Network Synchronization and information transmission under time transformations

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We investigate the effect of general time transformations on the phase synchronization phenomenon and the mutual information rate between pairs of nodes in dynamical networks. We demonstrate two important results concerning the invariance of both phase synchronization and the mutual information rate. Under time transformations phase synchronization can neither be introduced nor destroyed and the mutual information rate cannot be raised from zero. On the other hand, quantifiers of the phase synchronization as well as the absolute value of nonzero mutual information rate can be drastically modified. Finally, we discuss the relevance of our findings for communication in dynamical networks.

Time and complex dynamics play a major role in biological, social, economical, and physical systems. Cycles of different periods often govern their dynamical behavior and determine their intrinsic activity. A variety of processes require a precise timing between the oscillators cycles for a proper functioning, as for example, the respiratory and cardiac systems [1], spike discharges and information transmission [2, 3] in neuron networks, ecology [4], fireflies blinking after dark, and peacemaker cells of the human heart [5]. Synchronization is an efficient mechanism to generate such a timing [1, 2, 3, 4, 5, 6, 7, 8]. Among several types of synchronization recently found in complex systems [7], chaotic phase synchronization (PS) displays special importance because of its weak constraints on the dynamics and coupling strength. It has been reported that PS mediates the process of information transmission and collective behavior in neural and active networks [2, 9, 10], as well as communication processes in the human brain [2, 11, 12].

Coupled dynamical systems under time transformations are important in physics without an absolute time as well as technological applications, biological systems, and data analysis, in situations where the time cannot be directly obtained, as in the study of sedimental cores in the field of geophysics. In the latter case, the time at which the sedimentation took place is usually unknown. Only a proxy for the time can be derived from the measurements, which does not yield the "real" time but only a monotonous transformation of it [13]. In the study of synchronization phenomenon in such a system the natural question is whether not having access to the *real* time could effect synchronization.

Such time transformations (typically nonlinear) have attracted a great deal of attention. They cause no change in the topology of the dynamics, but the duration of the cycles can be drastically modified. An important problem is to analyze whether the dynamical properties are invariant under time transformations [14]. Recent results have shown that dynamical systems under time transformation can present nontrivial and counterintuitive properties. For example, a nonmixing dynamics can be converted to a mixing one [15].

In this work, we show that time coordinate transformations, satisfying simple conditions of integrability, can neither introduce nor destroy the phenomenon of PS. We also explore the natural connection between synchronization and information exchange in coupled oscillators. We uncover the transformation law for the mutual information rate (MIR), the rate with which information about a node can be retrieved in another node. If the MIR is zero in one time frame it will remain zero for any other. On the other hand, if the MIR is nonzero under a time transformation it can be drastically modified. *Surprisingly, if there is no synchronization (to any extend) between the nodes forming a network, time transformations containing information about a node of the network cannot be used to carry this information to another node.*

We first illustrate our findings for the paradigmatic example of two coupled Rössler oscillators: $\dot{x}_{1,2} = -\alpha_{1,2}y_{1,2} - z_{1,2} + \epsilon(x_{2,1} - x_{1,2}), \dot{y}_{1,2} = \alpha_{1,2}x_{1,2} + 0.15y_{1,2}, \dot{z}_{1,2} = 0.2 + z_{1,2}(x_{1,2} - 10)$, with $\alpha_1 = 1$, and $\alpha_2 = \alpha_1 + \Delta\alpha_2$. We shall denote $\mathbf{x}_j = (x_j, y_j, z_j)$, where $j = 1, 2$, and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2)$. Since these oscillators have a proper rotation, i.e., the trajectory rotates around a fixed point [Fig. (1)a], we can simply define a phase by $\tan\phi_j = y_j/x_j$, which yields $\phi_j(\mathbf{x}, t) = \int_0^t (\dot{y}_j x_j - \dot{x}_j y_j)(x_j^2 + y_j^2)dt$. Given a real number c, the condition for PS between $\mathbf{x}_1$ and $\mathbf{x}_2$ can be written as:

$$|\phi_1(t) - \phi_2(t)| < c. \qquad (1)$$

Furthermore, let us denote the time at which the oscillator $\mathbf{x}_j$ completes its ith cycle by t_j^i. Thus, PS implies

$$|t_1^i - t_2^i| \leq \kappa, \qquad (2)$$

where $\kappa \in \mathbb{R}$ is a finite number. More details concerning this equivalence shall be provided later on. The value of κ shows how well paced both oscillators are; the smaller the value of κ, the better the timing between $\mathbf{x}_1$ and $\mathbf{x}_2$.

For $\epsilon = 0.0015$ and $\Delta\alpha_2 = 0.001$, the two oscillators are in PS, which means that the phase difference is

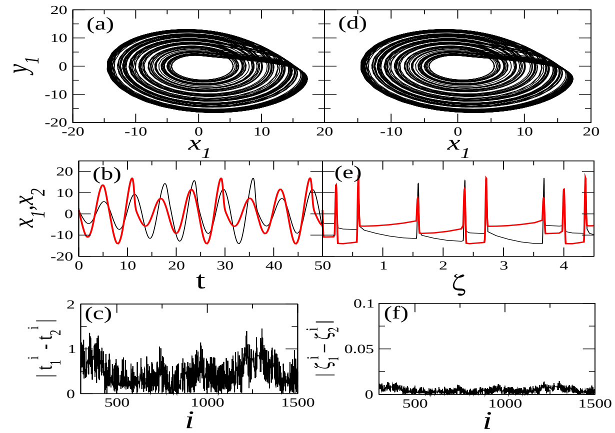

FIG. 1: [Color online] The effect of time coordinate change in two synchronous Rössler oscillators. In (a) we show the attractor projection onto the (x, y)-plane, and in (b) the time series x_1 and x_2 vs. t. In (c) we depict the time difference $|t_1^i - t_2^i|$. One can see that even though the quantity $|t_1^i - t_2^i|$ is bounded, it has a large fluctuation. In (d-f) we proceed a time-coordinate change $t \to \zeta$ given by Eq. (4). In (d) we show the attractor projection onto the (x, y)-plane, and the time series x_1 and x_2 vs. ζ in (e), while the amount $|\zeta_1^i - \zeta_2^i|$ is shown in (f). One can see that after the time-coordinate change the timing condition is drastically improved.

bounded for all times. Consequently, Eq. (2) holds [Fig. 1(c)]. In PS regimes the oscillators have the same mean frequency, namely $\langle\dot{\phi}_1(t)\rangle_t = \langle\dot{\phi}_2(t)\rangle_t \approx 1.035$, where $\langle\cdot\rangle_t$ is the time average with respect to t. The average period is given by $\langle T\rangle_t = 2\pi/\langle\dot{\phi}_j(t)\rangle_t \approx 6.067$. We have $max|t_1^i - t_2^i| \approx 3/2$ [Fig. 1(c)], corresponding approximately to $\langle T\rangle_t/4$, which can be rather problematic for a reliable communication system based on chaos synchronization, since both oscillators do not reach the Poincaré section with a precise timing [10].

The situation can be altered to improve substantially the timing condition in Eq. (2) changing the time coordinate $t \to \zeta$ by

$$d\zeta = \lambda(\mathbf{x}, t)dt, \tag{3}$$

such a transformation may distort directly the synchronization phenomenon acting on the times t_j^i. To improve the timing between the oscillators given a $\gamma \gg 1$, we perform the time coordinate change:

$$\lambda(\mathbf{x}, t) = \begin{cases} \gamma, & \text{if} \quad x_{1,2} > 0,\ x_{2,1} < 0, \text{ and } \dot{y}_1 > 0 \\ 1, & \text{otherwise} \end{cases} \tag{4}$$

which shrinks the time difference between t_1^i and t_2^i enhancing the pacing between the oscillators. γ may be chosen according to the desired pacing condition. For our purposes we fix $\gamma = 100$. The new time coordinate is given by $\zeta_j = \int_0^{t_j} \lambda(\mathbf{x}, t)dt$. This time transformation causes no changes in the state space, Figs. 1 (a) and (d). However, the time series of $x_1 \times t$ and $x_1 \times \zeta$ are drastically modified [Fig. 1(b,e)]. Although the time transformation is not able to interfere with the PS phenomenon [Fig. 1(f)], it changes the frequency of the oscillators $\langle\dot{\phi}_1(\zeta)\rangle_\zeta = \langle\dot{\phi}_2(\zeta)\rangle_\zeta \approx 1.350$, which implies that $\langle T\rangle_\zeta \approx 4.652$. On the other hand, we have that $max|\zeta_1^i - \zeta_2^i| \approx 0.015$, which corresponds approximately to $\langle T\rangle_\zeta/310$. *While the time transformation modifies the average period in about* 20%, *the timing is improved by a factor of 100.* Of course, Eq. (4) can be altered to have an even better timing. For instance, one could have a time dependent γ, *or even dependent on a stochastic process.*

We consider the general case of two oscillators $\dot{\mathbf{x}}_j = \mathbf{F}_j(\mathbf{x}_j)$, where $\mathbf{x}_j \in \mathbb{R}^n$ and $\mathbf{F}_j : \mathbb{R}^n \to \mathbb{R}^n$, and analyze a general coupling scheme:

$$\dot{\mathbf{x}}_{1,2} = \mathbf{F}_{1,2}(\mathbf{x}_{1,2}) + \mathbf{C}(\mathbf{x}, t)\mathbf{H}(\mathbf{x}), \tag{5}$$

where $\mathbf{H}(\mathbf{x}) = (\mathbf{H}_1(\mathbf{x}), \mathbf{H}_2(\mathbf{x}))$ is a coupling vector function, and $\mathbf{C}(\mathbf{x}, t)$ is the coupling matrix [16]. We suppose that each $\mathbf{x}_j$ has a stable attractor and a frequency $\dot{\phi}_j = \Omega_j(\mathbf{x}, t)$, where $\Omega_j(\mathbf{x}, t)$ is a continuous function (or Riemann integrable). Furthermore, we assume that there is a number M such that $\Omega_j(\mathbf{x}, t) \le M$. From now on, whenever there are no problems with the notations, we shall omit the dependence of the functions on the coordinates.

The relation between PS and the timing condition given by Eq. (2) follows from the fact that:

$$|\phi_1(t) - \phi_2(t)| \le M|t_1^i - t_2^i| + \Gamma, \tag{6}$$

where $\Gamma > 0$ is always bounded [17]. Therefore, a bounded time event difference $|t_1^i - t_2^i|$ implies the boundedness of the phase difference. It is easy to demonstrate that the boundedness of the phase difference implies Eq. (2).

PS is invariant under time coordinate transformations. To analyze the effect of general time coordinate change we assume $\lambda(\mathbf{x}, t)$ to be (i) at least Riemann integrable (ii) finite, and (iii) bounded away from zero. Holding these hypotheses *we put no further constrain on* $\lambda(\mathbf{x}, t)$, *so it could be any nonlinear function, or even a function depending on some random process.* The conditions (ii) and (iii) are equivalent to the existence of two numbers $\delta^{-1}, \eta \in \mathbb{R}_+$ such that $\delta^{-1} \le \lambda(\mathbf{x}, t) \le \eta$. Under the assumptions $(i{-}iii)$ we can prove that PS is invariant under time coordinate transformations. Hereafter, we postpone the detailed proof [19] and focus on the applications.

Distortions on the phase diffusion (coherence) by a time coordinate change and its effect on PS. Time transformations alter important characteristics of the dynamics. We can transform an oscillator that is originally endowed with strong phase diffusion into an oscillator with arbitrarily small phase diffusion (coherent behavior). The phase diffusion and the coherent properties are given by the distribution ρ of Δt_ℓ^i. By means of a time coordinate transformation we are able to deform ρ into an extremely sharp (delta-like) distribution.

We present the following analytical example to show that enhancing coherence in the oscillator does not introduce PS. Supposing that the oscillators $\mathbf{x}_1$ and $\mathbf{x}_2$ are not in PS, we write $t_1^i - t_2^i = \alpha \times i + \xi^i$, where $\alpha, \xi^i \in \mathbb{R}$ are chosen to hold the equality. By performing a time coordinate change we endow the oscillator $\mathbf{x}_1$ with zero phase diffusion. This means that we have a new time ζ with $\Delta\zeta_1^i = \zeta_1^i - \zeta_1^{i-1} = 1$, i.e. $\Delta\zeta_1^i = \Delta t_1^i / \Delta t_1^i$, where $\Delta t_1^i = t_1^i - t_1^{i-1}$. The new time coordinate is given by $\zeta_1^i = \sum_{n=0}^{i} \Delta t_1^n / \Delta t_1^n$ and $\zeta_2^i = \sum_{n=0}^{i} \Delta t_2^n / \Delta t_1^n$. We have $|\zeta_1^i - \zeta_2^i| = |\sum_i (\Delta t_1^n - \Delta t_2^n)/\Delta t_1^n|$. Next, consider the minimum Δt_1^n, namely $min_n \Delta t_1^n = \gamma^{-1}$. Thus, we have that $|\sum_i (\Delta t_2^n - \Delta t_1^n)/\Delta t_1^n| \geq \gamma |t_2^i - t_1^i|$, which can be written as: $|\zeta_1^i - \zeta_2^i| \geq \gamma(\alpha \times i + \xi^i)$. Therefore, as the number of periods tends to infinity, the time event difference $|\zeta_1^i - \zeta_2^i|$ diverges.

Introducing PS by violating hypotheses (ii) and (iii). Considering our former example where $t_1^i - t_2^i = \alpha \times i + \xi^i$, we could transform the time by $\lambda^{-1}(\mathbf{x}, t) = i$ if $t_1^i < t \leq t_1^{i+1}$. The timing condition is given by $|\zeta_1^i - \zeta_2^i| = |\int_{t_1^i}^{t_1^i - \alpha i - \xi^i} \lambda(\mathbf{x}, t)| \leq |\alpha i / i| + |\xi^i / i|$. Thus, $\lim_{i \to \infty} |\zeta_1^i - \zeta_2^i| \leq \alpha$. The time difference is bounded by α. This does not contradict our results, because this time transformation is not bounded, violating assumptions (ii) and (iii).

The effect of time transformations in the information transmission in networks. For every pair of oscillators $\mathbf{x}_j$ and $\mathbf{x}_k$ we can define a coordinate transformation $\mathbf{x}_{jk}^{\|} = \mathbf{x}_j + \mathbf{x}_k$ and $\mathbf{x}_{jk}^{\perp} = \mathbf{x}_j - \mathbf{x}_k$, that produces two positive conditional Lyapunov exponents (in units of bits/unit time) $\sigma^{\|}(t)$ and $\sigma^{\perp}(t)$. The mutual information rate (MIR) $I_C(t)$, is bounded from above by $\sigma^{\|}(t)_{-}\sigma^{\perp}(t)$ [9].

The main goal is to know how the mutual information rate behaves as we implement a time transformation. By choosing a proper nonlinear $\lambda(\mathbf{x}, t)$ in Eq. (3) we can introduce different time scales in the oscillators time series as well as endow the time transformation with as much information about the dynamics as we want. The main question is whether, under such nonlinear $\lambda(\mathbf{x}, t)$, the information contained in $\lambda(\mathbf{x}, t)$ could be transmitted to the oscillators.

To answer this question we need to uncover the general transformation law for I_C. Time transformations modify the conditional Lyapunov exponents in the same way as the Lyapunov exponents [14]. Therefore, we can uncover the transformation law of $I_C(t)$ for the time-ζ [18]:

$$I_C(\zeta) \leq \frac{I_C(t)}{\langle \lambda \rangle_t}, \tag{7}$$

where, again $\langle \cdot \rangle_t$ stands for the time average. Equation (7) shows an invariant character of I_C. *if $I_C(t) = 0$, what happens in the absence of synchronization (correlation) between oscillators, no time transformation that respects conditions $(i - iii)$ can raise $I_C(t)$ from zero.* Hence, no matter how much information is contained in $\lambda(\mathbf{x}, t)$, if there is no synchronization this information can not be used. If, on the other hand, $I_C(t)$ is positive, then $I_C(\zeta)$ can be made arbitrarily large.

To illustrate our findings, we consider a network of four identical Hindmarsh-Rose chaotic neurons electrically coupled in an all-to-all topology, $\dot{x}_j = y_j + 3x_j^2 - x_j^3 - z_j + I_j + \sum_k C_{jk}(x_k - x_j)$, $\dot{y}_j = 1 - 5x_j^2 - y_j$, $\dot{z}_j = -rz_j + 4r(x_j + 1.6)$, where C_{jk} stands for the coupling matrix. We use r=0.005, $I_i = 3.2$, and random initial conditions.

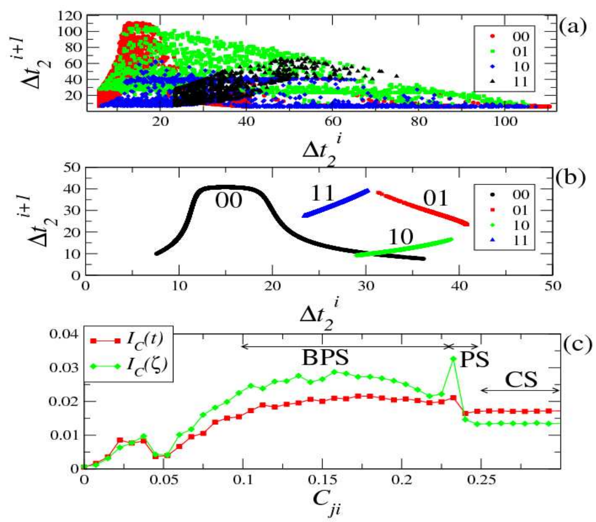

FIG. 2: [Color online] Encoded time intervals between two spikes in neuron $\mathbf{x}_2$ for C_{jk}=0 (a), and for C_{jk}=0.3 (b). In (c), the MIR between neurons $\mathbf{x}_1$ and $\mathbf{x}_2$, for the time-t (filled squares), and for the time-ζ (filles diamonds). BPS (burst phase synchronization) is found for C_{jk}=[0.1,0.23]. In this regime only the burst are phase synchronized. PS is found for C_{jk}=[0.23,0.25], and CS (complete synchronization) is found for C_{jk}=[0.25,0.3]. For $C_{jk} \in$ [0.05,0.23] $\lambda(\mathbf{x}, t)$ is smaller than 1, which provides an increasing in I_C up to 60%. For CS $\langle \lambda \rangle_t = 1.29$ providing a decreasing in I_C.

We define the following time transformation

$$\lambda(\mathbf{x}_1, t) = \begin{cases} \alpha, & \text{if } x_1 = 0 \text{ and } y_1 > -4.6, \\ \beta, & \text{if } x_1 = 0 \text{ and } y_1 \leq -4.6. \end{cases} \tag{8}$$

This shrinks the time between spikes when the $y_1 > -4.6$ and stretches when $y_1 > -4.6$ creating a frequency modulation between the spikes, which depends on the trajectory position. Hence, the transformation carries information about $\mathbf{x}_1$. In our analysis we keep fix $\alpha = 0.5$ and $\beta = 2$. t_j^i denotes the time at which occurs the ith crossing of the trajectory of $\mathbf{x}_j$ with the section $x_j = 0$ (an spike event). The time interval between two crossings is $\Delta t_j^i = t_j^{i+1} - t_j^i$.

We introduce a symbolic dynamics which exhibits rather easily the results for the distinct synchronization regimes. We can encode the binary information about the transformation $\lambda(\mathbf{x}_1, t)$ by setting α to the symbol "0" and β to "1". Hence, we have for two consecutive

$\lambda = \alpha$: '00'; one $\lambda = \alpha$ followed by $\lambda = \beta$: '01'; one $\lambda = \beta$ followed by $\lambda = \alpha$: '10'; and finally two consecutive $\lambda = \beta$: '11'. Whenever the time transformation is able to transmit the information about the symbols, we can access the information about $\mathbf{x}_1$ in the spike time intervals of the other neurons.

Figures 2(a-b) show return maps Δt_2^i vs. Δt_2^{i+1} of the neuron $\mathbf{x}_2$. We split this map into four return maps, depending on the value of the transformation $\lambda(\mathbf{x}_1, t)$. That is, distinguished by the different symbols '00','01','10','11'. The information about the values of $\lambda(\mathbf{x}_1, t)$ should be considered to be unknown, but here we make use of it to illustrate our ideas.

By measuring Δt_2^i we should be able to infer the time interval Δt_1^i, if the time transformation can transmit information. Figure 2(a) shows that return maps $\Delta t_2^i \, vs. \, \Delta t_2^{i+1}$ for the different values of $\lambda(\mathbf{x}_1, t)$ superimpose, and as a consequence it is impossible to discern whether the region that encodes for 00 is mapped to either 01 or 00, and so on. That leads to a complete uncertainty about $\lambda(\mathbf{x}_1, t)$ by measuring Δt_2^i. Therefore, there is no exchange of information between $\mathbf{x}_1$ and $\mathbf{x}_2$.

When the neurons are complete synchronized (for C_{ji}=0.3), we see in Fig. 2(b) that except for one point, the return maps $\Delta t_2^i \, vs. \, \Delta t_2^{i+1}$ for different values of $\lambda(\mathbf{x}_1, t)$ are disjoint, which means that by measuring Δt_2^i we have complete knowledge about the trajectory of the neuron $\mathbf{x}_1$.

Effect of $\lambda(\mathbf{x}_1, t)$ *on* $I_C(t)$. We keep fix $\lambda(\mathbf{x}_1, t)$ and vary C_{ij}. Eq. (7) states that whenever $\langle \lambda(\mathbf{x}_1, t) \rangle_t < 1$ the time transformation increases the MIR. In Fig. 2(c) we show the MIR between Δt_1^i and Δt_2^i using the Shannon mutual information [20], for the two time frames. $I_C(t)$ denotes the MIR in the time-t frame and $I_C(\zeta)$ the MIR in the time-ζ frame. For $C_{ij} \in [0.5, 0, 23]$ $\langle \lambda(\mathbf{x}_1, t) \rangle_t < 1$ which provides an effective increasing in the MIR.

In Eq. (8), λ is defined to contain information about $\mathbf{x}_1$. However, λ could be defined to contain information about an arbitrary information signal to be transmitted. In such a case, each disjoint region [as the ones shown in Fig. 2(b)] would encode information about this signal, which can be retrieved somewhere else in the network.

In conclusion, we have demonstrated that, whenever $\lambda(\mathbf{x}, t)$ is bounded bounded away from zero and Riemann integrable, the phenomenon of PS is invariant. Furthermore, the MIR is also invariant, in the sense that it cannot be raised from zero. The invariant character of PS and MIR points out a remarkable tight relationship between synchronization and information. If there is no synchronization (to any extend) in the network time transformations cannot be used to carry information from one node to another, regardless of how much information it contains. On the other hand, quantifiers such as the timing and the absolute value of a nonzero MIR can be drastically modified. This can be advantageously used for practical applications as communication in dynamical networks.

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[16] This procedure can be easily extended to general networks without lost of generality.

[17] We have $|\phi_1(t) - \phi_2(t)| = |\int_0^{t_1^i} \Omega_1 dt - \int_0^{t_2^i} \Omega_2 dt - \int_{t_2^i}^{t_1^i} \Omega_2 dt + \beta^i(t)|$, where $\beta^i(t) = \int_{t_1^i}^{t} \Omega_1 dt - \int_{t_1^i}^{t} \Omega_2 dt$. For $\phi_j(t_j^i) = i \times 2\pi$, and noting that $|t_j^i - t_j^{i-1}| \leq \Lambda$, implying $max|\beta^i| \leq 2M\Lambda < \Gamma$.

[18] The proof of the transformation law for I_C follow similar arguments to the ones of Ref. [14].

[19] T. Pereira, et al. to be published.

[20] We encode the spikes using the following rule. The i-th symbol of the encoding is by "1" if a spike is found in the time interval $[i\delta, (i+1)\delta[$, and "0" otherwise. A symbolic sequence is split into words of length L=4. The Shannon entropy is H = -$\sum_m P_m \log_2 P_m$, where P_m is the probability of finding one of the 2^L words. The MIR is estimated by $I_C = [H(\mathbf{x}_1) + H(\mathbf{x}_2) - H(\mathbf{x}_1; \mathbf{x}_2)]/(\delta \times L)$.

We choose $\delta \in [\min(\Delta t_j^i), \max(\Delta t_j^i)]$ to maximize I_C.