# ANOMALOUS TRANSPORT VIA KIRCHHOFF RADIATION

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**Abstract**

The principal toroidal transport anomalies, namely, (i) enhanced particle and thermal diffusivities, (ii) magnetic field dependence, (iii) isotope mass dependence, (iv) temperature dependence, and (v) density dependence, are traced to momentum exchange collisions induced via Kirchhoff thermal radiation. Kirchhoff radiation may also play an important role in power degradation in auxiliary heated plasmas, heat pinches, hot/cold pulse propagation, transport barriers, and L-H transitions.

## 1. Radiative collisionality

Collisionality occurring via radiative momentum exchange through thermally excited electrostatic cyclotron-harmonic modes far exceeds that due to the Fokker-Planck encounters [1]. The collisionality contribution of the Kirchhoff radiation is given by [1],

$$\nu_{\sigma\xi}^{rad} = \frac{T}{\pi^2 n_\sigma} \int_0^{\alpha_D k_D} \int_0^{\alpha_D k_D} \left| \frac{1}{m_\sigma v_{\parallel\sigma}} \frac{\Im\left[\omega^\sigma\right] \Im\left[\omega^\xi\right]}{\Re\left[\omega\right] \Im\left[\omega\right]} \right| k_\parallel k_\perp \; dk_\parallel dk_\perp \; , \tag{1}$$

where $T$ is the temperature in energy units, $\Re\left[\omega\right] + i\Im\left[\omega\right] = \omega(\mathbf{k})$; $\Im\left[\omega\right]$ is comprised of contributions $\Im\left[\omega^\sigma\right]$ from all the particle species $\sigma$ such that $\sum_\sigma \Im\left[\omega^\sigma\right] = \Im\left[\omega\right]$; $v_{\parallel\sigma}$ is the parallel velocity defined in (3); $\alpha_D \sim \mathcal{O}(1)$ defines the small-wavelength limit for the applicability of the Vlasov-Boltzmann equation; and $k_D = 2\pi/\lambda_D$ is the Debye wavenumber corresponding to the Debye length

$$\lambda_D = \left( \sum_\sigma \frac{n_\sigma q_\sigma^2}{T_\sigma} \right)^{-1/2} \; . \tag{2}$$

For a given $\mathbf{k}$, $\Im\left[\omega^\sigma\right]$ for each of the particle species consists of contributions from all the cyclotron-harmonic numbers $n$, such that $\Im\left[\omega^\sigma\right] = \sum_n \Im[\omega^\sigma]_n$. The corresponding particle parallel velocities in (1) are given by the relation

$$\omega - n\omega_{c\sigma} - k_\parallel v_{\parallel\sigma} = 0 \; . \tag{3}$$

The set of $\Im[\omega^\sigma]_n$ is obtained using the complete, homogeneous, cylindrical-geometry, hot-plasma, non-relativistic dispersion relation $D(\omega, \mathbf{k}) = 0$.

Unless otherwise stated, it is assumed that $T_e = T_i = T$, atomic mass number $A = 1$, and $\alpha_D = 1$ in the computational results of Figs. 1-6 for an electron-proton plasma ($n_e = 10^{20}\,\mathrm{m}^{-3}, T = 10\,\mathrm{keV}, B_0 = 6\,\mathrm{T}$) in thermodynamic equilibrium. One hundred cyclotron harmonics are included in the integration of (1).

## 2. Thermal plasmas

In (1) $\nu^{rad}$ scales approximately as,

$$\nu^{rad} \sim \frac{T^{1/2}}{n_e \lambda_D^4} \sim \frac{n_e T^{1/2}}{T_D^2} \,, \tag{4}$$

where $T$ is the temperature of the radiating plasma component, and $T_D$ is the temperature that determines the Debye length. In thermal equilibrium $T_D = T$, so that collisionality varies with temperature as $\sim T^{-3/2}$ (Fig. 1). As a result diffusivities $(D, \chi)$ increase as $T^{-1/2}$ near the plasma edge. This increase may be further accentuated by Debye length decrease due to the presence of impurities near the edge.

An important parameter affecting radiative collisionality is the ratio of gyroradius $\rho_{e,i}$ to the Debye length $\lambda_D$. For thermonuclear plasmas $\rho_e \sim \lambda_D$, whereas $\rho_i \gg \lambda_D$. Strong wave-particle interactions occur for perpendicular wavelengths $\lambda_\perp \sim \mathcal{O}(\rho)$. As the magnetic field is increased, $\rho_i$ shrinks towards $\lambda_D$, causing a $B_0^2$ dependence of $\nu^{rad}_{ii,ei,ie}$ (Fig. 2). A smaller increase $(\sim B_0)$ is registered by $\nu^{rad}_{ee}$ because $\rho_e$ is already close to $\lambda_D$. For $B_0 \gtrsim 2\,T$, $\chi_i$ dominates thermal conductivity, so that in thermonuclear plasmas, diffusivities $(D, \chi)$ are practically independent of $B_0$.

Identical considerations apply for density increase. Increasing density forces the Debye length further away from $\rho_i$ and occasions no alteration in $\nu^{rad}_{ii,ei,ie}$ (Fig. 3). Decreasing $\lambda_D$ does strengthen electron-electron interactions, but only as $n_e^{0.6}$ instead of the linear increase expected from (4).

Increasing isotope mass pushes $\rho_i$ further away from $\lambda_D$, causing a rapid decrease in $\nu^{rad}_{ii,ie} \sim A^{-3/2}$ and a more moderate fall in $\nu_{ei} \sim A^{-1/2}$ as shown in Fig. 4. Thus $(D, \chi_i) \sim A^{-1/2}$ as observed experimentally.

Figure 5 shows the strong dependence of $\nu^{rad}$ on $\alpha_D$. Figure 6 gives the parallel velocity distribution of $\nu^{rad}$. Since $\nu^{rad}_{ei}$ is dominantly contributed by the trapped electrons, the $\nu_{ei}$ enhancement does not affect parallel plasma resistivity.

## 3. Non-thermal plasmas

In this section we consider non-thermal plasmas characterized by (i) unequal electron/ion temperatures due to poor electron-ion coupling, and/or (ii) presence of non-Maxwellian tails due to auxiliary heating. In non-thermal plasmas the Debye length (and hence the number of modes) is determined by the cold plasma component, while the radiation temperature and diffusion step size are determined by the hot-plasma constituent.

Strictly speaking, Kirchhoff's law is inapplicable to such plasmas; however, valuable insights are possible by considering $T \neq T_D$ in (4). It is immediately apparent that the temperature of the cold-plasma component entering through $T_D$ in (4) is of critical importance. Minor changes in $T_D$ introduced through small variations in electron-ion coupling lead to dramatic changes in $\nu^{rad}$ and in diffusivities $(D, \chi)$.

Low-density Ohmically heated plasmas are characterized by cold ions due to poor electron-ion coupling. The small Debye length causes ever-increasing diffusivity enhancement as the plasma density is lower ed, giving rise to the observed linear (LOC) and saturated (SOC) Ohmic confinement regimes.

Auxiliary heating is invariably accompanied by energetic, hot tails. Assuming that the tail temperature rises linearly with injected power, and since the observed bulk-plasma temperature varies as $P^{1/2}$,

$$\chi \sim \nu T \sim \frac{T^{3/2}}{T_D^2} \sim \frac{P^{3/2}}{P} \sim P^{1/2} \, , \tag{5}$$

resulting in the observed power degradation in toroidal plasmas.

During ECR heating, kinetic temperature of the electron tail might exceed the bulk-plasma temperature at the axis. Energy can be transported against the bulk-plasma temperature gradient (heat pinch) without violating thermodynamical principles.

In tenuous, low-density, small-diameter plasmas, introduction of cold pellets causes density increase which promotes electron-ion coupling, leading to a rapid increase of Debye length with a concomitant reduction in diffusivities. This would cause temperature and density increase throughout the plasma column except in the edge region where local cooling due to the cold pellet predominates. A reverse sequence of events would lead to cooling near the plasma axis following local electron heating near the edge.

The considerations of this section show the critical role played by the Debye length alterations via electron-ion coupling in plasma transport due to radiative collisions. Conditions that promote electron-ion coupling reduce diffusivity. Since the Debye length is principally determined by the cold plasma component, even minor changes in electron-ion coupling lead to drastic changes in thermal as well as particle diffusivities. Such drastic changes may well have the appearance of bifurcations such as the ones causing transport barriers and L-H transitions.

## 4. Discussion

For thermal plasmas, radiative collisionality obtained using Kirchhoff's law is able to resolve the ubiquitous, yet hitherto untractable anomalies of toroidal plasma transport. The enhancement in $\chi_e$ is at the core of the transport riddle; its resolution alone establishes the importance of radiative collisionality. Among the consequences of $\nu_{ee}$ enhancement is the possibility of efficient Alfven wave current drive, since it is immaterial to which class of particles (trapped or circulating) the wave momentum is initially imparted.

Since Kirchhoff's law is inapplicable to non-thermal plasmas, the considerations of Sec. 3 are of a qualitative nature. Nevertheless, Kirchhoff radiation seems capable of resolving a diverse array of anomalous observations within a single unified framework.

## References

[1] S. Puri: Phys. Plasmas **5** (August 1998), *to be published*.

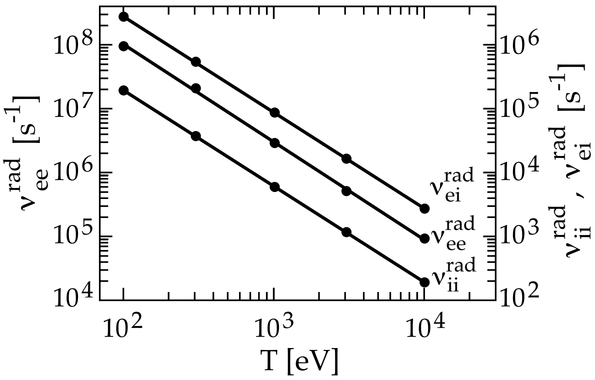


**Fig. 1.** $\nu^{rad}$ versus T

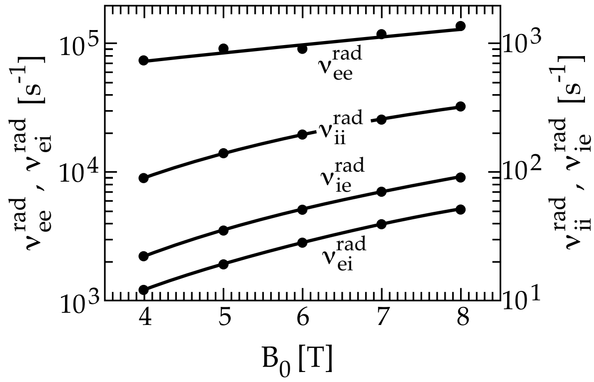


**Fig. 2.** $\nu^{rad}$ versus $B_0$

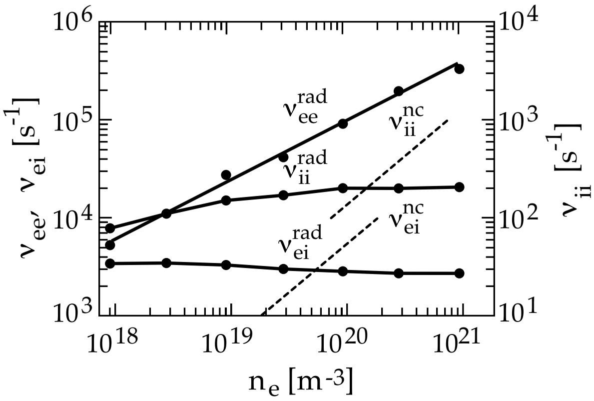


**Fig. 3.** $\nu^{rad}$ and $\nu^{nc}$ versus $n_e$

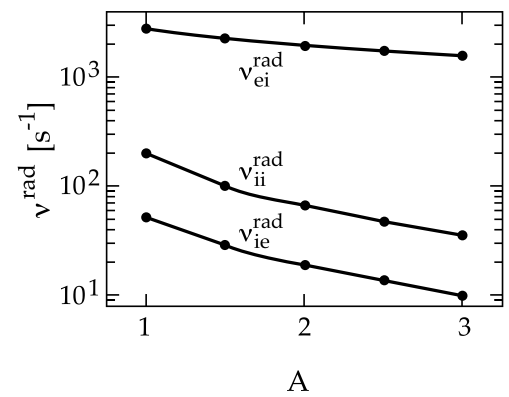


**Fig. 4.** $\nu^{rad}$ versus A

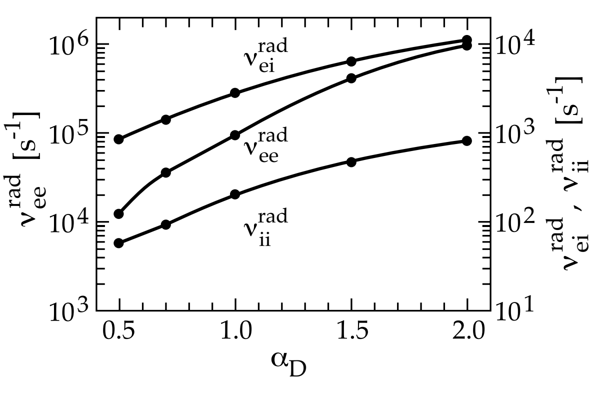


**Fig. 5.** $\nu^{rad}$ versus $\alpha_D$

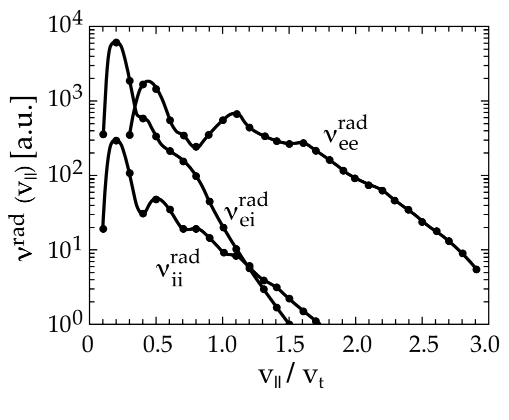


**Fig. 6.** $\nu^{rad}$ versus $\nu_{||}$