

An Efficient Output Error Bound for Model Order Reduction of Parametrized Evolution Equations

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Numerical simulation of large-scale complex systems is challenging, especially when such a task needs to be done many times under parameter variations, e.g., in the context of optimization, control, and parameter estimation. Model order reduction (MOR) is a useful technique for constructing a low-cost surrogate, i.e., a reduced order model (ROM), which can reproduce the input-output response of the original large-scale system, with compromise on the accuracy to an acceptable extent. To compute a ROM, an efficient a posteriori error estimation is crucial because it enables the computation to be reliable and automatic.

In this work we propose an efficient output error estimation for projection based MOR methods applied to parametrized nonlinear evolution problems. For (nonlinear) evolution equations, time-stepping schemes are often used to solve them, and error estimations for reduced basis methods have been studied in recent years [1, 3]. The error estimator may lose sharpness when a large number of time steps are needed, because the error estimator is actually a summation of the error over the previous time steps. To circumvent this problem, we introduce a dual system at each time instance in the evolution process. With the help of the dual system and under suitable assumptions, the output error of the reduced model can be estimated more sharply. The proposed output error bound is independent of the choice of the projection matrix, and hence is independent of the MOR methods.

The proposed error estimation is applied to two kinds of evolution problems arising from chromatographic separation processes. One is batch chromatography with bi-Langmuir isotherm equations, which is a nonlinear parametric evolution problem. The other is a linear parametric periodic switching system, the model of simulated moving bed chromatography with linear isotherm equations. Reduced basis methods are used to construct the reduced order model. More precisely, the POD-Greedy algorithm [2] is employed to generate the reduced basis. Figure 1 illustrates the decay of the error bounds and the corresponding true error during the reduced basis extension process for the batch chromatographic model. It is seen that the proposed output error bound (ErrorBound-2) is much sharper than ErrorBound-1 [1, 3]. The resulting reduced models are employed to solve the underlying optimization problems. Employing the reliable compact reduced models, the optimization problems are solved much faster with sufficient accuracy. The results of the optimization of batch chromatography are summarized in Table 1. It is seen that the optimal solution of the ROM based optimization converges to the full order model (FOM) based optimal solution, and the runtime is significantly reduced. The speedup factor (SpF) is 58.

Table 1: Results of the optimization based on the FOM ($\mathcal{N} = 1500$) and the ROM ($N = 45$).

Models	Objective	Optimal solution	#Iterations	Runtime [h]/SpF
FOM-Opt.	0.020264	(0.07964, 1.05445)	202	33.88 / -
ROM-Opt.	0.020266	(0.07964, 1.05445)	202	0.58 / 58

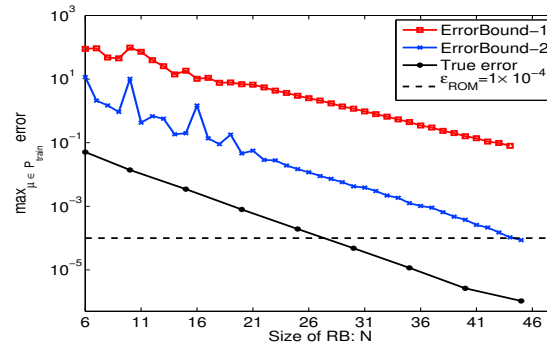


Figure 1: Illustration of the decay of the error bounds and the corresponding true error during the reduced basis extension process for the batch chromatographic model. ErrorBound-1 refers to the output error bound used in [3], while ErrorBound-2 is the proposed output error bound.

References

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