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(3+2)D Vlasov simulation of electron drift turbulence

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1 Introduction

Electron drift turbulence is one of the main causes for anomalous transport in the steep gradient zone of tokamaks. A careful investigation of the relevant plasma parameter regime shows that the electron motion parallel to the magnetic field is usually at most weakly collisional. As a consequence, electron drift turbulence cannot be treated by conventional (i.e., Braginskii-type) fluid models. In order to capture important wave-particle interactions – like electron Landau damping – a hybrid model of drift-kinetic electrons and cold ions is used instead. Effects of small but finite plasma beta are taken into account. The nonlinear basic equations are solved numerically on a massively parallel computer applying upwind methods on a stationary grid in (3+2)D phase space.

2 Model equations

The equations used to treat the nonlinear dynamics of collisionless drift waves in a three-dimensional sheared slab geometry are described in detail elsewhere [1, 2]. The time evolution of the perturbed electron distribution function $f(\mathbf{r}, w_{||}, \mu, t)$, the parallel ion velocity $u_{||}(\mathbf{r}, t)$, the electrostatic potential $\phi(\mathbf{r}, t)$, and the parallel vector potential $A_{||}(\mathbf{r}, t)$ is given in dimensionless form by

$$d_t f = \omega_T f_m v_x + \omega_T \alpha_e w_{||} f_m B_x - \alpha_e w_{||} \nabla_{||} f + \alpha_e [w_{||} f_m - (\delta/2)\, \partial_{w_{||}} f]\, [\nabla_{||}\phi + \hat{\beta}\, \partial_t A_{||}]\,, \quad (1)$$

$$a\hat{\epsilon}_s\, d_t u_{||} = -\nabla_{||}\phi - \hat{\beta}\, \partial_t A_{||}\,, \quad (2)$$

$$a\, d_t \nabla_\perp^2 \phi = \nabla_{||} J_{||}\,, \quad (3)$$

with the auxiliary equation

$$J_{||} \equiv u_{||} - \int \alpha_e w_{||} f\, d^3 w = -\nabla_\perp^2 A_{||}\,, \quad (4)$$

the differential operators

$$d_t \equiv \partial_t + v_x\, \partial_x + v_y\, \partial_y = \partial_t + \mathbf{z}\times\nabla\phi\cdot\nabla\,, \quad (5)$$

$$\nabla_{||} \equiv \partial_z + B_x\, \partial_x + B_y\, \partial_y = \partial_z - \hat{\beta}\, \mathbf{z}\times\nabla A_{||}\cdot\nabla\,, \quad (6)$$

$$\nabla_\perp^2 = (\partial_x + \hat{s} z\, \partial_y)^2 + \partial_y^2\,, \quad (7)$$

$f_m = \pi^{-3/2}\, e^{-w^2}$, $\omega_T = \omega_n + \omega_t\, (w^2 - 3/2)$, $\omega_n = L_\perp/L_n$, $\omega_t = L_\perp/L_{T_e}$, $\hat{\epsilon}_s = (qR/L_\perp)^2$, $\hat{s} = (dq/dr)/(q/r)$, $a = A_i/Z_i$ where A_i and Z_i are the ion mass and charge number, respectively, $L_n = |\nabla \ln n_0|^{-1}$, $L_{T_e} = |\nabla \ln T_{e0}|^{-1}$, and the drift parameter $\delta = \rho_s/L_\perp$. The coordinate system $\mathbf{r} = (x, y, z)$ is aligned to the background gradients and the sheared magnetic field: $\mathbf{x} \propto -\nabla n_0 \propto -\nabla T_{e0}$, $\mathbf{z} \propto \mathbf{B}$, and $\nabla x \times \nabla y \cdot \nabla z = 1$. Its non-orthogonality is reflected in the metric and in the parallel boundary condition $S(x, y, z+2\pi) = S(x, y - 2\pi\hat{s}x, z)$ for any scalar quantity S. In the perpendicular (x, y) plane, periodic boundary conditions are applied. The two most important model parameters are

$$\hat{\beta} = \left(\frac{c_s/L_\perp}{v_A/qR}\right)^2 , \quad \hat{\mu} \equiv 2\alpha_e^{-2} = \left(\frac{c_s/L_\perp}{v_e/qR}\right)^2 . \tag{8}$$

They determine the ratios of the three frequencies $\omega_\star \sim c_s/L_\perp$, $k_\| v_A \sim v_A/qR$, and $k_\| v_e \sim v_e/qR$ where $c_s = (T_{e0}/M_p)^{1/2}$ is the ion sound speed, $v_A = B_0/(4\pi n_0 M_p)^{1/2}$ is the Alfvén velocity, and $v_e = (T_{e0}/m_e)^{1/2}$ is the electron thermal velocity. The above equations in (3+2)D phase space are solved numerically on a massively parallel computer (T3E). In order to avoid the numerical noise inherent to particle simulations, we use a Vlasov approach based on finite-difference methods of the upwind and predictor-corrector type [1].

3 Nonlinear simulation results

Although collisionless drift waves in a sheared slab are linearly stable, we observe self-sustaining turbulence if it is initialized at sufficiently high amplitudes whereas it dies out otherwise. This nonlinear simulation result confirms a conjecture by Hirshman and Molvig [3] claiming that the linear mode structure might be too delicate to have relevance to turbulence. As was shown earlier, collisionless drift wave turbulence and its collisional counterpart are driven by the same inherently nonlinear destabilization mechanism [4]. This finding invalidates all linear or quasilinear results and emphasizes the need for nonlinear simulations.

Another important result concerns the effect of nonlinear electron Landau damping on turbulent transport [4]. This kinetic effect is associated with parallel trapping of electrons in fluctuations of the electrostatic potential (described by the velocity space nonlinearity in Eq. 1). Because it cannot be modeled by fluid equations, it has been mostly neglected in previous work. Many authors argued that this is justified because the fluctuating part of the distribution function, $\tilde{f}_e$, is much smaller than the Maxwellian, f_m, describing the equilibrium, and therefore we can expect $\partial_{v_\|}\tilde{f}_e \ll \partial_{v_\|} f_m$ (or $\delta\, \partial_{w_\|} f \ll w_\| f_m$ in the above dimensionless equations). It should be noted, however, that this statement implicitly assumes that $\tilde{f}_e(v_\|)$ varies only on velocity scales of the order of v_e which is generally not true. $\tilde{f}_e(v_\|)$ can vary very rapidly in $v_\|$ space for $v_\| \sim \omega/k_\|$, i.e., near resonances (see, e.g., Ref. [4]). Our simulation results show that nonlinear electron Landau damping can diminish the turbulent transport significantly in certain parameter regimes. As can be seen in Table 1, this effect gets stronger for increasing drift parameter δ and is fairly pronounced for $\delta \gtrsim 0.02$. As is inferred from runs 3 and 10, there is no clear correlation between the amount of transport supression and the turbulent amplitudes. Rather, we find that shear plays an important role in deciding whether nonlinear electron Landau

Table 1: Effect of nonlinear electron Landau damping on particle transport Γ for $\hat{\beta} = 0$. Γ' denotes the transport in the presence of the velocity space nonlinearity which is responsible for electrostatic particle trapping.

Run	$\hat{\mu}$	$\hat{s}$	δ	Γ	Γ'	$(\Gamma' - \Gamma)/\Gamma$
1	0.15	0.0	0.02	0.38	0.24	−37%
2	0.3	0.0	0.02	0.80	0.50	−38%
3	0.3	0.95	0.02	0.021	0.017	−19%
4	0.3	0.95	0.01	0.021	0.019	−10%
5	0.3	0.95	0.005	0.021	0.021	−0%
6	10	0.32	0.04	0.52	0.18	−65%
7	10	0.32	0.02	0.52	0.36	−31%
8	10	0.64	0.02	0.21	0.16	−24%
9	10	0.95	0.02	0.098	0.081	−17%
10	10	1.27	0.02	0.041	0.037	−10%

damping is an important effect for a given value of δ. This has to do with the presence or absence of direct coupling of density and potential fluctuations at $k_{\|} \approx 0$ [4]. For values of $\hat{s}$ and δ which are typical for a tokamak edge, $\hat{s} \gtrsim 1$ and $\delta \lesssim 0.04$, one can therefore expect a supression of turbulence and transport by up to a few 10 per cent. Consequently, the "classical" argument for leaving away the parallel nonlinearity is proven to be wrong.

It is an interesting feature of the present nonlinear model that it resolves fundamental contradictions between experimental results and (quasi-)linear theory, namely the dependence of turbulent transport on radial position and ion mass. In both cases the dependence on plasma parameters like the density profile scale length L_n and magnetic shear $\hat{s}$ is strong enough to reverse the trends one derives from the gyro-Bohm scaling. E.g., one finds from the simulations that the $\hat{\mu}$ dependence of the radial particle flux Γ (which is normalized to $D_{GB}\,(n_0/L_\perp)$ where $D_{GB} = cT_{e0}\rho_s/eB_0L_\perp)$ is given approximately by $\Gamma \propto \hat{\mu} \propto L_\perp^{-2}$ for $\hat{\mu} \lesssim 1$ [2]. Due to $D_{GB} \propto T_{e0}^{3/2}/L_\perp$ this leads to the scaling $D \propto T_{e0}^{3/2}/L_\perp^3$ for the (dimensional) diffusion coefficient D. Neglecting the $L_\perp$ dependence of Γ, the radially decreasing electron temperature would lead to a radially decreasing turbulent transport, in contrast to experimental findings. However, taking this effect into account results in a radial increase of D if $L_\perp$ falls off faster than $T_{e0}^{1/2}$. Furthermore, in our nonlinear numerical computations the turbulent transport is seen to *decrease* with increasing ion mass [2]. This finding is in accordance with experiments and overrides estimates from heuristic random walk models which predict $D \propto a^{1/2}$ where $a = A_i/Z_i$. The computed trend with a results mainly from a competition between two opposing tendencies, namely the gyro-Bohm scaling inherent in our normalization and the scaling with $\hat{s}$ (i.e. shear suppression). Therefore we need the full computational result to predict the net outcome.

Moreover, the electrostatic system exhibits a particle pinch, i.e., the turbulent particle flux is directed up-gradient for sufficiently high values of η_e (see Fig. 1). Its roots lie in the completely different perpendicular dynamics of fast and slow electrons, making it a genuinely kinetic effect which cannot easily be described by fluid models. The $w_{\|}$ spectra

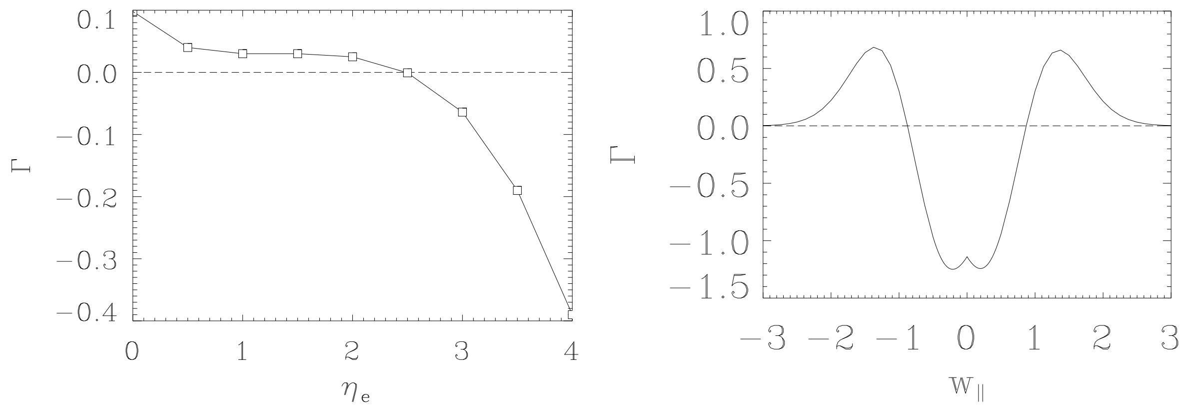

Figure 1: Turbulent particle flux Γ as a function of η_e and $w_{\|}$ spectrum of Γ for $\eta_e = 4$.

of Γ can be parameterized extremely well in terms of superpositions of two Maxwellian distributions associated, respectively, with a positive (down-gradient) fast particle part and a negative (up-gradient) slow particle part. From these results we can also conclude that in studies of collisionless drift-wave turbulence there is no shortcut to treating the entire velocity space dynamics. In particular, it is not permissible to use "adiabatic cutoff techniques", i.e. to choose a cutoff velocity above which all electrons are assumed to be adiabatic [5].

4 Future directions

Preliminary comparisons with a companion electromagnetic Landau-fluid model [6] are quite successful in that the observed transport levels are always within 50% of each other in the parameter regime under consideration [7]. More detailed comparisons in wider ranges of parameter space are underway. It is also planned to incorporate gyrokinetic ions in the near future.

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