Effects of Anisotropic Plasma Temperature on the Frequency Shift in Resonant Cavities

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Abstract: The anisotropic temperature effects of inhomogeneous plasma in resonant cavities are studied. For the TM_{OMO} mode, the wave equation can be treated exactly in both limit cases $T_{\parallel} \ll T_{\perp}$ and $T_{\parallel} \gg T_{\perp}$. It shows us how the parallel temperature may be responsible for the absence of frequency shift.

Resonant cavity techniques have long been a powerful tool for measuring density and collision frequency in a plasma[1,2,3]. However, little attention has hitherto been paid to the anisotropy effects of a warm inhomogeneous plasma. In this paper, it is shown for the special excitation mode TM_{OMO} (transverse magnetic), how the (integro-differential) wave equation reduces exactly to the equation for a cold plasma in the limit $T_{\parallel} \to 0$ (and $T_{\perp}$ finite) and, on the other hand, how it can lead to absence of frequency shift in the limit $T_{\perp} \to 0$ (and $T_{\parallel}$ finite).

Let us consider an infinitely long cylindical metallic cavity with a static magnetic field $\bar{B} = \hat{i}_z B_0$ along its axis. The distribution function for an anisotropic, low-ß, collisionless plasma is then:

$$f_0 = f_0(v_\perp^2; v_\parallel^2; r v_\varphi + \Omega r^2/2), \quad \Omega \equiv \frac{e B_0}{m c} \tag{1}$$

For transverse magnetic (TM) eigenmode solutions with radial dependence only:

$$\bar{E}(r) = \hat{i}_z E(r) e^{i\omega t} \tag{2}$$

it can be shown that the wave quation reduces exactly to the form:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dE(r)}{dr}\right] + \frac{\omega^2}{c^2}\left[1 - \sum_{e,i}\frac{\omega_p^2(r)}{\omega^2}\right]E(r) - \frac{1}{c^2}\sum_{e,i}\frac{4\pi e^2}{m}\int d^3v\, 2v_\parallel^2 \frac{\partial f_0}{\partial v_\perp^2}\int_{-\infty}^{0} du\, e^{i\omega u}\frac{\partial E(R)}{\partial u} = 0 \tag{3}$$

where the time integration is made along the unperturbed orbits (characteristics) $R = R(r, \bar{v}, u)$ with $u = t' - t$.

The main feature of eq. (3) ist that, in the limit of zero parallel temperature, it reduces to the equation of the cold plasma case, however large the perpendicular temperature may be.

In such a limit $T_{\parallel} \to 0$, it is known that the treatment of the eigenvalue problem necessarily leads to a positive frequency shift ($\Delta\omega/\omega > 0$) given approximately in the low-density limit ($|\omega_p/\omega| \ll 1$) by:

$$\Delta\omega \sim \mathcal{O}\left(\int_0^{R_0} n(r)\, dr\right) \tag{4}$$

On the other hand, in the limit $T_{\perp} \to 0$ (and $T_{\parallel}$ arbitrary), the wave equation reduces to the following:

$$\frac{1}{r}\frac{d}{dr}\left[r\left(1 + \sum_{e,i}\frac{\omega_p^2(r)}{\omega^2 - \Omega^2}\frac{v_{\parallel th}^2}{c^2}\left(1 + \mathcal{O}(T_\perp)\right)\right)\frac{dE(r)}{dr}\right] + \frac{\omega^2}{c^2}\left[1 - \sum_{e,i}\frac{\omega_p^2(r)}{\omega^2}\right]E(r) = 0, \quad \Omega \equiv \frac{eB_0}{mc} \tag{5}$$

with the boundary condition $E(r = R_0) = 0$.

We shall consider the density distribution $n(r)$ as a small parameter in order to treat the above equation (5) by means of the usual regular perturbation technique.

a) <u>Without plasma</u>

Equation (5) reduces to:

$$(A_0 + \lambda)E(r) = 0, \quad E(R_0) = 0 \tag{5a}$$

where $A_0 \equiv \frac{1}{r}\frac{d}{dr} r \frac{d}{dr}$ and $\lambda \equiv \omega^2/c^2$

The solution is: $E(r) = E_0(r) \sim J_0(\sqrt{\lambda}\, r)$

with $J_0(\sqrt{\lambda} R_0) = 0$, which implies $\lambda = \lambda_0$ (given in numerical tables).

b) <u>With plasma</u>

We write the new solution as $E(r) = E_0(r) + E_1(r)$

with the new eigenvalue $\lambda = \lambda_0 + \lambda_1$

so that eq. (5) can be written

$$\left[(A_0 + \lambda_0 + \lambda_1) + A_1(\lambda_0 + \lambda_1)\right](E_0 + E_1) = 0 \tag{5b}$$

with the boundary condition $E_1(r = R_0) = 0$

and

$$A_1(\lambda) \equiv \frac{1}{r}\frac{d}{dr} r \sum_{e,i}\frac{\omega_p^2(r)}{\omega^2 - \Omega^2}\frac{v_{\parallel th}^2}{c^2}\frac{d}{dr} - \sum_{e,i}\frac{\omega_p^2(r)}{c^2}$$

In lowest order, we get eq. (5a):

$$(A_0 + \lambda_0)E_0 = 0$$

and in the next order:

$$(A_0 + \lambda_0)E_1 + \lambda_1 E_0 + A_1(\lambda_0)E_0 = 0$$

Introducing

$$(f, g) \equiv \int_0^{R_0} r\, dr\, f g$$

it is easy to verify that

$$(E_0, A_0 E_1) = (A E_0, E_1)$$

so that eq. (5b) reduces to:

$$\lambda_1 = -(E_0, A_1(\lambda_0)E_0)/(E_0, E_0) \tag{6}$$

or, after some transformation:

$$\Delta(\omega^2) = \frac{\int_0^{R_0} r\, dr \sum_{e,i}\omega_p^2(r)\left\{E_0(r)^2 + \frac{v_{\parallel th}^2}{\omega^2 - \Omega^2}E_0'(r)^2\right\}}{\int_0^{R_0} r\, dr\, E_0(r)^2} \tag{7}$$

This gives us the frequency shift of a cylindrical cavity for the TM_{OMO} mode in the limit of small perpendicular temperature, but for arbitrary parallel temperature and density profile.

As can be seen, frequency shift may be absent. In a recent experiment[4], a microwave cavity operating in the TM_{OMO} mode was insensitive to the presence of a hot electron component of density. The authors attributed this result to some plasma temperature effect; but the parameters in this experiment ($T_{\parallel e} \ll T_{\perp e} \sim 1$ KeV ; $\omega \simeq 1.5$ GHz ; $\Omega_e \simeq 3.7$ GHz) clearly show that the explanation for this lack of frequency shift lies somewhere else.

<u>Bibliography</u>

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