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Scaling Relations in the Vortex State of Nodal Superconductors

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Abstract

In contrast to multigap superconductors (e.g. MgB_2), the low-temperature properties of nodal superconductors are dominated by nodal excitations. Here we extend for a variety of nodal superconductors the earlier work by Simon and Lee and Kübert and Hirschfeld. The scaling relations seen in the thermodynamics and the thermal conductivity will provide an unequivocal test of nodal superconductivity.

1 INTRODUCTION

Although nodal superconductors have been with us since 1979 [1], the systematic study of the gap symmetry of these new superconductors began only around 1994 with the establishment of d-wave symmetry of high-T_c cuprate superconductors through the angle resolved photoemission spectrum (ARPES) [2] and Josephson interferometry [3, 4]. Unfortunately, however, these powerful techniques have not been applied to other nodal superconductors like Sr_2RuO_4, heavy-fermion superconductors and organic superconductors.

Since 2001 Izawa et al have succeeded in determining the gap functions $|\Delta(\mathbf{k})|$'s in Sr_2RuO_4 [5], $CeCoIn_5$ [6], κ-$(ET)_2Cu(NCS)_2$ [7], YNi_2B_2C [8], $PrOs_4Sb_{12}$, [9, 10], and UPd_2Al_3 [11, 12] through measurements of the angle-dependent thermal conductivity in the vortex state. These experiments are only possible now since a) high-quality single crystals of these compounds are now available, b) low-temperature facilities which allow one to reach 1 - 0.1 K are available, and c)the necessary theoretical development following the seminal paper by Volovik [13].

Indeed Volovik's approach has been extended in a variety of directions, as reviewed in [14]. Also the angle dependent magnetothermal conductivity and the scaling relations [15] in the vortex state will provide a crucial test of nodal superconductivity. For example, the multigap superconductors do not exhibit the scaling relations we are going to discuss in general. Therefore, if any given superconductor exhibits a scaling relation discussed here, it is very likely that the material is a nodal superconductor. For example, the specific heat data of Sr_2RuO_4 by Deguchi et al [15] obeys the scaling relation given in [16]. Therefore the simplest choice of gap function in Sr_2RuO_4 is the chiral f-wave superconductor as pointed out in [5].

The scaling relations in the vortex state in d-wave superconductors were first proposed by Simon and Lee [17]. Then within the semiclassical approximation, à la Volovik [13] Kübert and Hirschfeld (KH) [18] have succeeded in deriving the scaling function for the quasiparticle density of states. KH then calculated the thermal conductivity in the scaling region [19]. An error in [19] was pointed out and corrected in [20]. However, in [20] only the asymptotic behavior of the thermal conductivity ($T \lll <|\mathbf{v}\cdot\mathbf{q}|>$, where $\mathbf{v}\cdot\mathbf{q}$ is the Doppler shift) has been worked out.

In the following we shall derive the scaling relations for a class of quasi-2D superconductors, where $|\Delta(\mathbf{k})| = \Delta|f|$ and f = $\cos(2\phi), \sin(2\phi)$ (d-wave superconductor), $f = e^{\pm i\phi}\cos\chi$ (chiral f-wave superconductor as in Sr_2RuO_4), $f = \cos(2\chi)$ (g-wave superconductor as in UPd_2Al_3 [12].) These superconductors have the same quasiparticle density of states as in d-wave superconductors [21]

$$N(E)/N_0 \quad = \quad G(x) \tag{1.1}$$

where

$$G(x) \quad = \quad \frac{2x}{\pi}K(x) \ \text{ for x} \leq 1 \tag{1.2}$$

$$\quad = \quad \frac{2}{\pi}K(x^{-1}) \ \text{ for x} > 1. \tag{1.3}$$

where $x = |E|/\Delta$ and K(k) is the complete elliptic integral of the second kind. In particular for $|E| < 0.3\Delta$ we have $G(E/\Delta) = \frac{|E|}{\Delta}$.

As discussed elsewhere [14], all the nodal superconductors so far discovered have G(E) $\sim |E|/\Delta$. Then one can establish a variety of scaling relations in the superclean limit [20], that is, for $(\Gamma\Delta)^{1/2} < T, E$; $\mathbf{v}\cdot\mathbf{q} < \Delta$ where T, E, Δ and Γ are the temperature, the quasiparticle energy, the maximal value of the energy gap and the quasiparticle scattering rate respectively. Therefore the scaling relations provide another test for nodal superconductivity.

2 QUASIPARTICLE DENSITY OF STATES

Let us limit ourselves to a class of quasi-2D systems with f listed in the preceding section. As already noted we have $G(E) \simeq |E|/\Delta$ for $E \ll \Delta$. In the presence of a magnetic field we find

$$G(E, \mathbf{H}) \quad = \quad <|E - \mathbf{v}\cdot\mathbf{q}|> \Delta^{-1} \tag{2.1}$$

where $\mathbf{v}\cdot\mathbf{q}$ is the Doppler shift and $\langle\ldots\rangle$ means the average over the Fermi surface and the vortex lattice. When $\mathbf{H} \parallel \mathbf{c}$ in the class of quasi-2D systems, the average can be performed analytically and we find [18]

$$G(E, \mathbf{H}) \quad = \quad \frac{4}{\pi}\frac{\epsilon}{\Delta}g(E/\epsilon) \tag{2.2}$$

where

$$g(s) \quad = \quad \frac{\pi}{4}s(1 + \frac{1}{2s^2}), s > 1 \tag{2.3}$$

$$\quad = \quad \frac{3}{4}\sqrt{1-s^2} + \frac{1}{4s}(1+2s^2)\arcsin(s), \qquad s \leq 1 \tag{2.4}$$

where $\epsilon = \frac{1}{2}v\sqrt{eH}$ and v is the Fermi velocity within the ab plane. The scaling function $\frac{\Delta}{\epsilon}G(E, \mathbf{H})$ is shown in Fig. 1. As is readily seen $G(E, \mathbf{H})$ for $\mathbf{H} \parallel \mathbf{c}$ cannot discriminate

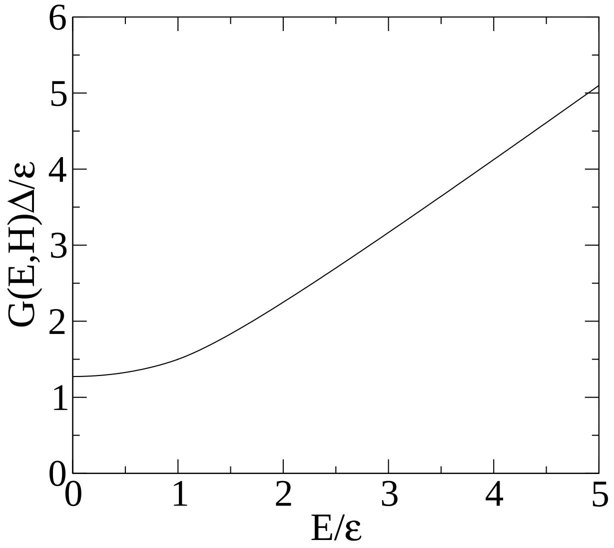

Fig. 1. The scaling function G(E, **H**)

between different $|\Delta(\mathbf{k})|$'s in the above class of nodal superconductors. Then the scaling function for the specific heat is given by [16]

$$C_s(T, \mathbf{H})/C_s(T, 0) = F(T/\epsilon) \tag{2.5}$$

where

$$F(T/\epsilon) = \frac{2}{9\pi\zeta(3)}(\frac{\epsilon}{T})^2 \int_0^\infty ds\, s^2 g(s) \mathrm{sech}^2\left(\frac{\epsilon s}{2T}\right) \tag{2.6}$$

$$\simeq 1 + \frac{\ln 2}{9\zeta(3)}(\frac{\epsilon}{T})^2, \text{for } \epsilon/\mathrm{T} \leq 1 \tag{2.7}$$

$$\simeq \frac{4\epsilon}{9\pi\zeta(3)T}[1 + \frac{1}{18}(\frac{\pi T}{\epsilon})^2 + \frac{7}{1800}(\frac{\pi T}{\epsilon})^4 + \ldots], \text{for } \epsilon/\mathrm{T} > 1 \tag{2.8}$$

The scaling function and the experimental data for Sr_2RuO_4 [15] are shown in Fig. 2. As is seen readily the scaling function gives an excellent description of the experimental data. As noted in [15], this clearly shows the superconductivity in Sr_2RuO_4 is consistent with the chiral f-wave superconductor as discussed in [22]. On the other hand, as noted in [23], this is incompatible with p-wave superconductivity.

Now when **H** is rotated within the a-b plane with an angle ϕ from the a axis, we can discriminate between $f = \cos(2\phi)$ and $f = \sin(2\phi)$ (the case of vertical nodes). We obtain for $f = \cos(2\phi)$

$$G(E, \mathbf{H}) = \frac{2}{\pi}\sum_{\pm}\left\langle \frac{\epsilon_\pm(\phi,\chi)}{\Delta} G(\frac{E}{\epsilon_\pm(\phi,\chi)}\right\rangle \tag{2.9}$$

$$= \frac{E}{\Delta}\left(1 + \frac{\epsilon^2}{2E^2}\right), \text{for } \frac{\epsilon}{\mathrm{E}} < 1 \tag{2.10}$$

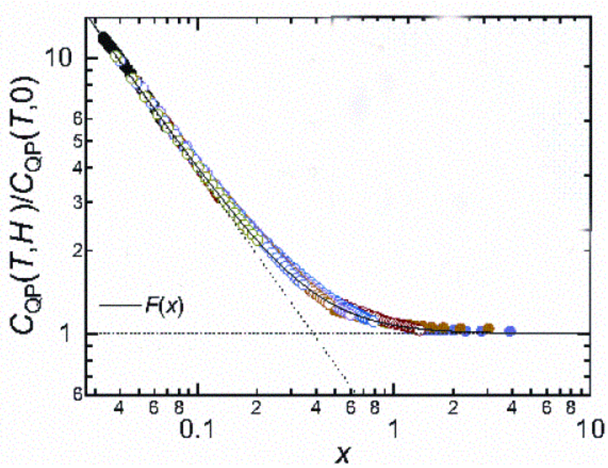

Fig. 2. The scaling function $F(T/\epsilon) = F(x)$ and Sr_2RuO_4 specific heat data from Ref. [15].

$$= (\frac{2}{\pi})^2 \frac{\epsilon}{\Delta} \sum_{\pm} (\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)} E\left(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}}\right) +$$

$$\frac{1}{6}(\frac{E}{\epsilon})^2 \left(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}}\right) K \left(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}}\right) + \ldots \quad (2.11)$$

$$\simeq \frac{4\epsilon}{\pi\Delta}(0.963 + 0.0205\cos(4\phi) + \frac{1}{6}(1.132 - 0.081\cos(4\phi)) \times (\frac{E}{\epsilon})^2 + \ldots) \text{for } \frac{\epsilon}{\mathrm{E}} > 1 \quad (2.12)$$

where $\epsilon = \frac{1}{2}\tilde{v}\sqrt{eH}$ and $\tilde{v} = \sqrt{vv_c}$. For f=sin(2ϕ) we have the same formulas as in Eqs. (12) and (13), except that $\cos(4\phi)$ in Eq.(14) should be changed to $-\cos(4\phi)$. Also the presence of the fourfold term in the specific heat has been studied by Revaz et al [24]. They found no fourfold term within an accuracy of 3%. This suggests strongly that the thermal conductivity provides a more sensitive test of the gap symmetry.

For superconductors with horizontal nodes (e.g. f $= \sin\chi, \cos(2\chi), \cos\chi$) the field configuration $\mathbf{H} \parallel b - c$ plane, with θ the angle $\mathbf{H}$ makes from the c-axis, is more appropriate. Then we find [12]

$$G(E, \mathbf{H}) = \frac{4}{\pi} \sum_{\pm} \left\langle \frac{\epsilon_{\pm}(\theta,\phi,\chi)}{\Delta} G(\frac{E}{\epsilon_{\pm}(\theta,\phi,\chi)}) \right\rangle \quad (2.13)$$

$$= \frac{E}{\Delta}(1 + \frac{\epsilon^2}{2E^2}), \text{for } \frac{\epsilon}{\mathrm{E}} < 1 \quad (2.14)$$

$$= (\frac{2}{\pi})^2 \frac{\epsilon}{\Delta} \sum_{\pm} \left(\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)} E(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}})\right) +$$

$$\frac{1}{6}(\frac{E}{\epsilon})^2 \left(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}}\right) K \left(\frac{1}{\sqrt{\frac{3}{2} \pm \frac{1}{2}\sin(2\phi)}}\right) \quad (2.15)$$

$$(2.16)$$

$$\simeq \frac{4\epsilon}{\pi\Delta}(0.963 + 0.0205\cos(4\phi) + \frac{1}{6}(1.132 - 0.081\cos(4\phi))) \times (\frac{E}{\epsilon})^2 + \ldots, \quad \frac{\epsilon}{\mathrm{E}} > 1 \tag{2.17}$$

$$\simeq \frac{4\epsilon}{\pi\Delta}\sqrt{x/2}(1 - \frac{1}{16}\sin^2\theta(\sin^2(\theta) + 16\alpha^2\cos^2\theta\sin^2\chi_0)x^{-2} + \frac{1}{3}(\frac{E}{\epsilon})^2 x^{-1}(1 + \frac{3}{16}\sin^2\theta(\sin^2(\theta) + 16\alpha^2\cos^2\theta\sin^2\chi_0)x^{-2}))\text{for } \frac{\epsilon}{\mathrm{E}} \geq 1 \tag{2.18}$$

where $\epsilon = v\sqrt{eH}$, $\alpha = v_c/v$ and $x = 1 + \cos^2\theta + 2\alpha^2\sin^2\theta\sin^2\chi_0$ and $\chi_0 = 0, \frac{\pi}{4}$ and $\frac{\pi}{2}$ for $f = \sin\chi, \cos(2\chi)$ and $\cos\chi$ respectively. Therefore in the present configuration the angle dependent thermal conductivity can discriminate different $\Delta(\mathbf{k})$'s with horizontal nodes.

So far we have completely ignored the effect of impurity scattering. As already indicated the present analysis is valid in the superclean limit [14, 20], i.e. for $(\Gamma\Delta)^{1/2} < T, E, \mathbf{v}\cdot\mathbf{q} < \Delta$ where Γ is the quasiparticle scattering rate in the normal state. Then the superclean limit appears to require $\Gamma/\Delta \leq 0.01$.

3 THERMAL CONDUCTIVITY

In the past few years the angle dependent magnetothermal conductivity (ADMTC) has proven itself the most powerful technique to probe the nodal structure of the gap function $\Delta(\mathbf{k})$. Also in many cases the nodal structure of $\Delta(\mathbf{k})$ is sufficient to deduce $\Delta(\mathbf{k})$ itself. We are concerned that much of the confusion and the controversy in the literature regarding the gap functions in Sr_2RuO_4, $PrOs_4Sb_{12}$ and UPd_2Al_3 may be largely due to a misunderstanding of Volovik's approach. References [14, 20] contain a detailed description of this approach. Generalizing the standard expression of the thermal conductivity given in [25, 26], the thermal conductivity of the class of nodal superconductors in the vortex state is given by [14]

$$\kappa_{zz} = \frac{n}{4mT^2}\int_0^\infty d\omega\,\omega^2 \left\langle \frac{h(\omega,\mathbf{H})}{\tilde{\Gamma}(\omega,\mathbf{H})} \right\rangle \mathrm{sech}^2(\omega/2T) \tag{3.1}$$

where

$$h = \frac{1}{2}\left(1 + \frac{|\tilde{\omega} - \mathbf{v}\cdot\mathbf{q}|^2 - \Delta^2 f^2}{|(\tilde{\omega} - \mathbf{v}\cdot\mathbf{q})^2 - \Delta^2 f^2|}\right) \tag{3.2}$$

and

$$\tilde{\Gamma} = \mathrm{Im}\sqrt{(\tilde{\omega} - \mathbf{v}\cdot\mathbf{q})^2 - \Delta^2 f^2} \tag{3.3}$$

Here $< \ldots >$ denotes the averages over the Fermi surface and vortex lattice [20]. In the superclean limit $\tilde{\omega}$ is given by

$$\tilde{\omega} = \omega + i\Gamma\left\langle \frac{|\tilde{\omega} - \mathbf{v}\cdot\mathbf{q}|}{\sqrt{(\tilde{\omega} - \mathbf{v}\cdot\mathbf{q})^2 - \Delta^2 f^2}} \right\rangle \tag{3.4}$$

$$\simeq \omega + i\Gamma G(\omega,\mathbf{H}) \tag{3.5}$$

in the Born limit. And in the unitary limit we find

$$\tilde{\omega} = \omega + i\Gamma G^{-1}(\omega,\mathbf{H}) \tag{3.6}$$

where $\mathrm{G}(\omega,\mathbf{H})$ has been defined in Eq.(5).

First we limit ourselves to the quasi-2D systems in a magnetic field $\mathbf{H} \parallel \mathbf{c}$. The in the Born limit we obtain

$$\kappa = \frac{n}{8mT^2\Gamma}\int_0^\infty d\omega\ \omega^2 \mathrm{sech}^2(\frac{\omega}{2T} = \frac{\pi^2 nT}{12m\Gamma} = \frac{1}{2}\kappa_n \tag{3.7}$$

where $\kappa_n = \frac{\pi^2 nT}{6m\Gamma}$ is the thermal conductivity in the normal state. In particular Eq.(28) gives the scaling function

$$F_B(T/\epsilon) = \frac{\kappa(T,\mathbf{H})}{\kappa(T,0)} = 1 \tag{3.8}$$

The last result agrees with the corresponding result given in Ref. [19] despite the use of a rather unphysical spatial average in this work. In the unitary limit, on the other hand, we obtain

$$\kappa = \frac{n}{8m(T\Delta)^2\Gamma}\int_0^\infty d\omega\,\omega^2 < |\omega - \mathbf{v}\cdot\mathbf{q}| >^2 \mathrm{sech}^2(\omega/2T) \tag{3.9}$$

$$= \frac{n}{8mT^2\Gamma}(\frac{\pi\epsilon}{4\Delta})^2\int_0^\infty d\omega\,\omega^2 G^2(\omega/\epsilon)\mathrm{sech}^2(\omega/2T) \tag{3.10}$$

where $G(\omega/\epsilon)$ has already been defined in Eqs. 5 and 6. This has asymptotics

$$\kappa = \frac{7n\pi^4T^3}{60m\Gamma\Delta^2}\left(1 + \frac{5}{7}(\frac{\epsilon}{\pi T})^2 + \frac{15}{28}(\frac{\epsilon}{\pi T})^4 + \dots\right),\ \text{for } \epsilon \ll \mathrm{T} \tag{3.11}$$

$$= \frac{\pi^2 nT}{12m\Gamma}(\frac{\pi\epsilon}{4\Delta})^2\left(1 + \frac{7\pi^2}{15}(\frac{T}{\epsilon})^2 + \dots\right),\ \text{for } \epsilon \gg \mathrm{T} \tag{3.12}$$

where $\epsilon = \frac{v\sqrt{eH}}{2}$. Then the scaling function is given by

$$F_u(T/\epsilon) = \frac{\kappa(T,\mathbf{H})}{\kappa(T,0)} = \frac{3}{2\pi^2T^3}\int_0^\infty d\omega\omega^2 G^2(\omega/\epsilon)\mathrm{sech}^2(\omega/2T) \tag{3.13}$$

$$= \left(1 + \frac{5}{7}(\frac{\epsilon}{\pi T})^2 + \frac{15}{28}(\frac{\epsilon}{\pi T})^4 + \dots\right),\ \text{for } \epsilon \ll \mathrm{T} \tag{3.14}$$

$$= \frac{5}{112}(\frac{\epsilon}{\Delta})^2\left(1 + \frac{7\pi^2}{15}(\frac{T}{\epsilon})^2 + \dots\right),\ \text{for } \epsilon \gg \mathrm{T} \tag{3.15}$$

These scaling functions are shown in Fig. 3. In this figure we also include the scaling function when the inversion symmetry is broken in the impurity scattering [27]. $F_I(T/\epsilon)$ describes the thermal conductivity data of the non-centrosymmetric triplet superconductor $CePt_3Si$ by Izawa et al [28]. The scaling function $F_u(T/\epsilon)$ is very different from the one given in Ref. [19] but describes consistently the scaling behaviors of the thermal conductivity of UPt_3 as reported by Suderow et al [29].

4 ANGLE DEPENDENT THERMAL CONDUCTIVITY TENSOR

Let us consider $d_{x^2-y^2}$-wave superconducivity as in the high-T_c cuprates, $CeCoIn_5$ [6] and κ-$(ET)_2(NCS)_2$ [7] in a magnetic field within the a-b plane. Then the thermal conductivity tensors within the ab-plane are given by

$$\kappa_{xx} = \kappa_{yy} = \frac{n}{8m\Gamma T^2}\int_0^\infty d\omega\ \omega^2\left\langle\frac{4\epsilon(\phi,\chi)}{\pi\Delta}G(\omega/\epsilon(\phi,\chi))\right\rangle^2 \mathrm{sech}^2(\omega/2T) \tag{4.1}$$

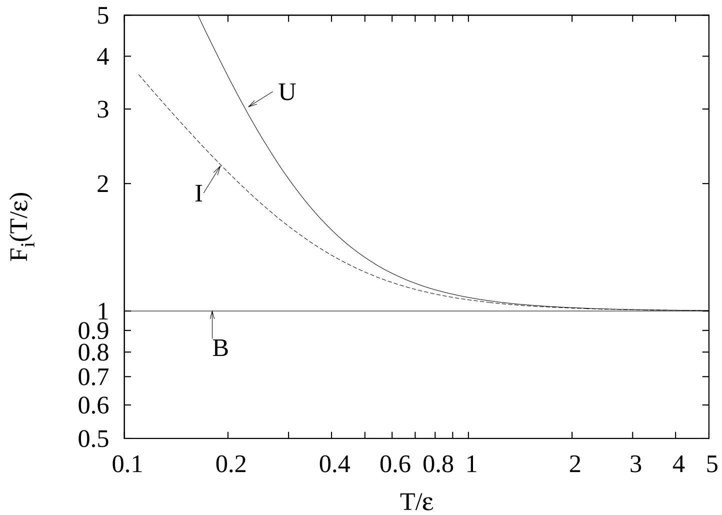

Fig. 3. The scaling functions F_I, F_B, and F_U

where $\epsilon(\phi,\chi) = \frac{\tilde{v}\sqrt{eH}}{2}(1 \pm \frac{1}{2}\sin(2\phi) - \frac{1}{2}\cos(2\chi))^{1/2}$ and $\langle \ldots \rangle$ means the average over $\pm$ and over χ. Here we assumed the unitary impurity scattering and the superclean limit in the present derivation. This gives the following asymptotics:

$$\kappa_{xx} = \kappa_{yy} = \frac{7nT}{60m\Gamma}(\frac{\pi T}{\Delta})^2\left(1 + \frac{5}{7}(\frac{\epsilon}{\pi T})^2 + \frac{15}{28}(\frac{\epsilon}{\pi T})^4 + \ldots\right), \text{ for } \epsilon \ll \text{T} \tag{4.2}$$

$$= \frac{4nT}{3m\Gamma}(\frac{\epsilon}{\Delta})^2[0.927 + 0.039\cos(4\phi) + \frac{7}{15}(\frac{\pi T}{\epsilon})^2[1.090 - 0.055\cos(4\phi)]\text{for } \epsilon \gg \text{T} \tag{4.3}$$

First of all, the present result is consistent with that in Ref. [20] for $T < \epsilon$. On the other hand, for $T > \epsilon$ there is no fourfold term. In other words the present theory in the superclean limit cannot describe the fourfold symmetry in κ_{xx} observed in YBCO for $T > 14K$ [30–32]. We have proposed the sign inversion of the fourfold term for $T > \epsilon$ in the clean limit in [33].

Also the Hall conductivity in the present geometry is given by [20]

$$\kappa_{xy} = \frac{n}{8m(T\Delta)^2\Gamma}\int_0^\infty d\omega\, \omega^2 \left\langle \sin(2\phi^{`})|\omega - \mathbf{v}\cdot\mathbf{q}|\right\rangle \langle|\omega - \mathbf{v}\cdot\mathbf{q}|\rangle\, \text{sech}^2(\omega/2T) \tag{4.4}$$

Further, we find

$$\left\langle \sin(2\phi^{`})|\omega - \mathbf{v}\cdot\mathbf{q}|\right\rangle = -\sin(2\phi)\frac{\epsilon^2}{2|\omega|}, \text{for } \epsilon \ll |\omega| \tag{4.5}$$

$$\simeq -\sin(2\phi)\frac{4\epsilon}{\pi\Delta}(0.535 - (\frac{\omega}{\epsilon})^2 0.14192) \text{ for } \epsilon \gg |\omega| \tag{4.6}$$

Inserting these into Eq.(4.4) we find

$$\kappa_{xy} = -\sin(2\phi)\frac{\pi^2 nT}{24m\Gamma}(\frac{\epsilon}{\Delta})^2(1 + \frac{3}{2}(\frac{\epsilon}{\pi T})^2, \text{for } \epsilon \ll \text{T} \tag{4.7}$$

$$= -\frac{4nT}{3m\Gamma}(\frac{\epsilon}{\Delta})^2\sin(2\phi)(0.5152 + 0.011\cos(4\phi)) - \frac{7}{30}(\frac{\pi T}{\epsilon})^2(0.213 + .0607\cos(4\phi))\text{for}\epsilon \gg \text{T} \tag{4.8}$$

Therefore in the superclean limit $\kappa_{xy} \sim -\sin(2\phi)H$ independent of ϵ/T. Also as $\frac{T}{\epsilon}$ increases the coefficient of -sin$(2\phi)H$ decreases almost 40%. The present result appears to be consistent with the Hall conductivity data of YBCO reported by Ocaña and Esquinazi [34] for $\epsilon/\Delta < 1$. Also in the superclean limit the sign of the Hall conductivity is the same for all T as long as $T < \Delta(T)$.

5 CONCLUDING REMARKS

We have shown a) the thermal conductivity in nodal superconductors for $T < 0.3T_c$ is dominated by the quasiparticles or nodal excitations, b) the quasiparticles in the vortex state are accurately described in terms of the semiclassical approximation. Thus the angle dependent magneto-thermal conductivity provides a powerful tool to determine the nodal structure of the gap function as demonstrated by a series of experiments by Izawa et al [5–12]. Also in most cases the nodal structure of the gap function is adequate to deduce $|\Delta(\mathbf{k})|$ itself. In addition we have shown that all these model superconductors exhibit a variety of scaling relations. We have proposed scaling relations for $PrOs_4Sb_{12}$ [35]. Furthermore, from the unusual scaling relation seen in the thermal conductivity in $CePt_3Si$ we can deduce anomalous impurity scattering in this system lacking crystalline inversion symmetry. Indeed the scaling relations in nodal superconductors provide a unique way to characterize this new class of superconductors.

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