

# Entropy of isolated horizons from quantum gravity condensates

Daniele Oriti,<sup>1,\*</sup> Daniele Pranzetti,<sup>2,†</sup> and Lorenzo Sindoni<sup>1,‡</sup>

<sup>1</sup>Max Planck Institute for Gravitational Physics (AEI), Am Mühlenberg 1, D-14476 Golm, Germany

<sup>2</sup>Scuola Internazionale Superiore di Studi Avanzati (SISSA), via Bonomea 265, 34136 Trieste, Italy

We construct condensate states encoding the continuum spherically symmetric quantum geometry of an isolated horizon in full quantum gravity, i.e. without any classical symmetry reduction, in the group field theory formalism. Tracing over the bulk degrees of freedom, we show how the resulting reduced density matrix manifestly exhibits an holographic behavior. We derive a complete orthonormal basis of eigenstates for the reduced density matrix of the horizon and use it to compute the horizon entanglement entropy. By imposing consistency with the isolated horizon boundary conditions and semi-classical thermodynamical properties, we recover the Bekenstein–Hawking entropy formula for any value of the Immirzi parameter. Our analysis supports the equivalence between the von Neumann (entanglement) entropy interpretation and the Boltzmann (statistical) one.

**Introduction.** — In this Letter we build and analyse, for the first time, spherically symmetric continuum states to model a quantum black hole horizon, working in the full theory. In doing so, we make no reference to a classical symmetry-reduced sector [1].

As quantum gravity states for continuum spherically symmetric geometries we use spin network condensates in the group field theory (GFT) formalism [2, 3] (see [4] for an application to cosmology). We impose on such states quantum conditions characterizing Isolated Horizons (IH) [5], and analyze their entanglement properties. In particular, we show that their entanglement entropy coincides with the Boltzmann entropy of horizon degrees of freedom and it satisfies an area law, which is a cornerstone of holography.

The major strength of our analysis is the possibility of keeping into account the sum over triangulations required in the coarse graining procedure leading, from the infinite number of microscopic degrees of freedom defining our continuum quantum states, to an effective macroscopic description, as well as the control over the interplay between the IH boundary condition and the calculation of entropy. In fact, we are able to control the states with a relatively small number of parameters, encoding the geometrical data of the continuum geometry: we are using hydrodynamic states. This construction allows us to explicitly compute the horizon density matrix and to prove the holographic nature of our states. Finally, the GFT formalism provides a uniform treatment of boundary and bulk degrees of freedom and allow us to remove some ambiguities in the canonical LQG approach [1].

The implications of these novel features are striking. The entanglement entropy can be computed exactly and it matches the Bekenstein–Hawking formula [6] for *any value* of the Immirzi parameter  $\gamma$  (see [7–9] for a recent discussion on its role), once consistency with semi-classical conditions is imposed. The calculation reduces to a state counting, with the microscopic d.o.f. encoded in the combinatorial structure of all possible horizon condensate graphs (for a fixed expectation value of the macroscopic area). This supports the entanglement in-

terpretation of black hole entropy suggested in [8].

Due to difficulties in extracting effective equations of motion for GFT condensates from the fundamental dynamics of a given GFT model, we will omit restrictions originating from the microscopic dynamics in this work. However, we will show the consequence of requiring the compatibility with the classical dynamics of isolated horizons and their thermodynamical properties.

**Construction.** — Our plan consists of the following steps. *i*) We define GFT condensate states (as constructed in [3]) for a spacelike, spherically symmetric geometry by acting with a class of refinement operators on a seed state. *ii*) We identify an isolated horizon shell by imposing appropriate boundary conditions and consistency with semi-classical properties of horizons. *iii*) We derive the reduced density matrix, tracing away all the remaining bulk degrees of freedom and find a complete orthonormal basis of its eigenstates. *iv*) We compute the entanglement entropy of the IH.

*Spherically symmetric quantum states.* — We define a spherical symmetric quantum geometry in terms of an appropriate gluing of homogeneous spherical shells to one another [3]. The states of each shell are constructed starting from a seed state for a given shell, upon which we act with refinement operators, increasing the number of vertices and keeping the topology fixed as the connectivity is changed. In this way, the GFT state for a given shell is given by a (possibly infinite) superposition of regular 4-valent graphs with given topology. Shells are then glued together to form a full 3d foliation.

To keep the topological structure under control, each 4-vertex carries a color  $t = \{B, W\}$  and each  $SU(2)$  group element  $g$  associated to a link of a given 4-vertex is labelled by a number  $I = \{1, 2, 3, 4\}$  (i.e. we use coloured 4-graphs [10]). Each shell is composed of three parts: an outer boundary, an inner boundary and a bulk in between. In order to distinguish these regions, we introduce a further colour  $s = \{+, 0, -\}$ , specifying whether a given vertex belongs to the outer boundary, to the bulk or to

the inner boundary, respectively. The initial seed state and the refinement operators are such that all the open radial links of each boundary have the same colour, different for the two boundaries. In order to glue shells together, and still be able to distinguish different shells, we add a label  $r \in \mathbb{N}$  to the shell wavefunction, which effectively plays the role of a radial coordinate.

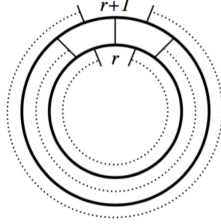


FIG. 1: Two shells,  $r$  and  $r + 1$ , glued together through their radial links.

The idea of GFT condensation posits that the same wavefunction  $\sigma$  should be associated to each new GFT excitation introduced in the state. This notion of wavefunction homogeneity *for each shell* captures the coarse grained homogeneity of continuum geometric data [3].

The field ladder operators for the vertex  $v$  are then

$$\hat{\sigma}_{r,t^v s^v}(h_I^v) = \int dg_I^v \sigma_r(h_I^v g_I^v) \hat{\varphi}_{t^v s^v}(g_I^v) \quad (1)$$

and its adjoint, satisfying the commutation relations

$$\left[ \hat{\sigma}_{r,t^v s^v}(h_I^v), \hat{\sigma}_{r',t^w s^w}^\dagger(h_I^w) \right] = \delta_{r,r'} \delta_{t^v,t^w} \delta_{s^v,s^w} \Delta_L(h_I^v, h_I^w). \quad (2)$$

Here we have defined the left invariant Dirac delta as:  $\Delta_L(h_I^v, h_I^w) = \int_{SU(2)} d\gamma \prod_{I=1}^4 \delta(\gamma h_I^v (h_I^w)^{-1})$ . The choice of the factor  $\delta_{r,r'}$  in the commutator is crucial: it implies that operators associated to different shells commute with each other. The commutator (2) was introduced in [3] for technical reasons, but we will show that it encodes crucial physical properties, as the form of (2) is at the origin of the holographic nature of our states.

A full space foliation can then be formed by glueing all the radial links of the outer boundary of the shell  $r$  with the (same number of) radial links of the inner boundary of the shell  $r + 1$ . Both sets of links must have the same colour. We are not going to explicitly define a refinement operator for the glued shells, as it plays no role in our entropy calculations. The general expression for the full states that we are interested into, then, is of the type:

$$|\Psi\rangle = \prod_r f_r(\widehat{\mathcal{M}}_{r,B}, \widehat{\mathcal{M}}_{r,W}) |seed\rangle, \quad (3)$$

where  $f_r$  is a function of the refinement operators  $\mathcal{M}_r$  of a given shell  $r$ . The action of the refinement operators can be represented pictorially:

$$\widehat{\mathcal{M}}_{r,B} : \begin{array}{c} | \\ \bullet \\ 3 \end{array} \begin{array}{c} 1 \\ | \\ 2 \end{array} \rightarrow \begin{array}{c} 1' \\ | \\ \bullet \\ 3' \end{array} \begin{array}{c} 2' \\ | \\ \bullet \\ 3' \end{array} \begin{array}{c} 1'' \\ | \\ \bullet \\ 4' \end{array} \begin{array}{c} 1 \\ | \\ \bullet \\ 3 \end{array} \begin{array}{c} 2 \\ | \\ \bullet \\ 2 \end{array} \quad (4)$$

$$\widehat{\mathcal{M}}_{r,W} : \begin{array}{c} 2 \\ | \\ \bullet \\ 3 \end{array} \begin{array}{c} 1 \\ | \\ \bullet \\ 4 \end{array} \rightarrow \begin{array}{c} 1 \\ | \\ \bullet \\ 2 \end{array} \begin{array}{c} 4' \\ | \\ \bullet \\ 3 \end{array} \begin{array}{c} 1'' \\ | \\ \bullet \\ 4' \end{array} \begin{array}{c} 2' \\ | \\ \bullet \\ 2' \end{array} \begin{array}{c} 1' \\ | \\ \bullet \\ 4 \end{array} \begin{array}{c} 2 \\ | \\ \bullet \\ 4 \end{array} \quad (5)$$

It corresponds to a dipole insertion, widely used in coloured tensor models and GFTs [10].

Geometric operators can then be computed for our GFT states, in a 2nd quantised language. For example, following [3], we define the horizon area operator

$$\hat{A}_{J,r,s} \equiv \sum_{t=B,W} \int (dg)^4 \hat{\sigma}_{r,ts}^\dagger(g_I) \sqrt{E_j^i E_j^j \delta_{ij}} \hat{\sigma}_{r,ts}(g_I), \quad (6)$$

where in this case  $s = \{+, -\}$  and  $J$  corresponds to the colour of the radial links dual to the boundary  $s$  of the shell  $r$  under examination. The expectation value of the area operator (6) on a shell boundary state gives

$$\langle \hat{A}_{J,r,s} \rangle = \langle \hat{n}_{r,s} \rangle a_{J,s}, \quad (7)$$

where  $a_{J,s}$  is the expectation value of the first quantized (LQG) area operator of a single radial link- $J$ , in the boundary  $s$  of the shell  $r$ , in a single-vertex state with wave function  $\sigma$ ;  $\hat{n}_{r,s}$  is the number operator defined as  $\hat{n}_{r,s} = \sum_{t=B,W} \int dh_I \hat{\sigma}_{r,ts}^\dagger(h_I) \hat{\sigma}_{r,ts}(h_I)$ . Due to the definition of the states, at each stage of refinement we always have  $n_{r,Bs} = n_{r,Ws} = \frac{n_{r,s}}{2}$ , where  $n \equiv \langle \hat{n} \rangle$ .

Notice that these expressions require regularization, as the states are, in general, not normalizable [3]. However, it is also easy to construct condensate states, peaked in some spin representation, for which all these steps can be followed rigorously, effectively reducing the analysis to the Abelian case discussed in [3].

These results are completely general. A factorization property similar to (7) holds for other one-body operators, like the 3-volume. The existence of a number operator in the GFT formulation of LQG represents a key difference with respect to the standard formulation, and it has a crucial role in the entropy calculation below.

*Imposition of IH conditions.* — In this context, we can model an isolated horizon with the imposition of the quantum version of the classical boundary condition [11]  $\mathcal{C}_{IH} = F^i(A) + \frac{\pi}{\mathcal{A}_{IH}} (1 - \gamma^2) \Sigma^i$ :

$$\hat{\mathcal{C}}_{IH} |\Psi\rangle = 0, \quad (8)$$

In the previous expression,  $\mathcal{A}_{IH}$  is the area of the isolated horizon,  $F^i$  the field strength of the Ashtekar-Barbero connection  $A^i$  on the IH and  $\Sigma^i$  the 2-form densitized triad pulled-back on the horizon. It becomes a condition on the condensate wave function  $\sigma$  for the outer shell.

Besides the IH condition, further restrictions on our states come from semiclassical conditions: the fluctuations of a set of operators, *e.g.* the area, should be small.

They restrict the possible superposition of graphs with different number of vertices, as it is evident from (7). Furthermore, we have to impose that the shells are thin, for the geometry to look smooth. This imposes a restriction on the expectation value of the volume per shell, the transverse area and the number of nodes. These conditions have operator equations counterparts, but we do not discuss them explicitly as they do not enter directly in our entropy calculations, the focus of the present work.

*Reduced density matrix.* — Now we focus on the computation of the entropy associated to the quantum isolated horizon, defined by our states. We do this in two steps: reduction to the density matrix associated to the outmost shell, and explicit computation of the entropy of the latter. The quantum state of our full IH states, described by the pure density matrix  $\hat{\rho} = |\Psi\rangle\langle\Psi|$ , consists of a (thin) shell and bulk degrees of freedom. We need only the degrees of freedom of the horizon, *i.e.* the ones of the outer boundary of the horizon shell  $r_0$ , described by a reduced density matrix obtained by appropriate traces.

A simple case will clarify the general procedure. Consider the graph  $A$  for the horizon outer boundary  $r_0$  and the graph  $B$  of the inner boundary of the neighboring shell  $r_0 + 1$ , glued along boundaries of colour 1. In order to be properly glued they must have the same number of vertices,  $n$ . The wavefunction is

$$\begin{aligned} \psi(g^{A_1}, \dots, g^{A_n}, g^{B_1}, \dots, g^{B_n}) &= \int \prod_{i=1}^n dh_I^{A_i} dh_I^{B_i} \\ &\times \sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \sigma_{B_i}(h_I^{B_i} g_I^{B_i}) \prod_{v,e} \delta(h_{v,e} h_{t_e^{-1},e}^{-1}), \end{aligned}$$

where the product over  $\delta$ 's encodes the connectivity of the total graph  $A \cup B$  via convolutions determined by the  $h_v^{-1}$ . We can thus write the total density matrix as

$$\begin{aligned} \rho^{(n)}(g^{A_1}, \dots, g^{A_n}, g^{B_1}, \dots, g^{B_n}; g'^{A_1}, \dots, g'^{A_n}, g'^{B_1}, \dots, g'^{B_n}) \\ &= \int \prod_{i=1}^n dh_I^{A_i} dh_I^{B_i} dk_I^{A_i} dk_I^{B_i} \\ &\times \left( \sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \sigma_{B_i}(h_I^{B_i} g_I^{B_i}) \prod_{v,e} \delta(h_{v,e} h_{t_e^{-1},e}^{-1}) \right) \\ &\times \left( \overline{\sigma_{A_i}(k_I^{A_i} g_I^{A_i}) \sigma_{B_i}(k_I^{B_i} g_I^{B_i})} \prod_{v,e} \delta(k_{v,e} k_{t_e^{-1},e}^{-1}) \right). \end{aligned}$$

We can trace away the B region of the graph using the following consequence of the commutation relations (2)

$$\int dg_I \sigma(h_I g_I) \overline{\sigma(k_I g_I)} = \int d\gamma \delta(\gamma h_I k_I^{-1}). \quad (9)$$

<sup>1</sup> The notation is designed to keep track of the combinatorics in terms of vertices  $v$  and edges  $e$  of the graph, so that  $t_e^{-1}$  will be the target vertex of the edge  $e$  departing from the vertex  $v$ .

The resulting reduced density matrix is

$$\begin{aligned} \rho_{red}^{(n)}(g^{A_1}, \dots, g^{A_n}; g'^{A_1}, \dots, g'^{A_n}) &= \int \prod_{i=1}^n dh_I^{A_i} dk_I^{A_i} \\ &\times \prod_{v,e} \delta(h_{v,e} h_{t_e^{-1},e}^{-1}) \delta(k_{v,e} k_{t_e^{-1},e}^{-1}) \sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \overline{\sigma_{A_i}(k_I^{A_i} g_I^{A_i})}. \end{aligned}$$

The mixed nature of the reduced density matrix is encoded *only* in the relation  $h_1^{A_i} = k_1^{A_i}$ .

This example shows a remarkable general property of these states: *the information about the combinatorial and geometric structure of the graph  $B$  is irretrievably lost, as a consequence of (9)*. This feature implements naturally the holographic features of null surfaces in classical gravity, and thus indirectly confirms the geometric interpretation of our GFT states. This happens even with no IH condition (8), and seems to follow directly from the hypothesis of condensation, encapsulated in the operators (1). Thus, it suggests that GFT condensates, as such, constitute a special class of holographic states.

**Entropy.** — The computation of the entanglement entropy can be done in detail, as we are able to diagonalize the reduced density matrix. We work at fixed (large) number of vertices, which is compatible with the semiclassicality conditions that we have mentioned (semiclassicality requires anyway good peakedness properties for the number operator). We comment at the end of the Section on relaxing this restriction. Using again (2), we see that the states

$$\Psi_A^{(n)} = \int \prod_{i=1}^n dg_I^{A_i} df_I^{A_i} \sigma_{A_i}(f_I^{A_i} g_I^{A_i}) \prod_{v,e} \delta(f_{v,e} f_{t_e^{-1},e}^{-1})$$

are eigenstates of the horizon density matrix  $\rho_{red}^{(n)}$ .

Therefore, we can write the reduced density matrix of the horizon for a given number  $n$  of boundary vertices as

$$\rho_{red-tot}^{(n)} = \frac{1}{\mathcal{N}} \sum_{s=1}^{\mathcal{N}} \rho_{red}^{(n)}(\Gamma_s), \quad (10)$$

where  $\mathcal{N}$  is the total number of horizon graphs for given number of vertices  $n$ , obtained with the refinement operators, and  $\rho_{red}^{(n)}(\Gamma_s)$  is the reduced density matrix for given graph. Orthogonality of the states for different graphs  $\Gamma_s, \Gamma_{s'}$ , which can be shown by direct computation, implies that the eigenvalues are:

$$\rho_{red}^{(n)}(\Gamma_s) \Psi_{r_0}^{(n)}(\Gamma_{s'}) = \begin{cases} \frac{1}{\mathcal{N}} & \text{if } s = s' \\ 0 & \text{if } s \neq s'. \end{cases} \quad (11)$$

The diagonal form of the density matrix allows us to compute the von Neumann entropy of the horizon. In particular, as a consequence of (11), *the horizon entanglement entropy is the same as the Boltzmann entropy*,

obtained by counting the graphs, whose combinatorics, due to the condensate hypothesis, encode all the microscopic degrees of freedom to be counted. For a state

$$\Psi_{r_0}^{(n)}(\Gamma_s) = \frac{\widehat{\mathcal{M}}_b^{N_b} \widehat{\mathcal{M}}_w^{N_w}}{N_b! N_w!} |seed\rangle \quad (12)$$

with  $N_b + N_w = 2n$ , the total number of graphs with  $n+1$  black and  $n+1$  white vertices that can be constructed by acting with the refinement operators is given by

$$\mathcal{N}(n, z) = \sum_{m=0}^{2n} \frac{(2n+1)!}{m!(2n-m)!} = (2n+1). \quad (13)$$

If we now include the degeneracy of the single vertex Hilbert space  $\Delta(a)$ , measuring the size of the space of wavefunctions satisfying the isolated horizon boundary conditions (and the other semiclassicality restrictions), the number of *states* to be counted is  $\tilde{\mathcal{N}}(n, z, a) = \mathcal{N}(n, z) \Delta(a)^{2n}$ . The Boltzmann entropy is

$$S(n, a) = 2n \log(\Delta(a)) + \log(2n+1). \quad (14)$$

Furthermore, the condition Eq. (9) is restrictive enough to allow the computation of  $a$  for the states satisfying it (see the Appendix of [3]). Combining this with the condition that the expectation value of the horizon area in Eq. (7) equals a macroscopic value  $\mathcal{A}_{IH}$ , we obtain:

$$S = \kappa \frac{\mathcal{A}_{IH}}{a} + \log\left(\frac{\mathcal{A}_{IH}}{a} + 1\right), \quad (15)$$

where  $\kappa = \log(\Delta(a))$  is a constant. *We thus obtain the desired area law from first principles.*

Notice that such area law for the entanglement entropy for any smooth closed codimension two surface emerges in various circumstances when considering quantum field theories in generic curved spacetimes [12]. In this sense then, as anticipated, the commutation relations (2) acquire a physical meaning, ensuring consistency between the quantum features of our GFT condensates and expected properties of classical smooth geometries, confirming their interpretation.

The proportionality constant  $\kappa/a$  can be explicitly computed from any specific choice of condensate state. It will reflect any restriction imposed on it by the effective equations of motion and the IH conditions, that we have not used yet. Leaving the detailed analysis to future work, our only working assumption at this stage is the compatibility of the classical dynamics with the GFT microscopic dynamics in our hydrodynamical approximation. This assumption is enough to determine (the general form of) the degeneracy  $\Delta$  as a function of  $a$ , once (8) has been imposed.

Generically, we expect  $\Delta(a)$  to be finite for any reasonable geometric state. Moreover, for fixed  $\mathcal{A}_{IH}$ , we are considering condensate states such that  $n$  is a large pure

number and, consequently,  $a$  is small (with respect to  $\mathcal{A}_{IH}$ ). In this limit, the IH boundary condition fixes the holonomy around each radial link to be flat; this happens only if the spin labels of the tangent links are 0. Therefore, in the limit  $a \rightarrow 0$ , the wavefunction  $\sigma$  should be a delta peaked on  $j_I = 0$ , which means  $\Delta(0) \sim 1$ . As soon as  $a > 0$ , then  $\Delta(a)$  should grow. For small values of  $a$ , therefore, we expect  $\Delta(a) \sim 1 + ca$  with  $c$  a (positive) constant. Therefore, the imposition of the IH boundary condition implies  $\kappa \sim ca + o(ca)$  and

$$S = c \mathcal{A}_{IH} + \log(\mathcal{A}_{IH}/a). \quad (16)$$

Using the continuum geometric interpretation of our states, and assuming that enough semiclassical features have been imposed on them, one can now use the compatibility with the thermodynamic relation  $\beta = \frac{\partial S}{\partial E}$ , to determine what the proportionality factor  $c$  should be. Here,  $\beta$  is the inverse Unruh temperature  $1/T_U = 2\pi\ell/\ell_P^2$ , for a stationary observer at distance  $\ell$  with a surface gravity  $k = 1/\ell$ , and  $E$  is the local energy defined in [15] for isolated horizons,  $E_{IH} = \frac{\mathcal{A}_{IH}}{8\pi\ell}$ . Here we use our assumption of convergence between macroscopic GR dynamics and effective equations of motion derived from the GFT dynamics. By doing so, we get the known condition:

$$c = 1/(4\ell_P^2) \quad (17)$$

as our thermodynamical consistency condition, which reproduces the semiclassical Bekenstein-Hawking entropy.

*Remarks.* — We stress once more that the proportionality factor can be computed for each specific choice of our microscopic GFT condensate states, and the agreement with the semiclassical value is to be seen as a constraint selecting those states, among those solving also the dynamics of the theory, which also admit a good semiclassical interpretation. We notice also that  $\ell_P$  appearing in  $c$  (and thus in the entropy formula), is going to be a function of the microscopic parameters of the theory, notably its dynamical coupling constants. These, in turn, are subject to renormalisation in going from the microscopic definition of the theory to the effective continuum semiclassical regime. To determine the flow of such parameters, for realistic models, is an active direction of current developments in the GFT approach [16].

Next, let us point out that the coefficient in front of the logarithmic correction depends directly on the form chosen for the refinement operators in the microscopic definition of our condensate states. Thus it can also be computed explicitly. In our case, i.e. Eq. (13) obtained from the actions (4), (5), the numerical coefficient of the logarithmic term is equal to one. A different numerical coefficient could be obtained by modifying the action of the refinement operators and consequently the counting of graphs for a given  $n$ . Moreover, it is possible to work with a more general mixed density matrix,  $\hat{\rho}_{red} = \sum_n w(n) \hat{\rho}^{(n)}$ , containing a mixture of states with

different number of vertices, coming from the trace of a generic state as in (3). For semiclassical mixtures (thus peaked around some value  $n_0$ ) the dominant area law contribution for these states is robust and independent from any detail of the mixture of graphs, the numerical coefficient of the logarithmic correction takes a different value, still of order unit, and an additional term appears, the Shannon entropy of the weights  $w_n$ .

Finally, it is important to notice that the leading term in entropy result (16), (17) does not depend explicitly on the Immirzi parameter  $\gamma$ . This is a striking consequence of the IH boundary conditions and of the GFT formalism. More precisely, the availability of a number operator (a purely GFT observable), and the possibility to construct and control condensate states incorporating a large (possibly infinite) superposition of graphs, rather than simple area eigenstates, represent key improvements over similar calculations in canonical LQG. Moreover, also the numerical coefficient in front of the logarithmic corrections does not depend on  $\gamma$ , due to its purely combinatorial origin, unless one modifies the construction by using  $\gamma$ -dependent weights  $w(n)$  for the mixture of states with different  $n$ .

**Acknowledgments.** LS has been supported by the Templeton Foundation through the grant number PS-GRAV/1401.

---

\* Electronic address: daniele.oriti@aei.mpg.de

† Electronic address: daniele.pranzetti@gravity.fau.de

‡ Electronic address: lorenzo.sindoni@aei.mpg.de

- [1] J. Diaz-Polo and D. Pranzetti, SIGMA **8**, 048 (2012) [arXiv:1112.0291 [gr-qc]].  
 [2] D. Oriti, arXiv:1408.7112 [gr-qc]; D. Oriti,

- arXiv:1310.7786 [gr-qc].  
 [3] D. Oriti, D. Pranzetti, J. P. Ryan and L. Sindoni, Class. Quant. Grav. to appear arXiv:1501.00936 [gr-qc].  
 [4] S. Gielen, D. Oriti and L. Sindoni, Phys. Rev. Lett. **111**, no. 3, 031301 (2013) [arXiv:1303.3576 [gr-qc]].  
 S. Gielen, D. Oriti and L. Sindoni, JHEP **1406**, 013 (2014) [arXiv:1311.1238 [gr-qc]].  
 [5] A. Ashtekar, C. Beetle and S. Fairhurst, Class. Quant. Grav. **17**, 253 (2000) [gr-qc/9907068].  
 [6] J. D. Bekenstein, Phys. Rev. D **7** (1973) 2333.  
 S. W. Hawking, Commun. Math. Phys. **43** (1975) 199.  
 [7] E. Frodden, M. Geiller, K. Noui and A. Perez, Europhys. Lett. **107**, 10005 (2014) [arXiv:1212.4060 [gr-qc]].  
 J. Ben Achour, A. Mouchet and K. Noui, JHEP **1506**, 145 (2015) [arXiv:1406.6021 [gr-qc]].  
 [8] D. Pranzetti, Phys. Rev. D **89**, no. 10, 104046 (2014) [arXiv:1305.6714 [gr-qc]].  
 [9] A. Ghosh and D. Pranzetti, Nucl. Phys. B **889**, 1 (2014) [arXiv:1405.7056 [gr-qc]].  
 D. Pranzetti and H. Sahlmann, Phys. Lett. B **746**, 209 (2015) [arXiv:1412.7435 [gr-qc]].  
 [10] R. Gurau and J. P. Ryan, SIGMA **8**, 020 (2012) [arXiv:1109.4812 [hep-th]].  
 [11] J. Engle, K. Noui, A. Perez and D. Pranzetti, Phys. Rev. D **82**, 044050 (2010) [arXiv:1006.0634 [gr-qc]].  
 [12] S. N. Solodukhin, Living Rev. Rel. **14**, 8 (2011) [arXiv:1104.3712 [hep-th]].  
 [13] W. Donnelly, Phys. Rev. D **85** (2012) 085004 [arXiv:1109.0036 [hep-th]].  
 [14] W. Donnelly, Class. Quant. Grav. **31** (2014) 21, 214003 [arXiv:1406.7304 [hep-th]].  
 [15] E. Frodden, A. Ghosh and A. Perez, Phys. Rev. D **87**, no. 12, 121503 (2013) [arXiv:1110.4055 [gr-qc]].  
 [16] J. Ben Geloun and V. Rivasseau, Commun. Math. Phys. **318**, 69 (2013) [arXiv:1111.4997 [hep-th]].  
 S. Carrozza, D. Oriti and V. Rivasseau, Commun. Math. Phys. **330**, 581 (2014) [arXiv:1303.6772 [hep-th]].  
 D. Benedetti, J. Ben Geloun and D. Oriti, JHEP **1503**, 084 (2015) [arXiv:1411.3180 [hep-th]].