we immediately get he decomposition of H_0 in terms of the mass eigenstates h_0 and h'

$$H_0 \;=\; \cos\beta\, h_0 \,-\, \sin\beta\, h' \tag{75}$$

whence the scattering amplitude would be well approximated by

$$\mathcal{A} \;\propto\; \frac{\cos^2\beta}{p^2 + m_{h_0}^2 + i\cos^2\beta\, m_{h_0}\,\Gamma_{SM}(m_{h_0}^2)} \;+ \\ + \frac{\sin^2\beta}{p^2 + m_{h'}^2 + i\sin^2\beta\, m_{h'}\,\Gamma_{SM}(m_{h'}^2)} \tag{76}$$

The existing experimental data suggest that $|\cos\beta|$ should be close to 1, if h_0 is to mimic the SM Higgs boson. The particle corresponding to h' has not been observed yet. The mixing will thus induce interactions of this new mass eigenstates with SM particles. In particular the decay channels of the standard Higgs boson are also open to the new scalar excitations, possibly leading to a kind of 'shadow Higgs' phenomenon, with decay amplitudes of approximately the same height but sharply reduced width [27]. In addition, depending on the mass values of the new scalars there may be extra decay channels involving new scalars, and possibly even heavy neutrinos, leading to a broadening of the resonance curve.

The existence of new scalar degrees of freedom mixing with the standard Higgs boson is the main *generic prediction* of the present model. It is a distinct signature that, though perhaps not so easy to confirm, can serve to discriminate the present model from other scenarios, in particular supersymmetric and two-doublet models which inevitably contain *non-sterile* scalars, or the νMSM model of [20], which does have a sterile scalar (and also keV range 'heavy' neutrinos), but absolutely nothing above m_{h_0} in the TeV range. Thanks to the mixing the new scalar(s) may eventually be seen at LHC, but the actual discovery potential for discovery depends, of course, on their masses, mixing angles etc. The mixing would also lead to a slight diminution in the decay width of the SM Higgs boson that can be measured in future precision tests at the Higgs resonance.

3.7 Numerical analysis

We conclude this paper by giving some numerical data which show that there exists a wide range of points in parameter space with the following properties:

- The quartic potential $\mathcal{V}^{quart}(H, \phi, \mu)$ is positive definite for all renormalization scales μ between M_t and the M_{Pl}, while all dimensionless couplings $c(\mu) = (\lambda(\mu), g(\mu), y(\mu))$ remain perturbative in this range (i.e. $|c(\mu)| < 4$ in our normalization conventions);

- the coefficients functions f_i of the quadratic divergences defined in (55) vanish at the Planck scale;

- For $\mu = M_t$ there exists a stationary point of the type (6), with $v_H \approx 174\,\text{GeV}$, which is the *global* minimum of the potential (4); moreover, the SM-like Higgs particle can be arranged to have $M_{h_0} = 125\,\text{GeV}$ such that $|t_\beta| < 0.3$, cf. (75);

- There exists a matrix Y^ν consistent with both Dashen's conditions and light neutrino data that yields $\eta_B \approx 6 \times 10^{-10}$ as well as a positive semi-definite pseudo-Goldstone boson mass matrix corresponding to (47).

Some representative numerical examples are listed in Tab. 1 with $y_M = y_M(\mu = M_t)$. We also show there decay width of the 'shadow Higgs' h' and the branching ratios of h_0 and h' into 'old particles' ($\equiv OP$), i.e. particles discovered prior to 2012. All points have $M_{h_0} = 125\text{GeV}$ and $v_H = 174\text{GeV}$.

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Table 1: Example points (all dimensional parameters are given in GeV)

$\|y_M\|$	M_N	$M_{h'}$	M_R	t_β	$\Gamma_{h'}$	$BR(h' \to OP)$	$BR(h_0 \to OP)$
0.56	545	378	424	-0.3	3.1	0.59	0.69
0.54	520	378	360	-0.3	3.1	0.59	0.68
0.75	1341	511	1550	0.25	6.2	0.73	0.91
0.75	2732	658	3170	-0.16	5.9	0.74	0.99
0.82	2500	834	2925	0.15	10.9	0.74	0.98

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