Bogoliubov spectrum of a cigar shaped Fermi superfluid in an optical lattice at the BEC-BCS crossover

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We study the Bogoliubov spectrum of an elongated Fermi superfluid confined in an one-dimensional superfluid along the Bose-Einstein-condensate (BEC)-Bardeen-Cooper-Schrieffer (BCS) crossover. We derive analytic expressions for the sound velocity, effective mass and the Bogoliubov excitation spectrum of the axial quasiparticles long the crossover based on the hydrodynamic theory. Our investigation reveal interesting signatures of BEC-BCS crossover in an optical lattice which deserve experimental investigation.

I. INTRODUCTION

The experimental realization [1, 2] of optical lattices for fermionic isotopes such as ^6Li or ^{40}K is stimulating new perspectives in the study of superfluidity in these systems. An increase in the superfluid transition temperature when using potentials created by standing light waves has been predicted [3]. It has been shown that for values of the Fermi energy above the first Bloch band the center of mass motion of a Fermi gas trapped in an one-dimensional periodic potential is overdamped in the collisional regime due to Umklapp process [4]. In such a fermionic system, it should be possible to adjust the interaction strength and light intensity to tune the system continuously between two limits: a Bardeen-Cooper-Schrieffer (BCS) type superfluid (involving correlated atom pairs in momentum space) and a Bose-Einstein condensate (BEC) in which spatially local pairs of atoms are bound together. This crossover between BCS-type superfluidity and the BEC limit for a dilute gas of fermionic atoms has been of recent theoretical interest [5]. In particular lot of theoretical attention has focussed on the collective excitations at the BEC-BCS crossover [6]. The first experimental results on the collective frequencies of the lowest axial and radial breathing modes of ultracold gases of ^6Li across the Feshbach resonance have also become available [7]. In atomic Fermi gases, tunable strong interactions are produced using the Feshbach resonance [8]. Feshbach resonance, occurs when the energy of a quasibound molecular state becomes equal to the energy of two free atoms. The magnetic field dependence of the resonance allows precise tuning of the atom-atom interaction strength. Across the resonance the s-wave scattering length goes from large positive to large negative values. The fermionic system becomes molecular BEC for strongly replusive interaction and transforms into a BCS superfluid when the interaction is attractive. Recent experiments have entered the crossover regime and yielded results of the interaction strength by the cloud size and expansion [9]. In atomic Fermi gas experiments, BEC-BCS crosover regime in optical lattices has not yet been demonstrated. Considering the fact that experiments in this area are making rapid progress, we were motivated to study for the first time the low energy Bogoliubov spectrum of a cigar shaped superfluid Fermi gas confined in an one-dimensional optical lattice along the BEC-BCS crossover regime using the hydrodynamic approach. For fermions confined in a trapping potential, the density profile changes slowly in space if the particle number of the system is large enough. Under such conditions, a local density approximation can be applied to the state and a hydrodynamic approach can be adopted to investigate the low energy collective modes. Expressions for the effective mass,sound velocity and various collective modes are new results of this work.

II. THE EFFECTIVE ACTION AND THE HYDRODYNAMIC EQUATIONS IN AN OPTICAL LATTICE

We consider a cigar shaped dilute ultracold fermionic gas trapped in an one-dimensional optical lattice. The optical lattice is formed by two counterpropagating laser beams, for example in the z direction.

$$V_{op}(z) = sE_R \sin^2\left(\frac{\pi z}{d}\right) \tag{1}$$

Here, d is the lattice period. s are the dimensionless amplitude of the lattice potential. $E_R = \dfrac{\hbar^2\pi^2}{2md^2}$ is the recoil energy ($\omega_R = \frac{E_R}{\hbar}$ is the corresponding recoil frequency) of the lattice. In addition to the optical lattice, we also have a harmonic trap $V_{ho}(r,z) = \dfrac{m}{2}\left(\omega_r^2 r^2 + \omega_z^2 z^2\right)$. Mass of the fermionic atom is m. ω_r and ω_z are the radial and the axial trap frequencies. We take $\omega_r > \omega_z$ so that an elongated cigar shaped BEC is formed. The harmonic oscillator frequency corresponding to small motion about the minima of the optical lattice is $\omega_s \approx \dfrac{\sqrt{s_1}\hbar\pi^2}{md^2}$. $\omega_s >> \omega_z$ so that the optical lattice dominates the harmonic potential along the z-direction and hence the harmonic potential is neglected. The strong laser intensity will give rise to an array of several quasi-two dimensional pancake shaped condensates. In writing down the effective action, we are follwing Wouters et al. [10]. The partition function for a system consisting of layers of 2D fermions at $T = 0$ is

$$Z = \int D\psi^\dagger_{j,\sigma}(\vec{x}) D\psi_{j,\sigma}(\vec{x}) exp\left(-S\left[\frac{\psi^\dagger_{j,\sigma}(\vec{x}), \psi_{j,\sigma}(\vec{x})}{\hbar}\right]\right) \tag{2}$$

where the action is given by

$$\begin{aligned} S\left[\psi^\dagger_{j,\sigma}(x), \psi_{j,\sigma}\right] &= \sum_j \sum_\sigma \int d^3x\, \psi^\dagger_{j,\sigma}(x)\left(\partial_\tau - \frac{\nabla^2}{2m} + V_{ho}(j) + \mu\right)\psi_{j,\sigma}(x) - \\ & U\psi^\dagger_{j,\uparrow}(x)\psi^\dagger_{j,\downarrow}(x)\psi_{j,\downarrow}(x)\psi_{j,\uparrow}(x) + J\psi^\dagger_{j,\sigma}(x)\psi_{j+1,\sigma}(x) + \psi^\dagger_{j+1,\sigma}(\vec{x})\psi_{j,\sigma}(x) \end{aligned} \tag{3}$$

Here, $x = (\vec{x}, \tau)$. The field $\psi_{j,\sigma}(x)$ belongs to a fermion of mass m in layer j and spin $\sigma(\uparrow$ or $\downarrow)$. U is the attractive strength between the fermions. The interlayer Josephson tunneling energy is $J = sE_R\left(\dfrac{\pi^2}{4} - 1\right) exp\left(-\sqrt{s}\left(\dfrac{\pi}{2}\right)^2\right)$[11]. In order to get rid of the quartic term in equation (3) one needs to perform a Hubbard-Stratonovic (HS) transformation and introduce the HS fields $|\Delta_j|$, θ_j, after which the integration over fermionic variables is performed. Our goal is an investigation of the superfluid properties of the ultracold Fermi system using the hydrodynamic approach. The hydrodynamic interpretation of $|\Delta_j|^2$ is the density of fermion pairs, whereas $\vec{v}(x) = \dfrac{\hbar\nabla_{\vec{x}}\theta_j(\vec{x})}{m}$ is interpreted as the superfluid velocity field.The final result for the effective action at $T = 0$ can be written as

$$S_{eff} = \sum_j \left\{-\hbar\, tr\left[\left(\frac{-G^{-1}}{\hbar}\right)\right] - \int d^3x\, \frac{|\Delta_j|^2}{g} - \int d^3x\, J_{j\to j+1}\cos\left(\theta_{j+1} - \theta_j\right)\right\} \tag{4}$$

where,

$$-G^{-1} = \sigma_0\left(\hbar\frac{\partial}{\partial\tau}\right) + \sigma_3\left(-\frac{\hbar^2}{2m}\nabla^2_{\vec{x}} - V_{ho}(j) + \mu\right) - \sigma_1\left(\hbar|\Delta_j|\right) \tag{5}$$

and σ_i are the pauli matrices.

$$J_{j\to j+1} = \frac{J^2\rho_j}{\frac{2\pi\hbar^2\rho_j}{m} + E_b^{2D}} \tag{6}$$

ρ_j is the density at each layer j. E_b^{2D} is the binding energy of the molecule in each layer. It is given by [12]

$$E_b^{2D} = 0.583\,\sqrt{s}\,E_R\, exp\left(\frac{\lambda}{\sqrt{2\pi}a_s s^{-1/4}}\right) \tag{7}$$

λ is the wavelength of the laser light and a_s is the scattering length. Since we are interested in the low-energy regime of S_{eff}, corrections that go beyond this regime are neglected [13]. Since we are dealing with $T = 0$ dynamics, the normal component is absent and the superfluid is stiff. This means that we neglect terms proportional to $\frac{\nabla^2\theta_j(x)}{m}$ which is equivalent to $\vec{\nabla}.\vec{v}=0$ (we consider steady flow). Having obtained the effective action S_{eff} that depends on $\theta_j(x)$ and $\rho_j(x)$, we get

$$\frac{\hbar}{2}\frac{\partial\rho_j}{\partial t}=-\frac{\hbar^2}{4m}\vec{\nabla}.\left(\rho_j\vec{\nabla}\theta_j\right)+J_{j,j-1}\sin(\theta_j-\theta_{j-1})-T_{j+1,j}\sin(\theta_{j+1}-\theta_j) \tag{8}$$

$$-\frac{\hbar}{2}\frac{\partial\theta_j}{\partial t}=\frac{\hbar^2}{8m}\left(\nabla\theta_j\right)^2+V_{ho}+\mu-\frac{\partial J_{j+1,j}}{\partial\rho_j}\cos(\theta_{j+1}-\theta_j)-\frac{\partial J_{j-1,j}}{\partial\rho_j}\cos(\theta_j-\theta_{j-1}) \tag{9}$$

In the next sextion our starting point will be the above hydrodynamic equations and solve the corresponding equations of motion for the density and velocity fluctuations.

III. MULTIBRANCH BOGOLIUBOV SPECTRUM

The equation of state enters through the density dependent chemical potential. We assume the power-law form of the equation of state as $\mu(\rho)=C\rho^\gamma$ [6]. γ is an effective polytropic index. The polytropic approximation has the advantage of allowing one to get analytical expressions for the eigenfunctions and eigenfrequencies of collective modes for various superfluid regimes in a unified way. At equilibrium, the two dimensional density profile takes the form at each layer $\rho_0(r)=\left(\frac{\mu}{C}\right)^{1/\gamma}\left(1-\tilde{r}^2\right)^{1/\gamma}$, where $\tilde{r}=\frac{r}{R}$ and $R=\sqrt{\frac{2\mu}{m\omega_r^2}}$. Linearizing around equilibrium, $\rho_j=\rho_0+\delta\rho_j$, $\theta_j=\delta\theta_j$ and $\mu(\rho_j)=\mu(\rho_0)+\frac{\partial\mu}{\partial\rho_j}|_{\rho=\rho_0}\delta\rho_j$. The equations of motion for the density and phase fluctuations are

$$\frac{\hbar}{2}\frac{\partial\delta\rho_j}{\partial t}=-\frac{\hbar^2}{4m}\vec{\nabla}.\left(\rho_0\vec{\nabla}\delta\theta_j\right)+\frac{J^2\rho_0}{\frac{2\pi\hbar^2\rho_0}{m}+E_b^{2D}}\left(2\delta\theta_j-\delta\theta_{j+1}-\delta\theta_{j-1}\right) \tag{10}$$

$$-\frac{\hbar}{2}\frac{\partial\delta\theta_j}{\partial t}=\frac{\partial\mu}{\delta\rho_j}|_{\rho_j=\rho_0}\delta\rho_j \tag{11}$$

In deriving the above equations,we have made the following assumptions

$$J_{j+1,j}=J_{j,j-1}=\frac{J^2\rho_0}{\left(\frac{2\pi\hbar^2\rho_0}{m}+E_b^{2D}\right)} \tag{12}$$

and

$$\frac{\partial J_{j\pm1,j}}{\partial\rho_j}=\frac{J^2E_b^{2D}}{2\left(\frac{2\pi\hbar^2\rho_0}{m}+E_b^{2D}\right)^2}<<\mu(\rho_j) \tag{13}$$

Let us consider the experiment of Greiner et al. [14]. They had prepared ultracold gas of fermionic ^{40}K atoms in the lowest energy spin states $|f=9/2,m_f=-7/2\rangle$ and $|f=9/2,m_f=-9/2\rangle$, where f is the total atomic angular momentum and m_f the magnetic quantum number. The Feshbach resonance occurs at a magnetic field $B_0=202.1\pm0.1\ G$ and has a width of $w=7.8\pm0.6\ G$ with $B<B_0$, the s-wave scattering length a_s is positive and a weakly bound molecular BEC is formed. At $B=B_0$ (unitary limit), $a_s\rightarrow\pm\infty$, corresponding to a very small binding energy (equation 7). Beyond the resonance ($B>B_0$), a_s is negative and the binding energies are related as $E_{bcs}^{2D}<E_{uni}^{2D}<E_{bec}^{2D}$. The scattering length a_s is determined as $a_s=174a_0\left(1+\frac{w}{(B_0-B)}\right)$ [14]. a_0 is the Bohr radius. Since we are interested in the crossover, we will focus our attention near the unitary limit.On the BEC side

of the unitary limit, if we take $B = 201.2\ G < B_0$, we find that $a_s^{bec} \approx 3.78 \times 10^{-7} m$ and $E_{bec}^{2D} \approx 4.26 E_R$. In the BEC region the chemical potential is $\mu = E_{bec}^{2D}$. Using these values, we find that $E_{bec}^{2D} >> \frac{2\pi\hbar^2\rho_0}{m}$ and $\mu >> \frac{\partial J_{j\pm1,j}}{\partial \rho_j}$. On the BCS side near the unitary limit, $B = 202.2\ G$,$a_s^{bcs} \approx -3.60 \times 10^{-7} m$ and $E_{bcs}^{2D} \approx 0.22 E_R$ and $\mu = E_F$(Fermi energy). Therefore in the BCS region, $E_{bcs}^{2D} \rightarrow 0$, consequently, $\frac{\partial J}{\partial \rho_j} \rightarrow 0$ and $\mu = E_F = \frac{\lambda^2 k_F^2}{(2\pi)^2} \propto \frac{1}{a_s^2}$. a_s is -ve and small and hence $\mu >> \frac{\partial J}{\partial \rho_j}$. Consequently we are justified in making the assumption (13). The second order equation of motion for the density fluctuation is given by

$$\frac{\hbar}{2}\frac{\partial^2 \delta\rho_j}{\partial t^2} = \frac{\hbar}{8m}\vec{\nabla}.\left(\rho_0 \vec{\nabla}\frac{\partial \mu}{\partial \rho_j}|\delta\rho_j\right) + \frac{2C\gamma\rho_0^{\gamma}F(\rho_0)}{\hbar}\left(-2\delta\rho_j + \delta\rho_{j+1} + \delta\rho_{j-1}\right) \tag{14}$$

Where

$$F(\rho_0) = \frac{J^2}{\left(\frac{2\pi\hbar^2\rho_0}{m} + E_b^{2D}\right)} \tag{15}$$

A further simplification yields

$$\frac{\partial^2 \delta\rho_j(\tilde{r})}{\partial t^2} = \frac{\mu\gamma}{mR^2}\vec{\nabla}_{\tilde{r}}.\left((1-\tilde{r}^2)^{1/\gamma}\vec{\nabla}_{\tilde{r}}(1-\tilde{r}^2)^{1-1/\gamma}\delta\rho_j(\tilde{r})\right) + \frac{4C\gamma\rho_0^{\gamma}(\tilde{r})F(\rho_0)}{\hbar^2}\left(-2\delta\rho_j(\tilde{r}) + \delta\rho_{j+1}(\tilde{r}) + \delta\rho_{j-1}(\tilde{r})\right) \tag{16}$$

We assume a normal mode solution of the form

$$\delta\rho_j(\tilde{r}, z, t) = \delta\rho_0(\tilde{r})\ exp\ i\left(\omega(k)t - jkd\right) \tag{17}$$

This yields

$$-\tilde{\omega}_\alpha^2\delta\rho_0(\tilde{r}) = \frac{\gamma}{2}\vec{\nabla}_{\tilde{r}}.\left((1-\tilde{r}^2)^{1/\gamma}\vec{\nabla}_{\tilde{r}}(1-\tilde{r}^2)^{1-1/\gamma}\delta\rho_j(\tilde{r})\right) - \frac{16\gamma C\rho_0^{\gamma}(\tilde{r})F(\rho_0)}{E_R^2}\sin^2\left(\frac{kd}{2}\right)\delta\rho_0(\tilde{r}) \tag{18}$$

where, $\tilde{\omega} = \frac{\omega}{\omega_R}$. Here α is a set of two quantum numbers:radial quantum number n_r and the angular quantum number m.

For $k = 0$, the energy spectrum is given by

$$\tilde{\omega}_\alpha^2 = |m| + 2n_r\left(\gamma(n_r + |m|) + 1\right) \tag{19}$$

The corresponding normalized eigenfunction is given by

$$\delta\rho_\alpha = A\left(1-\tilde{r}^2\right)^{(1/\gamma-1)}\tilde{r}^{|m|}P_{n_r}^{(1/\gamma-1,|m|)}(2\tilde{r}^2-1)\ exp\left(im\phi\right) \tag{20}$$

where $P_n^{a,b}(x)$ is a Jacobi polynomial of order n and ϕ is the polar angle. Also, the normalization constant A is given by

$$A^2 = \frac{2^{2-2/\gamma}}{\sqrt{\pi}R^2}\frac{[\Gamma(n_r+1)]^2\Gamma(1/\gamma)\Gamma(2/\gamma+2n_r+|m|)}{\Gamma(1/\gamma-1/2)[\Gamma(1/\gamma+n_r)]^2\Gamma(2n_r+|m|+1)}. \tag{21}$$

For $k \neq 0$, we expand the density fluctuation as

$$\delta\rho = \sum_\alpha b_\alpha\delta\rho_\alpha(\tilde{r}, \phi). \tag{22}$$

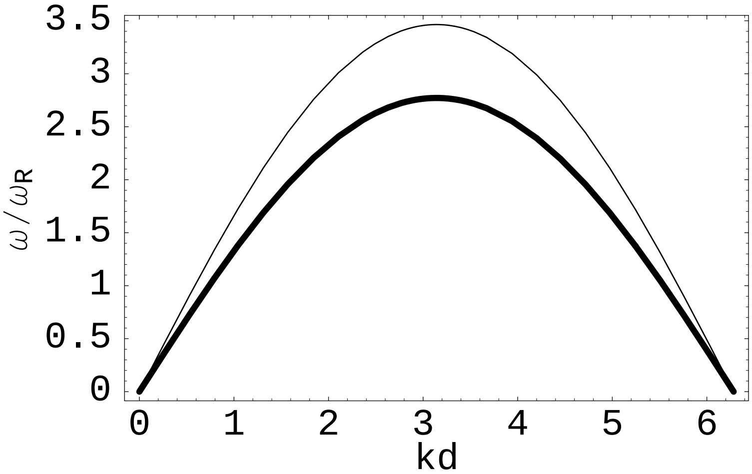

Figure 1: Plots of the phonon modes ($m=0, n_r=0$) in the BEC regime (thick line,$\mu = E_F$, $\frac{J}{E_R}=1, \gamma=0.8$) and BCS regime (thin line,$E_F = 4E^{2D}_{bcs}$, $\frac{J}{E_R}=1, \gamma=0.6$).

Substituting the above expansion into equation (18), we obtain

$$\begin{aligned} 0 &= [\tilde{\omega}_\alpha^2 - [|m| + 2n_r(\gamma(n_r+|m|)+1)] \\ &- B_0 \sin^2\left(\frac{kd}{2}\right)]b_\alpha + B_0 \sin^2\left(\frac{kd}{2}\right)\sum_{\alpha'} M_{\alpha\alpha'} b_{\alpha'}. \end{aligned} \quad (23)$$

where

$$B_0 = \frac{16\gamma\mu F(\rho_0)}{E_R^2} \quad (24)$$

The matrix element $M_{\alpha,\alpha'}$

$$\begin{aligned} M_{\alpha\alpha'} &= \tilde{A}^2 \int d^2\tilde{r}(1-\tilde{r}^2)^{2\gamma_0}\tilde{r}^{2+|m|+|m'|}e^{i(m-m')\phi} \\ &\times P_{n'_r}^{(\gamma_0,|m'|)}(2\tilde{r}^2-1)P_{n_r}^{(\gamma_0,|m|)}(2\tilde{r}^2-1), \end{aligned} \quad (25)$$

Equation (23) is the central result of this work which is derived for the first time.

where $\gamma_0 = 1/\gamma - 1$, $\tilde{A}^2 = \pi R^2 A^2$. In the BEC side of the unitary limit, $E^{2D}_{bec} >> \frac{2\pi\hbar^2\rho_0}{m}$ and $\mu = E^{2D}_{bec}$. Therefore $B_0^{bec} = \frac{16J^2}{E_R^2}$. On the other hand in the BCS side of the unitary limit, E^{2D}_{bcs} becomes comparable to $\frac{2\pi\hbar^2\rho_0}{m}$ and $\mu = E_F$. This gives $B_0^{bcs} = \frac{16J^2E_F}{E_R^2\left(E^{2D}_{bcs} + \frac{2\pi\hbar^2\rho_0}{m}\right)} \approx \frac{8J^2}{E_R^2}\left(\frac{E_F}{E^{2D}_{bcs}}\right)$. In typical experiments [9, 14], $E_F >> E^{2D}_{bcs}$.To calculate the sound velocity and the multibranch Bogoliubov spectrum, we need to know the adiabatic index γ. $\gamma = 2/3$ denotes the unitary as well as the BCS limit []. For the BEC side of the unitary limit $\gamma > 2/3$ and for the BCS side of the unitary limit we take $0.6 < \gamma < 2/3$. In Figure 1, we plot the lowest ($n_r = 0, m = 0$)mode for the BEC side of unitary limit (thick line, $\mu = E_F$, $\frac{J}{E_R} = 1, \gamma = 0.8$) and BCS side of unitary limit (thin line,$E_F = 4E^{2D}_{bcs}$, $\frac{J}{E_R} = 1, \gamma = 0.6$).Clearly, we find that in the entire Brillioun zone, the frequency of the phonon mode in the BCS side of unitary limit is greater than that in the BEC side of unitary limit. In the limit of long wavelength, the $n_r = 0$ mode is phonon like

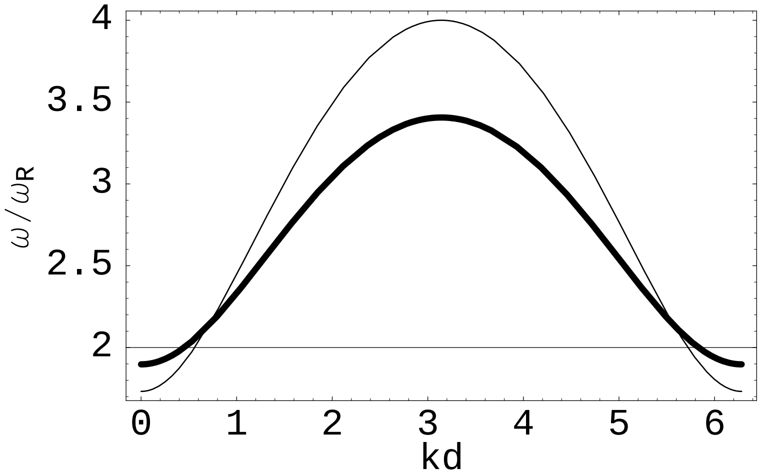

Figure 2: Plots of the monopole modes ($m = 0, n_r = 1$) in the BEC regime (thick line,$\mu = E_F$, $\frac{J}{E_R} = 1, \gamma = 0.8$) and BCS regime (thin line,$E_F = 4E_{bcs}^{2D}$, $\frac{J}{E_R} = 1, \gamma = 0.6$). One can clearly see the difference in the long wavelength and the short wavelength behaviour.

$$\omega_0 \approx k\sqrt{\frac{J}{m^*_{bec/bcs}}} \tag{26}$$

where $m^*_{bec} = \frac{\hbar^2}{2Jd^2\gamma(2-\gamma)}$ is the effective mass in the BEC side of unitary limit and $m^*_{bcs} = \frac{\hbar^2}{Jd^2\gamma(2-\gamma)}\left(\frac{E^{2D}_{bcs}}{E_F}\right)$ in the BCS side of unitary limit. Since $E_F >> E^*_{bcs}$ in the BCS side of the unitary limit, $m^*_{bcs} < m^*_{bec}$,this implies v_{bcs}(velocity of sound in the BCS side of unitary limit)$> v_{bec}$(velocity of sound in the BEC side of unitary limit). By gradually changing the magnetic field when we go from the BEC regime to the BCS regime, we obsere an increase in the phonon mode which is expected for a collisionless Fermi gas, where the elastic collision rate is strongly reduced by pauli blocking. In the absence of coupling between the axial modes with the density along the radial direction (for the $n_r = 0$ mode),the effect of Pauli blocking is strong and $\omega_{bec} < \omega_{bcs}$ in the entire Brillioun zone. the coupling between the axial modes and the radial density enhances the elastic collision rate.

In Figure 2, we show the monopole modes ($m = 0, n_r = 1$)in the BEC side of unitary limit (solid line) and the BCS side of unitary limit.The monopole modes show a very peculiar behaviour. In the long wavelength region, $\omega_{bec} > \omega_{bcs}$. As we move away from the center of the Brillioun zone, there is a cross over and $\omega_{bec} < \omega_{bcs}$. The difference between the monopole modes in the BEC regime and that in the BCS regime is maximum at the edge of the Brillioun zone. If we look at equation (23), we find that in the long wavelength region, the term $2n_r[\gamma(n_r+1)]$ (this part indicates the coupling between the radial density and the axial frequencies and is finite for $n_r \neq 0$)dominates over the part determined by the optical lattice $(1 - \sum_\alpha M_{\alpha,\alpha})\, B_0\frac{k^2d^2}{4}$. Consequently, the elastic damping rate is high and $\omega_{bec} > \omega_{bcs}$ as expected from experiments on elongated Fermi gases [7], because the term $2n_r[\gamma(n_r + 1)]$ is greater in the BEC region as compared to that in the BCS region (since $\gamma_{bec} > \gamma_{bcs}$). On moving away from the center of the Brillioun zone, the term proportional to $\sin^2\left(\frac{kd}{2}\right)$ starts dominating and as we go from the BEC to BCS regime, we probably enter the collisionless phase and the elastic collision rate decreases and as a result $\omega_{bec} < \omega_{bcs}$. Note that Pauli blocking reduces the binding energy because of which $E^{2D}_{bcs} < E^{2D}_{bec}$. For both the phonon modes and the monopole modes, the frequency (ω_{uni}) in the unitary limit($\gamma = 2/3$) will lie between ω_{bec} and ω_{bcs}. Eventhough $\gamma = 2/3$ in both the unitary and BCS limit, $\omega_{uni} < \omega_{bcs}$ because from equation (7) we notice that $E^{2D}_{uni} > E^{2D}_{bcs}$.

IV. CONCLUSIONS

We have studied the Bogoliubov spectrum of an elongated Fermi superfluid confined in an one-dimensional superfluid along the Bose-Einstein-condensate (BEC)-Bardeen-Cooper-Schrieffer (BCS) crossover. Using the hydrodynamic approach, we have analytically calculated the effective mass, velocity of sound and the multibranch Bogoliubov spectrum in the BEC and BCS side of the unitary limit. We have showed that as the system crosses over from the BCS side to the BEC side the velocity of sound decreases. The effective mass on the other hand increases as the system crosses from the BCS side to the BEC side. The Bogoliubov axial excitation frequencies on either side of the unitary limit show a strong dependence on the coupling with the radial density and the binding energy and thus provide valuable information on the physical behavior of the system. Near the center of the Brillioun zone, we show that the monopole frequency on the BEC side is greater than that on the BCS side but as we go towards the edge of the Brillioun zone, the monopole frequency on the BCS side becomes greater than that on the BEC side. The various Bogoliubov frequencies calculated here can be measured by Bragg scattering experiments.

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