EVALUATION OF AN ICRF WAVEGUIDE LAUNCHER INCORPORATING A POLARIZATION ROTATOR

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Waveguides are potentially attractive wave launchers for plasma heating in the ion cyclotron range of frequencies. A conventional vacuum waveguide cannot, however, be used to launch the fast wave in a toroidal device, owing to the relatively low frequencies used, and the need to position the launcher between toroidal field coils. In this paper I evaluate an alternative waveguide design, illustrated in Fig. 1, which has been proposed by Vdovin [1; see also 2] for the ITER device. The waveguide is dimensioned with $a > c/(2f) > b$, where $f$ is the excitation frequency and $c$ the speed of light, so that only the TE$_{1,0}$ mode propagates. This mode is excited by a horizontal coupling probe. A wire grid oriented at an angle $\psi$ to the vertical wall rotates the wave polarization from $\pi/2$ to $\pi/2 - \psi$, through excitation of a large evanescent TE$_{0,1}$ component. A horizontal Faraday screen at the waveguide mouth acts as an analyzer, reflecting the remaining TE$_{1,0}$ component, and allowing the TE$_{0,1}$ component to be coupled to the plasma. I first analyze separately the coupling from the waveguide to a large plasma through the Faraday screen, and the reflection and transmission of waveguide modes by the grid. I then consider the combined effect of these discontinuities, and coupling from a transmission line to the waveguide through the coupling probe.

WAVEGUIDE–PLASMA COUPLING

I describe the fields $\vec{E}^w$ and $\vec{H}^w$ in the waveguide as a sum of TE$_{m,n}$ modes (denoting $(m,n)$ by the subscript $l$). For example,

$$E_y^w = U(y,z) \sum_l G_l(y,z)[A_l \exp(-j\beta_l x) + B_l \exp(j\beta_l x)]$$

(1)

where $\beta_l = \sqrt{\omega^2/c^2 - (m\pi/a)^2 - (n\pi/b)^2}$, $G_l(y,z) = \cos(m\pi y/a)\sin(n\pi z/b)$, $U(y,z) = 1$ if $0 < y < a$ and $0 < z < b$, 0 otherwise. I approximate the plasma by a straight cylinder (with $r, \theta, z$ coordinate system) periodic in $z = 2\pi R$, where $R$ is the major radius of the torus. The plasma fields $\vec{E}^p$ and $\vec{H}^p$ may be Fourier analyzed to give, for example, $E^p_\theta = \frac{1}{4\pi^2} \sum_{\mu,\nu} E^p_{\mu,\nu}(r)e^{i\mu\theta}e^{i\nu z/R}$. The boundary conditions at the vessel wall (i.e., at $z = 0, r = r_w$): $E^p_\theta(r_w) = E^w_\theta(0)$ and $U(r_w\theta, z)H^p_z(r_w) = H^w_z(0)$, and the orthogonality of the waveguide modes, give

$$A_q - B_q = \sum_l L_{ql}(A_l + B_l), \quad L_{ql} = (\pi^2 ab r_w RD^w_q)^{-1} \sum_\mu \sum_\nu G^*_{ql,\mu,\nu} Y^p_{\mu,\nu} G_l \mu,\nu$$

(2)

and $G_l \mu,\nu = \int_0^a dy \int_0^b dz G_l(y,z) e^{-i\mu \theta} e^{-i\nu z/R}$. 

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\[ D_q^w = \beta_q / (\mu_0 \omega) \]

is the waveguide admittance of the mode \( q \), the asterisk denotes complex conjugate. Note that this expression differs from that given by Lam et al. [3], who mistakenly excluded the \( U(r, \theta, z) \) factor from the \( H_x \) boundary condition. The plasma edge admittance \( Y_{\mu, \nu}^p = H_{\mu}^p / \mu_\nu / B_\nu^p \mu_\nu \) is calculated for a plasma of proposed ITER parameters \((B = 5 \text{ T}, R = 5.8 \text{ m}, r_w = 2.4 \text{ m}) \) using the method of Theilhaber and Jacquinot [4]. All power is assumed to be absorbed in a central region of constant density \( n_{eo} = 1 \times 10^{20} \text{ m}^{-3} \), which is surrounded by a parabolic region of width 1.4 m, and an edge region of width \( r_e \) in which the density decays exponentially from 0.1\( n_{eo} \) to 0.01\( n_{eo} \) at the wall. I use \( f = 76 \text{ MHz} \) (twice the deuteron cyclotron frequency). The reflection coefficients \( R_{lq} = K_q B_q / (K_l A_l) \), where \( K_l \) and \( K_q \) are constants used to give the waveguide fields the same normalization as defined in the next section, may be found using Eqn. (1).

Fig. 2 shows the power reflection coefficient \( |\Gamma_{(0,1)}(0,1)|^2 \) of the TE\(_{0,1} \) mode calculated for different waveguide dimensions and plasma-wall separations, using up to 20 TE modes. For 0.05 \( \leq b/a \leq 0.1 \), excellent transmission to the plasma occurs for a wide range of \( r_e \) (less than 1% of the input power is reflected as other modes in this range of \( b/a \)). The TE\(_{1,0} \) mode is fully reflected with a phase reversal due to horizontal orientation of the Faraday shield.

Polarization Rotating Grid

I approximate the grid as \( N \) parallel, infinitely thin, perfectly conducting wires. Following Collin [5], the transverse waveguide fields \( \tilde{E}_l^T \) and \( \tilde{H}_l^T \) of mode \( l \) are normalized so that \( \int \int_{0}^{a} \int_{0}^{b} \tilde{E}_l^T \times \tilde{H}_q^T \cdot \hat{x} dy \, dz = \delta_{lq} \), where \( \delta_{lq} \) is the Kronecker delta. The amplitude of the scattered field of mode \( q \) due to an incident field of mode \( l \) is

\[ S_{lq} = \frac{1}{2i} \sum_{k=1}^{N} \int I_l(L_k) \left( \sin \psi \hat{y} + \cos \psi \hat{z} \right) \cdot \tilde{E}_q^T \, dL_k \quad (3) \]

where \( I_l(L_k) \) is the current excited along the \( k \)th wire by mode \( l \) (proportional to \( \tilde{E}_l^T(L_k) \)) and the integral is along the wire. The corresponding reflection and transmission coefficients are \( R_{lq} = S_{lq} \) and \( T_{lq} = \delta_{lq} + S_{lq} \) respectively.

Fig. 3 shows the dependence of the transmission amplitude coefficients \( T_{(1,0)q} \) on \( \psi \) for an incident TE\(_{1,0} \) wave. The amplitude of the transmitted TE\(_{0,1} \) mode is maximum at \( \psi \sim 20^\circ \). The corresponding value of \( \psi \) increases with both \( a \) and \( b \), and is independent of \( N \) for \( N \geq 20 \). \( T_{(1,0)q} = 0 \) for modes \( q = (m, n) \) when \( m + n \) is even.

Grid–Plasma System

The calculation of the combined effect of the polarization rotating grid and the waveguide–plasma boundary requires consideration of the scattering coefficients of both propagating and evanescent modes at both discontinuities, and the propagation coefficients of the modes. The resultant matrices for the reflection coefficient from the
grid \([R_{(1,0)q}]\) and incident field on the Faraday shield \([A_{(1,0)q}]\) due to the \(\text{TE}_{1,0}\) mode incident on the grid are

\[
[R_{(1,0)q}] = [R_{(1,0)q}] + \sum_{S=0}^{\infty} [T_{(1,0)q}p_q] [\Gamma_{iq}p_q] ([R_{iq}p_q] [\Gamma_{iq}p_q])^S [T_{iq}]
\]

\[
[A_{(1,0)q}] = \sum_{S=0}^{\infty} [T_{(1,0)q}p_q] ([\Gamma_{iq}p_q] [R_{iq}p_q])^S
\]

where \(p_q = \exp(-\gamma_q l_q)\), \(\gamma_q = j\beta_q\) for propagating modes and \(|\beta_q|\) for evanescent modes, and \(l_q\) is the grid–Faraday shield separation. The power transmitted to the plasma can then be calculated from \([A_{(1,0)q}]\) and \([\Gamma_{iq}]\) using energy conservation.

Fig. 4 shows the \(l_q\) dependence of the fraction of the the incident power transmitted to the plasma for different values of \(\alpha\), \(\beta\), and \(\psi\). This fraction is far lower than would be expected from separate consideration of the values of \(T_{iq}\), \(R_{iq}\), \(\Gamma_{iq}\) and \(p_q\). This is because, after both the first and subsequent reflections from the Faraday shield, the \(\text{TE}_{1,0}\) mode reaches the grid almost exactly out of phase with the initial incident mode, and the thereby reflected \(\text{TE}_{0,1}\) component is thus also out of phase with the initially transmitted \(\text{TE}_{0,1}\) component. The cancellation is most complete when the \(\text{TE}_{0,1}\) and \(\text{TE}_{1,0}\) components excited by the initial scattering of the incident \(\text{TE}_{1,0}\) by the grid are of equal amplitude, which unfortunately occurs at the same \(\psi\) at which the excitation of the \(\text{TE}_{0,1}\) mode is strongest (see Fig. 3). The cancellation effect can be reduced by increasing the phase shift undergone by the \(\text{TE}_{1,0}\) mode in travelling between the grid and Faraday shield (by increasing \(l_q\) or \(\alpha\)) or by decreasing the ratio \(T_{(1,0)(1,0)}/T_{(1,0)(0,1)}\) (by increasing \(\psi\)). Note that increasing \(l_q\) increases the effect of the \(\text{TE}_{0,1}\) mode's evanescence; this can be countered by increasing \(\beta\). The efficacy of these measures can be seen in Fig. 4. In all cases, however, only a very small proportion of the power initially incident on the grid is transmitted to the plasma. It may be possible to improve this situation through the interposition of a horizontal inductive element between the grid and the Faraday shield. This would change the phase of the \(\text{TE}_{1,0}\) mode, without affecting the \(\text{TE}_{0,1}\) mode.

**COUPLING TO THE WAVEGUIDE THROUGH THE COUPLING PROBE.**

Expressions for the coupling from a transmission line to a waveguide via shorted \((d = b)\) and open-ended \((d < b)\) cylindrical coupling probes (see Fig. 1) have been calculated by Collin [5] and Lam et al. [3]. Using these expressions, I find that it is possible to match a 50 \(\Omega\) transmission line to the waveguide using an open-ended probe, despite the large reflection coefficient of the \(\text{TE}_{1,0}\) mode from the grid–plasma system. For example, in the case with the best coupling to the plasma of those shown in Fig. 4, for which the \(\text{TE}_{1,0}\) amplitude reflection coefficient is \(-0.990 + j0.086\), matching can be achieved using \(t = 0.06\) m, \(d = 0.18\) m, \(l = 3.22\) m and \(l_p = 2.78\) m. The standing wave ratio in the waveguide would, however, be extremely high, leading to substantial losses in the waveguide walls, and possibly to breakdown problems at high power. Without modification, the waveguide would thus be unsuitable as a high power ICRF launcher.
REFERENCES


Fig. 1—The waveguide launcher.

Fig. 2—b/a dependence of the power reflection coefficient of the TE0,1 mode from the waveguide-plasma boundary.

Fig. 3—ψ dependence of the transmission amplitude coefficients of different transmitted modes q, for a TE1,0 mode incident on the polarization rotating grid, with a = 3.0 m, b = 0.3 m, N = 51.

Fig. 4—lq dependence of the ratio of power transmitted to the plasma to power incident on the grid, for the combined grid-plasma system.