

Entropy-based control of continuous fluidized bed spray granulation processes

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Abstract: This paper is concerned with control of a continuous fluidized bed spray granulation. On the basis of an entropy function a control Lyapunov function will be derived. In order to facilitate the control design procedure this entropy-based control Lyapunov function will be approximated by its second order Taylor expansion.

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1. INTRODUCTION

Fluidized bed spray granulation is a particulate process, where a bed of particles is fluidized, while simultaneously injecting a solid matter solution. Due to high process air temperature, the fluid evaporates and the remaining solid material either contributes to growth of already existing particles or forms new nuclei. As product particles should have a certain minimum size an additional product classification is required. In this contribution a process configuration applying an air sifter with countercurrent flow as depicted in Fig. 1 will be studied. Another possibility is for example the application of an external classification using sieves with corresponding recycle of the over- and undersized fraction [2]. In order to allow a continuous process operation part of the withdrawn product particles will be milled and fed back as nuclei to the granulation chamber. It is well known that continuous granulation processes in general and in particular configurations applying a mill cycle tend to instability and the occurrence of nonlinear oscillations of the particle size distribution. These oscillations give undesired time behavior of product quality [4, 3, 2]. Similar patterns of behavior have been observed for other particulate processes as e.g. crystallization processes (e.g. [10]). In order to control these several approaches have been proposed ranging from linear finite dimensional control (e.g. [9, 7]) to nonlinear infinite dimensional control methods [8]. Especially the later, i.e. the discrepancy based control design, has been successfully applied to different particulate processes.

In this contribution control design exploiting the thermodynamic structure, i.e. the entropy, of the process will be studied. Therefore, an entropy-based control Lyapunov function will be derived and used for control design.

2. CONTINUOUS FLUIDIZED BED SPRAY GRANULATION

A continuous fluidized bed spray granulator with an additional mill as depicted in Fig. 1 consists of a granulation chamber, where the particle population is fluidized through an air stream and coated by injecting a suspension. The particle growth associated to the layering process has been described in [1].

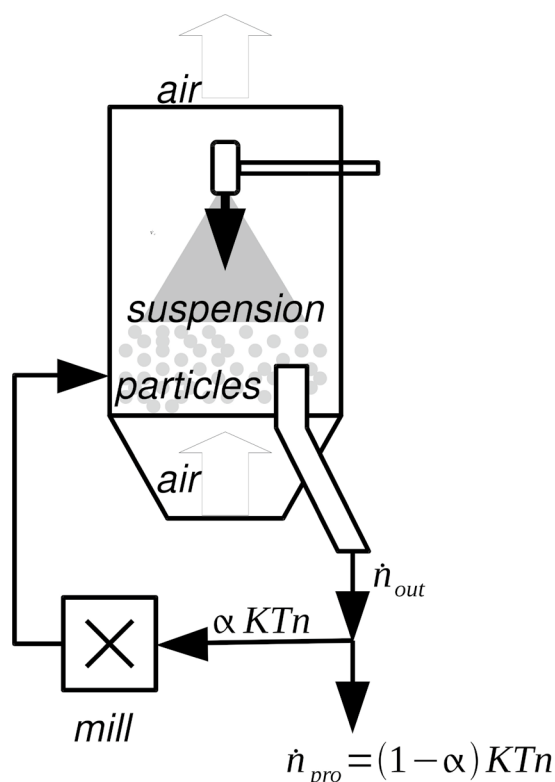


Fig. 1. Process scheme

$$G = 2 \frac{\dot{V}_e}{\pi \int_0^\infty L^2 n dL} \quad (1)$$

It should be mentioned that the particle growth rate is inversely proportional to the overall particle surface area and hence the second moment of the particle size distribution. Product particles are continuously removed through an air sifter with countercurrent flow. Here, due to the particle size specific sinking velocity, small particles with $L < L_2$ are reblown into the granulation chamber, while large product particles with $L \geq L_2$ pass the air sifter. The associated ideal separation function T is given as follows.

$$T(L) = \sigma(L - L_2) \quad (2)$$

The outlet flow is hence

$$\dot{n}_{out} = KT(L)n. \quad (3)$$

where K is the drain, depending mainly on the air velocity and the ration between granulation chamber and sifter cross section. In order to guarantee a continuous process operation nuclei have to be continuously supplied. This can be achieved using an additional mill. For simplicity it can be assumed that the mill is mass conserving and generates a rectangular distribution.

$$B = \alpha(\sigma(L - L_0) - \sigma(L - L_1)) \frac{\int_0^\infty L^3 KT ndL}{\int_0^\infty L^3 ndL} \quad (4)$$

$$= \alpha(\sigma(L - L_0) - \sigma(L - L_1)) \frac{\int_0^\infty L^3 KT ndL}{4(L_1^4 - L_0^4)} \quad (5)$$

To describe the process, a population balance model for the particle size distribution can be stated consisting of the following particle fluxes

- B particle flux from the mill,
- \dot{n}_{out} particle flux due to particle removal,

and size independent particle growth associated with the particle growth rate G .

$$\frac{\partial n}{\partial t} = -G \frac{\partial n}{\partial L} - \dot{n}_{out} + B \quad (6)$$

For numerical simulation the model equations are semi-discretized with the finite volume method (1st order up-wind flux discretization) with 310 grid points. The model parameters used are given in Table 1.

\dot{V}_e	$1.5 \cdot 10^5 \frac{mm^3}{s}$
L_0	0.1mm
L_1	0.2mm
L_2	0.7mm
K	$1 \cdot 10^{-3} \frac{1}{s}$
α_0	$5 \cdot 10^{-3} \frac{1}{s}$

Table 1. Plant parameters

3. ENTROPY-BASED CONTROL LYAPUNOV FUNCTION

In the following a control Lyapunov functional will be derived based on an entropy function for the population balance model. The candidate being proposed in this contribution is strongly related to the one studied recently in [6] for a continuous crystallizer.

$$S = \int_0^\infty -Cn \ln ndL = \int_0^\infty s(n)dL \quad (7)$$

Here, C is a positive constant, which will be specified later. As we are interested in deviations from and control design for a desired steady state n_0 the following quantities can be defined

$$\Delta S = S_0 - S, \quad (8)$$

$$\Delta Z = Z_0 - Z, \quad (9)$$

where $Z = kn$. The conjugate variable of Z can be calculated as

$$\omega_0 = \left. \frac{\partial s}{\partial Z} \right|_{Z=Z_0} \quad (10)$$

$$= -\frac{C}{k}(\ln n_0 + 1). \quad (11)$$

Using Z , its conjugate and the entropy S a control Lyapunov function can be derived [6, 5].

$$V = -\int_0^\infty \Delta Z \omega_0 dL + \Delta S \quad (12)$$

Using eq. (11) yields

$$V = \int_0^\infty \Delta n (C(\ln n_0 + 1)) dL, \quad (13)$$

$$- \int_0^\infty C(n_0 \ln n_0 - n \ln n) dL, \quad (14)$$

$$= \int_0^\infty C(\Delta n - n(\ln n_0 - \ln n)) dL. \quad (15)$$

In order order to simplify the control design for the continuous fluidized bed granulation process this control Lyapunov candidate will be approximated by its second order Taylor approximation.

$$V = \int_0^\infty \frac{C}{n_0} \Delta n^2 dL \quad (16)$$

Choosing $C = \frac{1}{2}n_0$ yields

$$V = \frac{1}{2} \int_0^\infty \Delta n^2 dL. \quad (17)$$

4. CONTROL DESIGN

In the following the second order approximation of the entropy-based Lyapunov candidate (17) will be used in order to design an appropriate feedback control law. Calculating the time derivative \dot{V} yields

$$\dot{V} = \int_0^\infty \Delta n \left(\frac{\partial n_0}{\partial t} - \frac{\partial n}{\partial t} \right) dL \quad (18)$$

$$= - \int_0^\infty \frac{G}{2} \frac{\partial \Delta n^2}{\partial L} + \Delta n [B_0 - B - KT\Delta n] dL \quad (19)$$

$$= - \frac{G}{2} \Delta n^2 \Big|_0^\infty - \int_0^\infty \Delta n [B_0 - B - KT\Delta n] dL \quad (20)$$

Here, the first two terms vanish due to the boundary conditions $n(L=0) = \lim_{L \rightarrow \infty} n(L) = 0$, i.e. there are no particles of size zero and infinitely large particles. Hence, the time derivative of the Control Lyapunov functional V does not depend on the growth rate G and thus all results will be robust with respect to variations in the suspension injection rate.

$$\dot{V} = \int_0^\infty \Delta n [B_0 - B - KT\Delta n] dL \quad (21)$$

Inserting the equations for the particle outlet (3) and the mill flux (5) results in the following

$$\dot{V} = - \int_{L_0}^{L_1} \Delta n \frac{\int_{L_2}^{\infty} \alpha_0 L^3 K n_0 - \alpha L^3 K n dL}{4(L_1^4 - L_0^4)} dL \quad (22)$$

$$- K \int_{L_2}^{\infty} \Delta n^2 dL \quad (23)$$

For convenience a virtual control input $u_{virt} = \alpha \int_{L_2}^{\infty} L^3 K n dL$ is introduced.

$$\dot{V} = -(u_{virt,0} - u_{virt}) \frac{\int_{L_0}^{L_1} \Delta n dL}{4(L_1^4 - L_0^4)} - K \int_{L_2}^{\infty} \Delta n^2 dL \quad (24)$$

In order to achieve negative definiteness of the time derivative of the entropy-based candidate Lyapunov functional V the following virtual control law is chosen

$$u_{virt} = u_{virt,0} + \frac{4(L_1^4 - L_0^4)}{\int_{L_0}^{L_1} \Delta n dL} \left[c \int_0^{\infty} \Delta n^2 dL \right], \quad (25)$$

which results in the control law for α

$$\alpha = \frac{u_{virt}}{\int_{L_2}^{\infty} L^3 K n dL}. \quad (26)$$

The time derivate of the entropy-based control Lyapunov function hence is

$$\dot{V} = -c \int_0^{\infty} \Delta n^2 dL - K \int_{L_2}^{\infty} \Delta n^2 dL \quad (27)$$

$$\leq -c \int_0^{\infty} \Delta n^2 dL = -2cV \quad (28)$$

guaranteeing asymptotic stability and exponential convergence. The control law is tested starting at an initial particle size distribution $n(t=0, L) = 1.5 \cdot n_0(L)$ depicted in Fig. 2 and with a tuning factor $c = 2 \cdot 10^{-6}$. As can be seen in Fig. 3 and 4 the proposed control law succeeds in controlling the desired particle size distribution with reasonable control effort (Fig. 5 and Fig. 6). For comparison the open loop process behavior is depicted in Fig. 7 showing considerably more oscillations and higher peaks in the particle size distribution. In order to evaluate the control performance the third moment of the particle size distribution in open and closed loop operation is shown in Fig. 8.

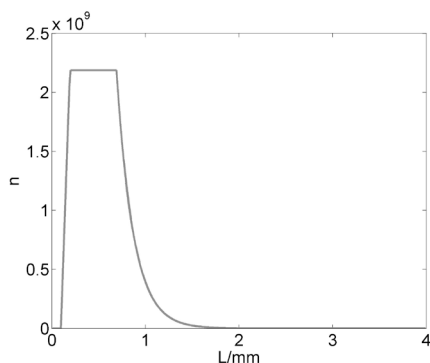


Fig. 2. Initial particle size distribution

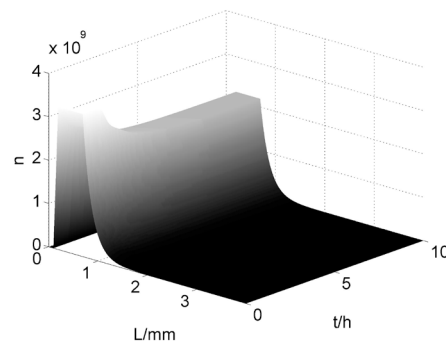


Fig. 3. Particle size distribution in closed loop operation n

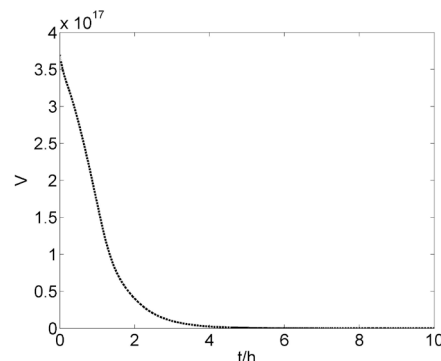


Fig. 4. Entropy-based Lyapunov function V

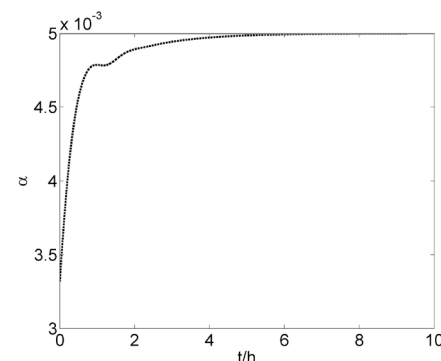


Fig. 5. Control input α

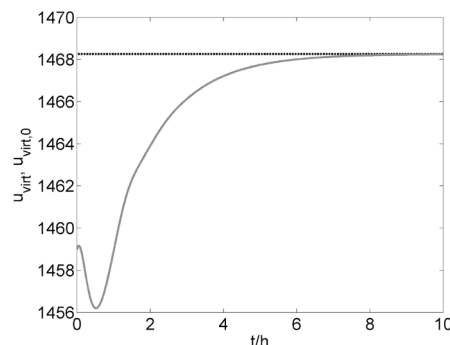


Fig. 6. Virtual control input u_{virt}

5. CONCLUSION

For a continuous fluidized bed spray granulation with mill cycle an appropriate control law has been derived and

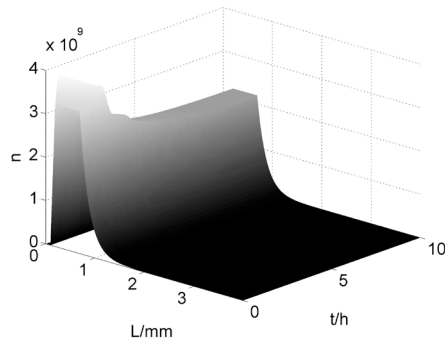


Fig. 7. Particle size distribution in open loop operation n

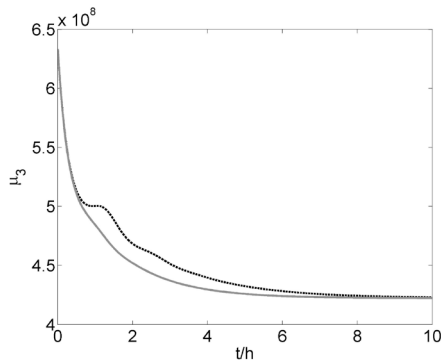


Fig. 8. Third moment of the particle size distribution μ_3 in open (black dashed) and closed loop operation (gray solid)

tested. The main idea is to derive a Lyapunov functional on the basis of an entropy function. In order to facilitate the control design procedure this entropy-based Lyapunov functional has been approximated by its second order Taylor expansion. Future work will be concerned with a thorough comparison between the here presented entropy-based control approach giving physical insight and the earlier proposed discrepancy based control design [8]. Applying the latter control laws are designed on the basis of a generalized distance measure, the discrepancy, focusing on the process input output behavior.

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