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# Routes towards Emergent Gravity

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**Abstract.** We discuss a toy model for an emergent non-relativistic gravitational theory. Within a certain class of Bose–Einstein condensates, it is possible to show that, in a suitable regime, a modified version of non-relativistic Newtonian gravity does effectively describes the low energy dynamics of the coupled system condensate/quasi-particles. Furthermore, we study the role of Lorentz and diffeomorphism invariance in emergent gravity scenarios by developing a toy model showing an emergent Lorentzian signature from a Euclidean setting and simultaneously and emergent gravitational dynamics of the Nördstrom type (scalar gravity). Some lessons about the crucial challenges awaiting in the future the emergent gravity proposal are finally drawn.

## 1. Introduction: BEC dynamics

In recent years the emergent gravity approach has constantly gained momentum [1]. In this particular perspective, the gravitational field, encoded in the spacetime geometry and its dynamics, is seen as a kind of large number/thermodynamical limit of some more fundamental theory.

While there is no proof that this is the case, there are evidences to support this point of view. First of all, let us remind the striking correspondence existing between the laws of black hole mechanics in General Relativity (GR) and the laws of thermodynamics [2]. This correspondence has inspired a deeper study of the thermodynamical aspects of gravitation. In particular, it has been shown that Einstein's equation can be seen as some sort of equation of state of a thermodynamical system at equilibrium [3, 4, 5, 6]. Furthermore, from the study of the properties of gravitational collapse and the resulting black hole formation, very specific patterns have been recognized in the parameter space of the initial conditions which resemble the behavior of critical phenomena (phase transitions) [7, 8].

In addition, it has been realized that within several condensed matter systems it is possible to distinguish some peculiar regimes in which the effective degrees of freedom are represented by fields propagating over effective pseudo-Riemannian structures [9].

For instance, in the case of perfect, irrotational and barotropic fluids, it can be proved that the perturbations in the velocity potential (*i.e.* the scalar function  $\theta$  whose gradient gives the velocity of the fluid,  $\vec{v} \propto \vec{\nabla}\theta$ ) do obey a massless Klein–Gordon equation in a curved effective spacetime whose metric tensor is given by the so-called acoustic metric,

$$g_{\mu\nu} = \frac{\rho}{c_s^2} \begin{pmatrix} -(c_s^2 - v^2) & \vdots & v^i \\ \cdots & \ddots & \cdots \\ v^i & \vdots & \delta_{ij} \end{pmatrix}, \quad (1)$$

where  $\rho$  is the local density of the fluid,  $c_s$  it the (local) speed of sound and  $v_i$  is the velocity field of the fluid flow.

Analogue models have been used to understand (and possibly to test in a laboratory) some peculiar aspects of physics in curved spacetimes, otherwise inaccessible (*e.g.* Hawking radiation). For a review of the subject see [9]. For the large majority, these analogue models for gravity do offer the possibility of studying some kinematical aspects of physics of curved spacetimes, leaving aside the issue of dynamics.

Despite the fact that, to date, there are no completely successful models of emergent gravity, *i.e.* non-gravitational system possessing an effective GR like limit, it is interesting to study simple toy models in which some aspects of the gravitational interaction can be mimicked. Here we present a toy model [10], based on a well-known condensed matter system, a Bose–Einstein condensate of dilute, weakly interacting gas of bosons [11]. While this toy model is very far from representing an analogue of a realistic theory of gravity, it gives nonetheless some insights on some interesting features of emergent theories.

The behaviour of dilute, weakly interacting Bose gases can be described with the formalism of second quantization, by introducing the field operators describing the gas atoms ( $V$  is the volume of the box in which we consider the gas)

$$\hat{\Psi}(\mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (2)$$

and a suitable Hamiltonian:

$$\hat{H}_0 = \int d^3x \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{-\hbar^2 \nabla^2}{2m} - \mu + \frac{\kappa}{2} |\hat{\Psi}(\mathbf{x})|^2 \right) \hat{\Psi}(\mathbf{x}). \quad (3)$$

As it is well known, for  $\mu, \kappa$  positive, this Hamiltonian leads to a condensation, *i.e.* the ground state of the system is characterized by a macroscopic occupation number of a single particle state. Within the formalism, this corresponds to the field operator  $\hat{\Psi}$  having a non-vanishing vacuum expectation value,

$$\hat{\Psi}(\mathbf{x}) \approx \psi(\mathbf{x})\mathbb{I} + \hat{\chi}(\mathbf{x}), \quad \langle \Omega | \hat{\Psi}(\mathbf{x}) | \Omega \rangle = \psi(\mathbf{x}), \quad (4)$$

where  $\psi$  is a classical complex scalar field, the condensate wavefunction, describing the mean field, and  $\hat{\chi}$  is a quantum operator describing the residual quantum fluctuations around the condensate, or, in more physical terms, the atoms which are out of the condensate.

Obviously, the nonlinearity in the Hamiltonian (3) makes the analysis of the physical properties of this system rather involved. As a first approximation, it is useful to assume that the non-condensed fraction is small compared to the condensed fraction.

The lowest order equation describes the condensate wave-function alone, without the backreaction terms induced by the  $\hat{\chi}$  field. It is called the Gross–Pitaevski equation and it is giving the first approximation to the condensate wavefunction dynamics. It is obtained from the equations of motion for the system by the replacement  $\hat{\Psi} \rightarrow \psi$ .

The most simple solutions to this equations are the constant ones, describing homogeneous condensates, which, in the analogue model perspective, correspond to a flat acoustic metric, *i.e.* to Minkowski spacetime, and therefore will play a particular role in the following. The weak field limit of GR will correspond to the weak field limit around this constant configuration. In fact, as a direct calculation show, the correspondence between the acoustic metric  $g_{\mu\nu}$  and the complex scalar field  $\psi$  is such that:

$$\left. \begin{aligned} g_{\mu\nu} &\approx \eta_{\mu\nu} + h_{\mu\nu} \\ \psi &\approx (\kappa/\mu)^{1/2}(1 + u(x) + iv(x)) \end{aligned} \right\} \Rightarrow h_{00} \propto u(x) \quad (5)$$

so that the perturbations in the number density encoded in  $u(x)$  are the natural candidate to represent some sort of Newtonian gravitational field.

The next order equation is an equation for the noncondensed fraction  $\hat{\chi}$  in the external field  $\psi$ . The equation of motion has the shape:

$$i\hbar \frac{\partial}{\partial t} \hat{\chi} = -\frac{\hbar^2}{2m} \nabla^2 \hat{\chi} + \mu \hat{\chi} + \mu \hat{\chi}^\dagger. \quad (6)$$

The Hamiltonian (3) does possess a global  $U(1)$  symmetry,  $\hat{\Psi} \rightarrow e^{i\alpha} \hat{\Psi}$ , which is spontaneously broken by the condensation mechanism (leading to the  $\psi \neq 0$  ground state). As a consequence of the Goldstone theorem, the excitations will be gapless. (This can also be proved directly by diagonalizing the equation (6) using the Bogoliubov transformations.) Finally, it can be showed that, in some low energy regime, quasiparticles are described by a massless scalar field propagating on an effective acoustic geometry of the same shape of (1).

### 1.1. The coupled system

In order to see how some sort of gravitational dynamics is encoded in the BEC, a suitable dynamical framework must be set up in order to see how the quasiparticles backreact over the condensate. This formalism consists in considering an improved version of the Gross–Pitaevski equation which consistently takes into account the effect of the particles out of the condensate. The Gross–Pitaevski (GP) equation is replaced by the so-called Bogoliubov–de Gennes (BdG) equation

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi - \mu \psi + \kappa |\psi|^2 \psi + 2\kappa \mathbf{n} \psi + \kappa \mathbf{m} \psi^*, \quad (7)$$

where  $\mathbf{n}, \mathbf{m}$  are given by the expectation values:

$$\mathbf{n} = \langle \Xi | \hat{\chi}(x)^\dagger \hat{\chi}(x) | \Xi \rangle, \quad \mathbf{m} = \langle \Xi | \hat{\chi}(x)^2 | \Xi \rangle, \quad (8)$$

where the state  $|\Xi\rangle$  is the particular state one is considering. Notice that, if this state were the Fock vacuum state for particles, these expectation values would be identically zero. Notice also that one is implicitly taking a normal ordering in the particle operator, so that an unphysical (divergent) zero point energy is removed automatically.

This equation, suitably modified, will give rise to the dynamics for the field  $u(x)$ , and hence to an analogue of the Poisson equation.

To construct some analogue of Newtonian gravity, we need massive particles as sources of the gravitational field (massless particles do not gravitate in Newtonian gravity). Therefore, the quasiparticles must not be Goldstone bosons, but instead pseudo-Goldstone: the  $U(1)$  symmetry

has to be broken explicitly at the level of the Hamiltonian. This is achieved by adding a term of the form  $-\lambda\hat{\Psi}^\dagger\hat{\Psi}^\dagger + \text{h.c.}$  to the Hamiltonian (3), which in turn implies an extra  $-\lambda\psi^*$  term on the right hand side of Eq. (7).<sup>1</sup>

The analysis of the properties of the quasiparticles in the case of homogeneous background (see [10] for details) leads to the conclusion that the quasiparticles dispersion relation is

$$E = \left( \mathcal{M}^2 c_s^4 + p^2 c_s^2 + \frac{p^4}{4m^2} \right)^{1/2}, \quad (9)$$

where  $c_s$  and  $\mathcal{M}$  are respectively the speed of sound and the mass of the quasiparticles (which are function of the microscopic parameters  $\lambda, \mu, m$ ).

In the case of small momenta, and when the condensate wavefunction is not exactly homogeneous (*i.e.* when  $u(\mathbf{x}) \neq 0$ ), the Hamiltonian for the quasiparticles takes the shape

$$\hat{H}_{q.p.} \approx \mathcal{M}c_s^2 - \frac{\hbar^2 \nabla^2}{2\mathcal{M}} + \frac{C(\lambda, \mu, m)}{\mathcal{M}c_s^2} u(\mathbf{x}), \quad (10)$$

(with  $C(\lambda, \mu, m)$  denotes a certain function of the various coupling constants, see [10] for the details) which leads to the identification of a “gravitational potential”:

$$\Phi_{grav}(\mathbf{x}) = D(\lambda, \mu, m)u(\mathbf{x}), \quad (11)$$

with  $D(\lambda, \mu, m)$  being a constant depending on the microphysics of the system.

*1.1.1. The emergent gravitational system* Having presented the main ideas and required tools, we pass to the results. Consider the Hamiltonian with the  $U(1)$  breaking term. In the limit in which the backreaction of the condensate is small, *i.e.* in the limit in which there are few quasiparticles, when the condensate is almost homogeneous, the Bogoliubov–de Gennes equation can be rewritten as:

$$\left( \nabla^2 - \frac{1}{L^2} \right) \Phi_{grav} = 4\pi G_N \rho_{matter} + \Lambda, \quad (12)$$

where  $G_N$  and  $L$ , the analogue of the Newton’s constant and the healing length, are function of the various coupling constants, and

$$\Lambda \equiv B(\lambda, \mu, m) \left( \mathbf{n}_\Omega + \frac{1}{2} \mathbf{m}_\Omega \right), \quad (13)$$

were again  $B(\lambda, \mu, m)$  is a function of the microscopic parameters characterizing the model [10].

Notice the peculiar splitting of the source term. A detailed analysis [10] shows that the expectation values (8) always split into two contributions, one nonlocal term due to the quasiparticles,  $\rho_{matter}$ , and an unavoidable vacuum contribution,  $\Lambda$ , due to the inequivalence between the Fock vacuum for particles and the Fock vacuum for quasiparticles.

The reader will immediately realize that this interaction is very far away from a realistic Newtonian gravitational interaction: indeed, the would-be Poisson equation includes a term which makes the interaction short ranged. In particular, this range is set by the healing length  $L$ , which is an UV scale for the physics of the BEC (very much like the Planck scale in quantum gravity). This might have been guessed from the beginning, given that the healing length represents the typical scale for the dynamics of the condensate. Despite the fact that this system is not an analogue for a realistic form of gravitational interaction, it does offer some intriguing inspirations which we develop now.

<sup>1</sup> The breaking of the  $U(1)$  symmetry has an obvious interpretation: the number of bosons is no longer a conserved charge (although it is conserved on average). This could lead to some issues about the physical realizability of such a system. For a critical discussion on this point see [10].

## 2. Beyond BECs

The discussion of the BEC model has shown that by limiting the analysis to condensed matter systems there are rather strong constraints on the kind of gravitational models which is reasonable to simulate. For example, in the above investigation we had a single scalar field: it would be interesting to see what happens if several different species are present. In that case, besides the issue of having a short range rather than a long range interaction, also the coupling to the gravitational field must be carefully discussed. Indeed, in order to have some sort of equivalence principle, all the fields must be coupled to the gravitational field in the same way.

The natural setup to discuss these issues is the 2-BEC model [12, 13]: in fact in this case one could treat a multi-particle system whose richness could allow a closer mimicking of Newtonian gravity with a long range potential. However, the fact that emergent gravity has to be Newtonian in a BEC-based analogue model seems to be unavoidable since the gravitational potential depends on the condensate, which is typically described by non-relativistic equations. A possible way to avoid this issue is either to consider relativistic BEC [14, 15] (however in this case we would still expect to get only some type of scalar gravity), or to change completely paradigm and identify gravity not as the condensate but as linked, together with matter, to the perturbations around the condensate. We will consider later this second point of view in a different model.

Furthermore, there is another important issue that requires attention. In our treatment we neglected the quantum potential, i.e. we have deliberately worked in the hydrodynamic limit of the theory, carefully avoiding the issue of the breakdown of acoustic Lorentz invariance in the system at suitably high energies of the quasi-particles. Presumably the breakdown of this emergent spacetime symmetry, namely local Lorentz invariance, will be linked also to some relevant regime change in the gravitational dynamics (which is anyway affected by the presence of a Lorentz symmetry breaking scale, the healing length, which ends up setting the graviton mass scale). Should we take Lorentz symmetry breaking as a crucial ingredient of the emerging gravity paradigm or as an accident of the condensed matter analogue models? In the first case, how the breakdown of such spacetime symmetry affects the symmetries of the gravitational dynamics and in particular diffeomorphism invariance? Furthermore, does this imply that an emergent gravity scenario should give up the relativity principle and bring us back to Newton's absolute space and time?

In order to explore these issues we can start investigating the role of Lorentz invariance in emergent gravity scenarios by considering the most well known “no-go theorem” against them, i.e. the so called Weinberg–Witten theorem [16].

### *2.1. Lorentz invariance and emergent gravity: the Weinberg–Witten theorem*

The idea of having the graviton as a composite particle/emergent field is certainly a fascinating idea. However, there are limitations to what it is possible to do. In particular, there is a theorem, due to Weinberg and Witten [16], which is often presented as a crucial (fatal, in fact) obstruction for a successful emergent gravity program.

The theorem states precise limits for the existence of consistent theories with massless particles. It has two parts, and it says that (quoting from [16]):

- (i) A theory that allows the construction of a Lorentz-covariantly conserved four-vector current  $J^\mu$  cannot contain massless particles of spin  $j > 1/2$  with nonvanishing values of the conserved charge  $\int J^0 d^3x$ .
- (ii) A theory that allows for the construction of a conserved Lorentz covariant energy-momentum tensor  $\theta^{\mu\nu}$  for which  $\int \theta^{0\nu} d^3x$  is the energy-momentum four-vector cannot contain massless particles of spin  $j > 1$ .

For a careful discussion of the proof of the theorem, and for references, see [17]. For additional comments, see [18, 19]

Crucial ingredients for the proof of this theorem are Lorentz invariance and the nonvanishing of the charges obtained from Lorentz covariant vectors and tensors. Interestingly, the gauge bosons like the gluons and the graviton are not forbidden since the current for the gluons is not Lorentz-covariant conserved, and the graviton does not possess a covariant stress-energy tensor (but rather a pseudo-tensor).

This theorem, then, poses rather strong constraints on the possible theories that can be built in Minkowski spacetime. Of course, gravity is not just the theory of a spin-2 particle in Minkowski spacetime. Nevertheless, it surely makes sense to consider the linearized theory in sufficiently small neighborhoods. In this limit, then, the theorem does apply.

With this caveat in mind, we can say that in an emergent gravity program this theorem must be taken appropriately into account and appropriately evaded. There are (at least) two “obvious” way out:

- allow for Lorentz symmetry breaking, or
- make the spacetime manifold to emerge as well.

The first option is rather straightforward, and it is essentially what could be pursued within scenarios like the one considered in analogue models, in which a preferred time function is specified.

The possibility of Lorentz Violation (LV) is not at all an exotic one. Specific hints of LV arose from various approaches to Quantum Gravity (QG). Examples include string theory tensor VEVs [20], spacetime foam [21], semiclassical spin-network calculations in Loop QG [22], non-commutative geometry [23, 24, 25], some brane-world backgrounds [26] and condensed matter analogues of “emergent gravity” [27]. Although none of these calculations proves that Lorentz symmetry breaking is a necessary feature of Planck scale physics, they did stimulate research aimed at understanding the possible measurable consequences of LV [28, 29, 30]. Furthermore, recent investigations strongly suggested that an high energy breakdown of Lorentz invariance might strongly improve the renormalizability of field theories [31, 32] including gravitation [33].

Therefore, the option of introducing Lorentz symmetry violation deserves at least some consideration. However, there is apparently also a (conceptually high) price to pay: a step back from Minkowski spacetime to the notions of absolute space and time. Moreover, and most importantly, there is the issue of recovering a low energy approximate Lorentz invariance. We shall come back later on this point.

The second option is probably the most viable, conceptually appealing, but most demanding in terms of new concepts to be introduced. If no reference is made to a background Minkowski spacetime, but rather the graviton emerges in the same limit in which the manifold emerges, then there is no obvious conflict with the Weinberg-Witten theorem. Simply, what is called the gauge symmetry in terms of fields living of spacetime is the manifestation of an underlying symmetry acting on the fundamental degrees of freedom in the limit when they are reorganized in terms of a spacetime manifold and fields (gauge fields and gravitons in particular).

There are already two examples of this possibility, namely matrix models and quantum graphity models. In both cases, the very notion of spacetime manifold is immaterial for the foundations of the theory. The manifold and the metric are derived concepts, obtained in precise dynamical regimes of the theory. The interested reader can find additional comments and references in [34, 35].

### **3. A concrete example: emergent Lorentzian signature and Nordström gravity**

To give support to the ideas presented so far, let us discuss some results concerning a toy model which we have used to get some further insight. We are not going to reproduce the full calculations, which can be found in [36]. Rather, we will discuss the main outcomes.

Initially, we have considered fields  $\psi_i$  that live in a Euclidean space, and showed that there exists a class of Lagrangians (essentially purely kinetic K-essence ones) such that the perturbations  $\varphi_i$  around some classical solutions  $\bar{\psi}$  propagate in a Minkowski spacetime. In this case  $\bar{\psi}$  is essentially picking up a preferred direction, so that we have a spontaneous symmetry breaking of the Euclidean symmetry. The apparent change of signature is free of the problems usually met in signature change frameworks since the theory is fundamentally Euclidean. Lorentz symmetry is only approximate, and in this sense it is *emergent*.

The main lesson we want to emphasize here is that *Lorentzian signature can emerge from a fundamental Euclidean theory* and this process can in principle be reconstructed by observers living in the emergent system. In fact, while from the perturbations point of view it is a priori difficult to see the fundamental Euclidean nature of the world, this could be guessed from the fact that some Lorentz symmetry breaking would appear at high energy (in our case in the form of a non-dynamical ether field). In this sense, we have a toy-model for the emergence of the Poincaré symmetries. This construction can be seen as a generalization of the typical situation in analogue models of gravity [9] where one has Poincaré symmetries emerging from fundamental Galilean symmetries [9]. However, let us stress that in our case no preferred system of reference is present in the underlying field theory given that the fundamental Lagrangian is endowed with a full Euclidean group  $ISO(4)$ .

In identifying an emergent gravitational dynamics our starting point is then the first order truncated Lagrangian for the perturbations  $\varphi_i$ . This comes out to be of the simple form

$$\mathcal{L}_{\text{eff}}(\varphi_1, \dots, \varphi_N) = \sum_i \eta^{\mu\nu} \partial_\mu \varphi_i \partial_\nu \varphi_i. \quad (14)$$

which can simply be rewritten in terms of the (real) multiplet  $\varphi = (\varphi_1, \dots, \varphi_N)$  as

$$\mathcal{L}_{\text{eff}}(\varphi) = \eta^{\mu\nu} (\partial_\mu \varphi)^T (\partial_\nu \varphi). \quad (15)$$

This system has a global  $O(N)$  symmetry which has emerged as well from the initial Lagrangian [36]. It is hence quite natural to rewrite the multiplet  $\varphi$  by introducing an amplitude characterized by a scalar field  $\Phi(x)$  and a multiplet  $\phi(x)$  with  $N$  components such that<sup>2</sup>

$$\begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_N \end{pmatrix} = \Phi \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix}, \quad \text{with } |\phi|^2 \equiv \sum_i \phi_i^2 = \ell^2. \quad (16)$$

$\ell$  is an arbitrary length parameter to keep the dimension right. In particular,  $\Phi$  is dimensionless and  $\phi$  has the dimension of a length.  $\Phi$  is the field invariant under  $O(N)$  transformations, whereas  $\phi$  does transform under  $O(N)$ . As we shall see, this field redefinition will provide us the means to identify gravity and matter degrees of freedom. The Lagrangian for the perturbations (15) reads now as<sup>3</sup>

$$\mathcal{L}_{\text{eff}}(\varphi_1, \dots, \varphi_N) \rightarrow \mathcal{L}_{\text{eff}}(\Phi, \phi_1, \dots, \phi_N) = \ell^2 \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \sum_i \Phi^2 \eta^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \lambda(|\phi|^2 - \ell^2), \quad (17)$$

where  $\lambda$  is a Lagrange multiplier. We recognize in particular the action for a non-linear sigma model given in terms of the fields  $\phi_i$ . The associated equations of motion are

$$\eta^{\mu\nu} (\ell^2 \partial_\mu \partial_\nu \Phi - \Phi \sum_i \partial_\mu \phi_i \partial_\nu \phi_i) = 0, \quad (18)$$

$$\eta^{\mu\nu} (2\partial_\mu \Phi \partial_\nu \phi_i + \Phi^2 \partial_\mu \partial_\nu \phi_i + \frac{1}{\ell^2} \partial_\mu \phi_j \partial_\nu \phi_k \delta^{jk} \phi_i) = 0, \quad (19)$$

$$|\phi|^2 - \ell^2 = 0. \quad (20)$$

<sup>2</sup> Our field redefinition is the generalization of the so-called Madelung representation [9].

<sup>3</sup> We use the normalization condition  $|\phi|^2 = \ell^2$ , which implies in particular  $\sum_i \phi_i \partial_\mu \phi_i = 0$ .



If we introduce the (conformally flat) metric

$$g_{\mu\nu}(x) = \Phi^2(x)\eta_{\mu\nu}, \quad (21)$$

the equations of motion (19) can be simply rewritten as

$$(\sqrt{-g})^{-1}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi_i) + \frac{1}{\ell^2}g^{\mu\nu}\partial_\mu\phi_j\partial_\nu\phi_k\delta^{jk}\phi_i = \quad (22)$$

$$\square_g\phi_i + \frac{1}{\ell^2}g^{\mu\nu}\partial_\mu\phi_j\partial_\nu\phi_k\delta^{jk}\phi_i = 0, \quad (23)$$

where we have introduced the d'Alembertian  $\square_g$  for the metric  $g$  and used that  $\sqrt{-g} = \Phi^4$  and  $g^{\mu\nu} = \Phi^{-2}\eta^{\mu\nu}$ . Notice that equation (19) can be rewritten in the form (23) using the metric redefinition (21) only in four dimensions. To be consistent, the change of variable  $\Phi \rightarrow g_{\mu\nu}$  should be completed with the constraint that  $g_{\mu\nu}$  is conformally flat, that is

$$C_{\alpha\beta\gamma\delta}(g) = 0, \quad (24)$$

where  $C_{\alpha\beta\gamma\delta}$  is the Weyl tensor.

Eq. (23) suggests that *the gravitational degree of freedom should be encoded in the scalar field  $\Phi$ , whereas matter should be encoded in the  $\phi_i$* . We are therefore aiming at a scalar theory of gravity with actions:

$$S_{\text{eff}} = \int dx^4 \sqrt{-\eta} \mathcal{L}_{\text{eff}} = S_{\text{grav}} + S_{\text{matter}}, \quad (25)$$

$$S_{\text{grav}} = \ell^2 \int dx^4 \sqrt{-\eta} \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi, \quad (26)$$

$$S_{\text{matter}} = \int dx^4 \sqrt{-\eta} \left( \sum_i \Phi^2 \eta^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \lambda(|\phi|^2 - \ell^2) \right), \quad (27)$$

where we have explicitly written the volume element  $\sqrt{-\eta} = 1$  so to make clear that these actions are given in flat spacetime.

It is easy to see that the very same actions can be recast in the form of actions in a curved spacetime endowed with the metric (21). In particular for the matter action in (27) one has

$$S_{\text{matter}} = \int dx^4 \left( \sum_i \Phi^2 \eta^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \lambda(|\phi|^2 - \ell^2) \right) = \quad (28)$$

$$\int \sqrt{-g} dx^4 \left( \sum_i g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \lambda'(|\phi|^2 - \ell^2) \right), \quad (29)$$

where we have suitably rescaled the Lagrange multiplier to  $\lambda'$ . This allows to construct the stress-energy tensor  $T_{\mu\nu}$  for the non-linear sigma model, and its trace  $\mathbf{T}$  with respect to the metric  $g$ :

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}} = \sum_i \left( \partial_\mu \phi_i \partial_\nu \phi_i - \frac{1}{2} g_{\mu\nu} (g^{\alpha\beta} \partial_\alpha \phi_i \partial_\beta \phi_i) \right), \quad (30)$$

$$\mathbf{T} = g^{\mu\nu} T_{\mu\nu} = -\Phi^{-2} \sum_i \eta^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i. \quad (31)$$

Finally, the above result, together with the recognition that the Ricci scalar  $\mathbf{R}$ , associated to the metric  $g_{\mu\nu}$ , can be written as  $\mathbf{R} = -6\Box_\eta\Phi/\Phi^3$ , allows us to rewrite Eq. (18) as the Einstein–Fokker equation

$$\Box_\eta\Phi = \frac{1}{\ell^2}\eta^{\mu\nu}\Phi\sum_i\partial_\mu\phi_i\partial_\nu\phi_i \quad \Leftrightarrow \quad \mathbf{R} = \frac{6}{\ell^2}\mathbf{T}. \quad (32)$$

In summary, we can gather together the equations of motion (23, 24, 32), obtained by introducing the metric (21), we have

$$\mathbf{R} = \frac{6}{\ell^2}\mathbf{T}, \quad C_{\alpha\beta\gamma\delta} = 0. \quad (33)$$

$$\Box_g\phi_i + \frac{1}{\ell^2}g^{\mu\nu}\partial_\mu\phi_j\partial_\nu\phi_k\delta^{jk}\phi_i = 0, \quad |\phi|^2 - \ell^2 = 0. \quad (34)$$

We recognize the equations of motion as those for Nordström gravity

$$\mathbf{R} = 24\pi G_N \mathbf{T}, \quad C_{\alpha\beta\gamma\delta} = 0, \quad (35)$$

coupled to a non-linear sigma model. Indeed, the rewriting of (18)-(20) into the form (33)-(34), is a special case of the procedure suggested by Einstein and Fokker so to cast Nordström gravity in a geometrical form [37].

We see from the above equation that the Newton constant  $G_N$  in our model has to be proportional to  $\ell^{-2}$ . However, in identifying the exact relation between the two quantities, some care has to be given to the fact that the stress-energy tensors appearing respectively in equation (33) and equation (35) do not share the same dimensions. This is due to the fact that the fields  $\phi_i$  have the dimension of a length rather than the usual one of an energy. This implies that in order to really compare the expressions one has to suitably rescale our fields with a dimensional factor,  $\Xi$ , which in the end would combine with  $\ell$  so to produce an energy,  $\dim[\ell\Xi] = \text{energy}$ . In particular, is easy to check that one has to assume  $4\pi\ell^2\Xi^2 \equiv E_{\text{Planck}}^2$  in order to recover the standard value of  $G_N$  (assuming  $c$  as the observed speed of signals and  $\hbar$  as the quantum of action). As a final remark, we should stress that the scale  $\ell$  is completely arbitrary within the emergent system and in principle should be derived from the physics of the “atoms of spacetime” whose large  $N$  limit gives rise to the initial fundamental Lagrangian (the one for the initial fields  $\psi_i$ ).

Accidentally, the above discussion also shows that, once the fields are suitably rescaled so to have the right dimensions, the constraint appearing in Eq. (34) is fixing the norm of the multiplet to be equal to the square of the Planck energy. This implies that the interaction terms in the aforementioned equation are indeed Planck-suppressed and hence negligible at low energy. This should not be a surprise, given that in the end  $\ell\Xi$  is the only energy scale present in our model. It is conceivable that more complicate frameworks, possibly endowed with many dimensional constants, will introduce a hierarchy of energy scales and hence break the degeneracy between the scale of gravity and the scale of matter interactions.

In the second part of the discussion of this toy model, using a natural field redefinition adapted to the symmetries of the system, we have identified from the perturbations  $\varphi_i$ , a scalar field  $\Phi$  encoding gravitational degrees of freedom and a set of scalar fields  $\phi_i$  (a non-linear sigma model) encoding matter fields. In this sense, gravity and matter are both emergent at the same level. This approach is then rather different from the one of analogue models of gravity where one usually identifies the analogue of the gravitational degrees of freedom with the “background” fields, *i.e.* the condensate or the solution  $\psi$  of the equations of motion. Indeed, following this line of thought in looking for a theory of gravitational dynamics, we would be led to require that the fundamental field theory must be endowed with diffeomorphisms invariance from the very start — the symmetries of the background are identical by construction to the ones of the

fundamental theory. This would imply that one would have to obtain gravity from a theory which is already diffeomorphisms invariant and hence most probably with a form very close to some known theory of gravitation.

For these reasons, we do expect that if an emergent picture is indeed appropriate for gravitation, then it should be of the sort presented here, with both matter and gravity emerging at the same level. Of course, it is not possible to exclude that a full fledged theory of gravity could emerge, together with the notion of manifold, in a single step from the eventual semiclassical/large number limit of the fundamental objects. In this case, however, we would still have a very different picture from the one envisaged in analogue models of gravity.

In particular, this allows not only for an emergent local Lorentz invariance for the perturbations dynamics but it leads as well to an emergent diffeomorphisms invariance. In fact, the (lowest order) equations of motion for the perturbations can be naturally rewritten in an evidently diffeomorphisms invariant form, from the point of view of “matter fields observers”. In fact, following the standard hole argument (see [38] for a careful discussion of the various issues related to diffeomorphism invariance), this also implies that the coordinates  $x^\mu$ , used to parameterized our theory, do not have any physical meaning from the point of view of the  $\phi_i$  “matter observers”. They are merely parameters. In agreement with the fact that diffeomorphisms invariance is emergent in our system, it can be noted that the higher order contributions contributions ends up breaking it at the same level it breaks Lorentz invariance.

Furthermore, Nordström gravity is also a nice framework for discussing the subtle distinction between background independence and diffeomorphisms invariance [39]. We call background some geometrical degrees of freedom that are not dynamical. For example, in GR the topology of the manifold and its dimension, or the signature of the metric, can be considered as (trivial) background quantities. We can therefore have some specific background structures while still having diffeomorphisms invariance. Nordström gravity is encoded in conformally flat metrics. If one considers fields which are conformally coupled to the metric (such as the electromagnetic field), these fields only see the metric  $\eta_{\mu\nu}$  which is of course not dynamical. The Minkowski metric can be seen then as a background structure, this is what one may call a “prior geometry” (*e.g.* see [40]). One may hence say that diffeomorphism invariance is somewhat of a weaker form in Nordström gravity with respect the one present in GR.

In particular, while the essence of diffeomorphism invariance in GR is encoded in the associated Hamiltonian constraints, these are not defined in the present formulation of Nordström gravity. Furthermore, in the most general implementations of Nordström theory, quantities can be built which manifestly include the background structure  $\eta_{\mu\nu}$  and hence are not diffeomorphism invariant. However, within our model, the prior geometry cannot be detected. Indeed, in order to detect the Minkowski background, one should be able to propose a method to pinpoint the conformal factor  $\Phi^2$  in the relation  $g_{\mu\nu} = \Phi^2 \eta_{\mu\nu}$ . However, a careful analysis shows that this is actually impossible. Let us elaborate on this point. If we perform a conformal transformation,  $x^\mu \rightarrow \bar{x}^\mu(x)$ , the (lowest order) equations of motions for the perturbations (in their original parametrization) are transforming like

$$\square_\eta \varphi_i = 0 \rightarrow \square_{\bar{\eta}} \varphi_i = 0, \quad (36)$$

where  $\eta$  and  $\bar{\eta}$  are two different Minkowski metrics related by some conformal factor  $\lambda(x)$ . Therefore,  $\eta$  and  $\bar{\eta}$  are indistinguishable, due to conformal invariance the equations of motion for  $\varphi_i$ . Hence, what appears to be a background structure, namely  $\eta_{\mu\nu}$ , is ambiguously defined, and the coordinates  $x^\mu$  in which the equations of motion for the fields  $\varphi_i$  are written have no operational meaning, they are mere labels. Furthermore, this ambiguity in the definition of what would be called a background structure implies an ambiguity on the definition of the conformal factor relating the physical metric to the would-be background structure. In this sense, within this very specific implementation of the model which has conformal invariance,

there is no Minkowski geometry as a background. There is a background structure, which is the conformal structure of Minkowski spacetime. This is a mild limitation of our simple toy model as a diffeomorphism invariant, background independent system.

Of course, the above discussion holds only at the lowest order in the fields  $\varphi_i$ . As previously discussed, higher orders in perturbation theory will generate terms leading to a breaking of the conformal symmetry and hence the appearance of the background structures, *i.e.* the Euclidean space and the  $\partial_\mu\psi$  which have selected the timelike direction.

Finally, it is interesting to discuss in details the features that allowed the construction of such a toy model. In particular, it is important to stress the role of symmetries, in order to make clear the way in which they enter at the various levels. As in the case of selecting Riemannian geometry out of Finsler geometry (see e.g. [41] for a pedagogical introduction), here there are some symmetries which are absolutely essential: it is only due to their presence that we do have an emergent gravitational system possessing a geometrical nature.

We have seen that in order to produce a working model, a number of properties must be assumed. First of all, there is an underlying  $ISO(4)$  symmetry which allows us to use particularly simple affine solutions. This  $ISO(4)$ , when spontaneously broken, can lead to an approximate Poincaré invariance. Moreover, the masslessness of the resulting modes is promoting this Poincaré invariance to a full conformal invariance, which is approximate as well. This conformal invariance is the key symmetry which hides the background structure, forbidding a low energy observer to detect a background metric structure (there is only a background conformal structure).

Conformal invariance seems to be deeply intertwined with the possibility of writing down the resulting equations of motion in the form of a system of diffeomorphism invariant equations, as we have seen. However, in order for the Lagrangian (14) to be conformal invariant, there must be an overall  $O(N)$  symmetry between the fields. This symmetry is just the other side of the coin of the mechanism leading to the monometricity. If two fields move in different metrics, clearly this  $O(N)$  is broken and the entire model fails to provide a geometric picture, let alone a diffeo-invariant one.

In general, one should expect that in any situation in which the metric is an emergent structure, there should be a mechanism taking care of the fact that different matter fields propagate over the same geometry. In this picture, where a manifold is given from the beginning, the role of internal and spacetime symmetries is crucial. The behavior we have described is not general at all. Of course, one could conclude that this kind of models is somehow contrived and unnatural.

However, there is also a positive side: given that symmetries (both of the equations of motion and of the ground state) play a crucial role in the emergence mechanism, the fact that our universe seems to be ruled, at large scales, by GR and locally by special relativity, suggests that not all the pre-geometric scenarios are viable, and that there are rather strong constraints on what are the possible mechanism of emergence. In particular we want to conclude this analysis with a discussion about the constraints related to the breakdown of Lorentz invariance.

#### 4. The naturalness problem

Should we take these results as a strong hint that Lorentz symmetry breaking should be a part of any working emergent gravity scenario? It is at this stage unclear if we can be that bold. Surely one open issue is the naturalness of theories endowed with Lorentz symmetry breaking. In general, radiative corrections lead to a dangerous “percolation” in the infrared regimes of the Lorentz breaking [42, 43, 44], something strongly constrained by current observations [28, 29, 30].

More precisely, it was found that it is generic that even starting with an effective field theory with only Lorentz breaking operators in the Lagrangian of mass dimension 5 and 6 for free particles, radiative corrections due to particle interactions will generate lower-dimension LV

terms that will then become dominant [42], as their dimensionless coefficients are of the same order as the higher dimension ones ( $O(1)$ , given our previous assumption, see [45]). Thus, either a symmetry (or some other mechanism) protects the lower dimension operators from large LV, or the suppression of the non-renormalizable operators indeed will always be greater than that of the renormalizable ones.

SuperSymmetry (SUSY) was proposed as a possible candidate for a custodial symmetry doing this job [46, 47]. SUSY is closely related to Poincaré invariance: the composition of two SUSY generators is proportional to the momenta, the generators of space-time translations. However, the idea is that SUSY could still be an exact symmetry even in the presence of LV and it can serve as a custodial symmetry, preventing certain operators from appearing in LV field theories.

The effect of SUSY on LV is to prevent dimension  $\leq 4$ , renormalizable LV operators to be present in the Lagrangian. Moreover, it has been demonstrated [46, 47] that the renormalization group equations for Supersymmetric QED plus the addition of dimension 5 LV operators à la Myers & Pospelov do not generate lower dimensional operators, if SUSY is unbroken. However, this is not the case for our low energy world, of which SUSY is definitely not a symmetry. The effect of soft SUSY breaking was again investigated in [46, 47]. As expected, it was found that, when SUSY is broken, the renormalizable operators appear in the Lagrangian. In particular, dimension  $\kappa$  operators arise from the percolation of dimension  $\kappa + 2$  LV operators<sup>4</sup>. The effect of SUSY soft-breaking is, however, to introduce a suppression of order  $m_s^2/M_{pl}$  ( $\kappa = 3$ ) or  $(m_s/M_{pl})^2$  ( $\kappa = 4$ ), where  $m_s$  is the scale of soft SUSY breaking. Given the present constraints, the theory in which  $\kappa = 3$  must be fine-tuned to be viable, because the SUSY-breaking-induced suppression is not powerful enough to eliminate linear modifications in the dispersion relation of electrons. However, if  $\kappa = 4$ , the induced dimension 4 terms are sufficiently suppressed, provided that  $m_s < 100$  TeV.

Let us stress, however, that all this studies have implemented SUSY in the standard (Lorentz invariant) form and it is not clear if and how a SUSSY group can be found in a Lorentz breaking context. In this sense the naturalness problem of Lorentz breaking theories is probably the most important issues to be solved not only for the quantum gravity phenomenology field but also for the whole emergent gravity proposal. We hope that studies like the one reported here will help in putting this issue under the spot of future research in the field.

## 5. Conclusions

In summary, we hope that the two examples of emergent gravitational dynamics presented here have suitably illustrated the potentialities of emergent gravity models inspired by the analogue gravity perspective. The first case, the one of a BEC system, has shown us that a gravitational-like dynamics (with a small cosmological constant) seems to be a natural by product of a condensation mechanism. However, the analogy with the real world was not only limited by the Lorentz breaking scale but also from the fact that gravity and matter seem to be living at rather different levels (gravity is the condensate while matter is associated to the quasi-particle states).

There are important conceptual and practical obstructions which forbid the BEC system to be used to discuss analogue of the gravitational field. In this sense, it is an attempt doomed to fail from the very beginning. First of all, the model is non-relativistic. Despite some encouraging results towards the extension to a relativistic theory made in [36], it is still not clear how to evade the Weinberg–Witten theorem [16, 17, 19] in order to produce a viable model of emergent spin-2 gravity. Nevertheless, the brief discussion of the BEC-based toy model offers the possibility of showing very nice properties which could be of help in understanding some of the puzzles we find in the study of the gravitational field.

<sup>4</sup> We consider only  $\kappa = 3, 4$ , for which these relationships have been demonstrated.

For instance, the naturalness of the small cosmological constant term present in a BEC. As we have seen, the cosmological constant term is proportional to the depletion factor, *i.e.* by the ratio between the number of particles on excited states and the number of particles in the ground state (the condensate). Whenever this ratio is small, the cosmological constant is similarly suppressed.

The toy model discussed in the second part of this proceedings is aimed at overcoming some of the problems encountered in the case of BECs. The model shows Lorentz violation but only in the limit of large fluctuations of the fundamental fields  $\varphi_i$ . This implies that the gravitational dynamics is no more endowed with a massive graviton whose mass scale is set by the UV Lorentz breaking scale of the system. Furthermore, this toy models shows how time and diffeomorphism invariance might emerge. In particular the latter is allowed by the special symmetries of the system. As we have said previously, similar symmetries are probably needed anyway to protect the IR limit of the theory from large violations of Lorentz invariance of the equations. Given that the presence of such violations seems to be a very natural way around to the Weinberg-Witten theorem obstruction, it might be that the next step towards a satisfactory emergent gravity scenario might have to consist in finding which sort of mechanism, possibly a custodial symmetry, could simultaneously guarantee the background independence of the emerging dynamics as well as a very accurate Local Lorentz invariance of the emergent spacetime. We hope to address these questions in future work.

## 6. Acknowledgments

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