Horizon entropy from quantum gravity condensates

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We construct condensate states encoding the continuum spherically symmetric quantum geometry of an horizon in full quantum gravity, i.e. without any classical symmetry reduction, in the group field theory formalism. Tracing over the bulk degrees of freedom, we show how the resulting reduced density matrix manifestly exhibits an holographic behavior. We derive a complete orthonormal basis of eigenstates for the reduced density matrix of the horizon and use it to compute the horizon entanglement entropy. By imposing consistency with the horizon boundary conditions and semi-classical thermodynamical properties, we recover the Bekenstein–Hawking entropy formula for any value of the Immirzi parameter. Our analysis supports the equivalence between the von Neumann (entanglement) entropy interpretation and the Boltzmann (statistical) one.

Introduction. — In this Letter we build and analyse, for the first time, spherically symmetric continuum states to model a quantum black hole horizon, working in the full theory. In doing so, we make no reference to a classical symmetry-reduced sector [1].

As quantum gravity states for continuum spherically symmetric geometries we use spin network condensates in the group field theory (GFT) formalism [2, 3] (see [4] for an application to cosmology). We impose on them conditions characterising horizons (including isolated horizons [5]), and analyze their entanglement properties. We show that their entanglement entropy coincides with the Boltzmann entropy of horizon degrees of freedom (dof) and it satisfies an area law, a cornerstone of holography.

The major strength of our analysis is the possibility of keeping into account the sum over triangulations required in the coarse graining procedure leading, from the infinite number of microscopic dof defining our continuum quantum states, to an effective macroscopic description, as well as the control over the interplay between horizon boundary conditions and the calculation of entropy. In fact, we are able to control the states with a relatively small number of parameters, encoding the geometrical data of the continuum geometry: we are using hydrodynamic states. This construction allows us to explicitly compute the horizon density matrix and to prove the holographic nature of our states. Finally, the GFT formalism provides a uniform treatment of boundary and bulk dof and allow us to remove some ambiguities in the canonical LQG approach [1].

The implications of these novel features are striking. The entanglement entropy can be computed exactly and it matches the Bekenstein–Hawking formula [6] for *any value* of the Immirzi parameter γ (see [7–9] for a recent discussion on its role), once consistency with semiclassical conditions is imposed. The calculation reduces to a state counting, with the microscopic d.o.f. encoded in the combinatorial structure of all possible horizon condensate graphs (for a fixed expectation value of the macroscopic area). This supports the entanglement interpretation of black hole entropy suggested in [8].

Due to difficulties in extracting effective equations of motion for our generalised GFT condensates from the fundamental dynamics of a given GFT model, we will omit restrictions originating from the microscopic dynamics in this work. However, we will rely on the use a maximum entropy principle to capture a few essential dynamical features, and as a partial characterisation of horizon geometries, and we will show the consequence of requiring the compatibility with the classical dynamics of isolated horizons and their thermodynamical properties.

Construction. — Our plan consists of the following steps. *i*) We define GFT condensate states (as constructed in [3]) for a spacelike, spherically symmetric geometry by acting with a class of refinement operators on a seed state, and with appropriate semiclassicality restrictions. *ii*) We derive the reduced density matrix, tracing away the remaining bulk dof and find a complete orthonormal basis of its eigenstates. *iii*) We compute the entanglement entropy, coinciding with the statistical entropy of the boundary dof, and show how the result is affected by different choices of boundary conditions.

Spherically symmetric quantum states. — We define a spherical symmetric quantum geometry in terms of a gluing of homogeneous spherical shells to one another [3]. The states of each shell are constructed starting from a seed state for a given shell, upon which we act with refinement operators, increasing the number of vertices and keeping the topology fixed as the connectivity is changed. In this way, the GFT state for a given shell is given by a (possibly infinite) superposition of regular 4-valent graphs with given topology. Shells are then glued together to form a full 3d foliation.

To keep the topological structure under control, each 4-vertex carries a color $t = \{B, W\}$ and each SU(2) group element g associated to a link of a given 4-vertex is labelled by a number $I = \{1, 2, 3, 4\}$ (i.e. we use coloured 4-graphs [10]). Each shell is composed of three parts: an outer boundary, an inner boundary and a bulk in between. In order to distinguish these regions, we introduce a further colour $s = \{+, 0, -\}$, specifying whether a given vertex belongs to the outer boundary, to the bulk or to the inner boundary, respectively. The initial seed state and the refinement operators are such that all the open radial links of each boundary have the same colour, different for the two boundaries. In order to glue shells together, and still be able to distinguish different shells, we add a label $r \in \mathbb{N}$ to the shell wave-function, which effectively plays the role of a radial coordinate. The idea

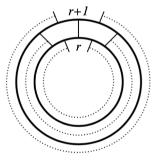


FIG. 1: Two shellsglued through their radial links.

of GFT condensation posits that the same wave-function σ should be associated to each new GFT excitation introduced in the state. This notion of wave-function homogeneity *for each shell* captures the coarse grained homogeneity of continuum geometric data [3].

The field ladder operators for the vertex v are then

$$\hat{\sigma}_{r,t^v s^v}(h_I^v) = \int \mathrm{d}g_I^v \; \sigma_{r,s^v}(h_I^v g_I^v)\, \hat{\varphi}_{t^v}(g_I^v) \tag{1}$$

and its adjoint, satisfying the commutation relations

$$\left[\hat{\sigma}_{r,t^v s^v}(h_I^v), \hat{\sigma}^\dagger_{r',t^w s^w}(h_I^w)\right] = \delta_{r,r'}\delta_{t^v,t^w}\delta_{s^v,s^w}\Delta_L(h_I^v, h_I^w). \tag{2}$$

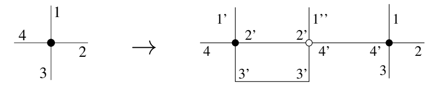
Here we have defined the left invariant Dirac delta as: $\Delta_L(h_I^v, h_I^w) = \int_{SU(2)} d\gamma \prod_{I=1}^4 \delta(\gamma h_I^v (h_I^w)^{-1})$. The choice of the factor $\delta_{r,r'}$ in the commutator is crucial: it implies that operators associated to different shells commute with each other. The commutator (2) was introduced in [3] for technical reasons, but we will show that it encodes crucial physical properties, as the form of (2) is at the origin of the holographic nature of our states.

A full spatial foliation can then be formed by glueing all the radial links of the outer boundary of the shell r with the (same number of) radial links of the inner boundary of the shell $r+1$. Both sets of links must have the same colour. We are not going to explicitly define a refinement operator for the glued shells, as it plays no role in our entropy calculations (but see [3] for the tools used in the construction). The general expression for the full states that we are interested into, then, is of the type:

$$|\Psi\rangle = \prod_r f_r(\widehat{\mathcal{M}}_{r,B}, \widehat{\mathcal{M}}_{r,W})\, |seed\rangle \;, \tag{3}$$

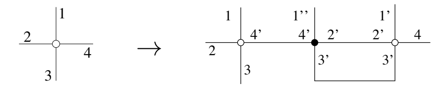
where f_r is a function of the refinement operators $\mathcal{M}_r$ of a given shell r. The action of the refinement operators can be represented pictorially:

$\widehat{\mathcal{M}}_{r,B}$:



(4)

$\widehat{\mathcal{M}}_{r,W}$:



(5)

It corresponds to a dipole insertion, widely used in coloured tensor models and GFTs [10].

Geometric operators can then be computed for our GFT states, in a 2nd quantised language. For example, following [3], we define the horizon area operator

$$\hat{\mathbb{A}}_{Jr,s} \equiv \sum_{t=B,W} \int (dg)^4 \hat{\sigma}^\dagger_{r,ts}(g_I) \sqrt{E^i_J E^j_J \delta_{ij}}\, \hat{\sigma}_{r,ts}(g_I)\,, \tag{6}$$

where in this case $s = \{+, -\}$ and J corresponds to the colour of the radial links dual to the boundary s of the shell r under examination. The expectation value of the area operator (6) on a shell boundary state gives

$$\langle \hat{\mathbb{A}}_{Jr,s} \rangle \;=\; \langle \widehat{n}_{r,s} \rangle \, a_{J,s}, \tag{7}$$

where $a_{J,s}$ is the expectation value of the first quantized (LQG) area operator of a single radial link-J, in the boundary s of the shell r, in a single-vertex state with wave-function σ; $\widehat{n}_{r,s}$ is the number operator defined as $\widehat{n}_{r,s} = \sum_{t=B,W} \int dh_I \, \hat{\sigma}^\dagger_{r,ts}(h_I)\hat{\sigma}_{r,ts}(h_I)$. Due to the definition of the states, at each stage of refinement we always have $n_{r,Bs} = n_{r,Ws} = n_{r,s}/2$, where $n \equiv \langle \widehat{n} \rangle$.

Notice that, in general, these expressions require regularization, as our condensate states are not always normalizable [3]. However, it is easy to construct condensate states, peaked in some spin representation, for which all these steps can be followed rigorously, effectively reducing the analysis to the Abelian case discussed in [3]. The full space of solutions to the equations characterising the condensate wave-function and the refinement move kernel is not known, and we can only exhibit a few explicit solutions. The existence of several other solutions is plausible, which then leaves a certain amount of freedom in the specification of the vertex wave-function.

These results are completely general. A factorization property similar to (7) holds for other one-body operators, like the 3-volume. The existence of a number operator in the GFT formulation of LQG represents a key difference with respect to the standard formulation, and it has a crucial role in the entropy calculation below.

Further restrictions. — In this context, we have two possible ways to characterize our shell condensate as a quantum horizon. One possibility would be to impose the quantum version of the classical isolated horizon boundary condition [11]. This can be done locally, at the level of each single vertex, by relating the curvature around the link dual to the boundary face to the flux associated the it, leading to a restriction on the vertex wave-function. A second way to define the horizon shell is through the condition that the reduced states maximize the entropy. Imposition of these two constraints in general does not commute, and will give different results for the entropy. We will come back on the strategy we follow below, after deriving the general result.

Further restrictions on our states come from semiclassicality conditions: the fluctuations of a set of operators, *e.g.* the area, should be small. They restrict the possible superposition of graphs with different number of vertices, as it is evident from (7). Furthermore, we have to impose that the shells are thin, for the geometry to look smooth. This imposes a restriction on the expectation value of the volume per shell, the transverse area and the number of nodes. These conditions have operator equations counterparts, but we do not discuss them explicitly as they do not enter directly in our entropy calculations.

Reduced density matrix. — Now we focus on the computation of the entropy associated to the quantum horizon, as defined by our states. We do this in two steps: reduction to the density matrix associated to the outmost shell, and explicit computation of the entropy of the latter. Our complete quantum state, described by the pure density matrix $\hat{\rho} = |\Psi\rangle\langle\Psi|$, consists of a (thin) shell and bulk dof. We need only the dof of the horizon, *i.e.* the ones of the outer boundary of the horizon shell r_0, described by a reduced density matrix obtained by appropriate traces.

A simple case will clarify the general procedure. Consider the graph A for the horizon outer boundary r_0 and the graph B of the inner boundary of the neighboring shell r_0+1, glued along boundaries of colour 1. In order to be properly glued they must have the same number of vertices, n. The wave-function is

$$\psi(g^{A_1},...,g^{A_n},g^{B_1},...,g^{B_n}) = \int \prod_{i=1}^{n} dh_I^{A_i} dh_I^{B_i}$$
$$\times\, \sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \sigma_{B_i}(h_I^{B_i} g_I^{B_i}) \prod_{v,e} \delta(h_{v,e} h^{-1}_{t_e{}^v,e}),$$

where the product over δ's encodes the connectivity of the total graph $A \cup B$ [1]. The total density matrix is

$$\rho^{(n)}(g^{A_1},...,g^{A_n},g^{B_1},...,g^{B_n};g'^{A_1},...,g'^{A_n},g'^{B_1},...,g'^{B_n})$$
$$= \int \prod_{i=1}^{n} dh_I^{A_i} dh_I^{B_i} dk_I^{A_i} dk_I^{B_i}$$
$$\times \left(\sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \sigma_{B_i}(h_I^{B_i} g_I^{B_i}) \prod_{v,e} \delta(h_{v,e} h^{-1}_{t_e{}^v,e}) \right)$$
$$\times \left(\overline{\sigma_{A_i}(k_I^{A_i} g_I'^{A_i}) \sigma_{B_i}(k_I^{B_i} g_I'^{B_i})} \prod_{v,e} \delta(k_{v,e} k^{-1}_{t_e{}^v,e}) \right).$$

[1] The notation is designed to keep track of the combinatorics in terms of vertices v and edges e of the graph, so that t_{e^v} will be the target vertex of the edge e departing from the vertex v.

We can trace away the B region of the graph using the following consequence of the commutation relations (2)

$$\int dg_I \sigma(h_I g_I) \overline{\sigma(k_I g_I)} = \int d\gamma \delta(\gamma h_I k_I^{-1}). \qquad (8)$$

The resulting reduced density matrix is

$$\rho^{(n)}_{red}(g^{A_1},\dots,g^{A_n};g'^{A_1},\dots,g'^{A_n}) = \int \prod_{i=1}^{n} dh_I^{A_i} dk_I^{A_i}$$
$$\times \prod_{v,e} \delta(h_{v,e} h^{-1}_{t_e{}^v,e}) \delta(k_{v,e} k^{-1}_{t_e{}^v,e}) \sigma_{A_i}(h_I^{A_i} g_I^{A_i}) \overline{\sigma_{A_i}(k_I^{A_i} g_I'^{A_i})}.$$

The mixed nature of the reduced density matrix is encoded *only* in the relation $h_1^{A_i} = k_1^{A_i}$.

This example shows a remarkable general property of these states: *the information about the combinatorial and geometric structure of the graph B is irretrievably lost, as a consequence of* (8). This feature implements naturally the holographic features of null surfaces in classical gravity, and thus indirectly confirms the geometric interpretation of our GFT states. This happens even with no characterization of our states as a quantum horizon states, and it seems to follow directly from the hypothesis of condensation, encapsulated in the operators (1). Thus, it suggests that GFT condensates, as such, constitute a special class of holographic states.

Entropy. — The computation of the entanglement entropy can be done in detail, as we are able to diagonalize the reduced density matrix. We work at fixed (large) number of vertices, which is compatible with the semiclassicality conditions (semiclassicality requires anyway good peakednesss properties for the number operator, as this translates into good peakedness of extensive geometric observables). Using again (2), we see that the states

$$\Psi^{(n)}_A = \int \prod_{i=1}^{n} dg_I'^{A_i} df_I^{A_i} \sigma_{A_i}(f_I^{A_i} g_I'^{A_i}) \prod_{v,e} \delta(f_{v,e} f^{-1}_{t_e{}^v,e})$$

are eigenstates of the horizon density matrix $\rho^{(n)}_{red}$.

Therefore, we can write the reduced density matrix of the horizon for a given number n of boundary vertices as

$$\rho^{(n)}_{red-tot} = \frac{1}{\mathcal{N}} \sum_{s=1}^{\mathcal{N}} \rho^{(n)}_{red}(\Gamma_s), \qquad (9)$$

where $\mathcal{N}$ is the total number of horizon graphs for given number of vertices n, obtained with the refinement operators, and $\rho^{(n)}_{red}(\Gamma_s)$ is the reduced density matrix for given graph. Orthogonality of the states for different graphs $\Gamma_s, \Gamma_{s'}$, which can be shown by direct computation, implies that the eigenvalues are:

$$\rho^{(n)}_{red}(\Gamma_s) \Psi^{(n)}_{r_0}(\Gamma_{s'}) = \begin{cases} 1 & \text{if } s = s' \\ 0 & \text{if } s \neq s'. \end{cases} \qquad (10)$$

The diagonal form of the density matrix allows us to compute the von Neumann entropy of the horizon. In particular, as a consequence of (10), *the horizon entanglement entropy is the same as the Boltzmann entropy*, obtained by counting the boundary graphs, whose combinatorics, due to the condensate hypothesis, encode all the relevant microscopic dof. For a state

$$\Psi_{r_0}^{(n)}(\Gamma_s) = \frac{\widehat{\mathcal{M}}_b^{N_b}\widehat{\mathcal{M}}_w^{N_w}}{N_b! N_w!}\,|seed\rangle \tag{11}$$

with $N_b + N_w = 2n$, the total number of graphs with $n+1$ black and $n+1$ white vertices that can be constructed by acting with the refinement operators is given by

$$\mathcal{N}(n) = \sum_{m=0}^{2n} \frac{(2n+1)!}{m!(2n-m)!} = 2^{2n}(2n+1)\,. \tag{12}$$

In the counting that leads to the result (12) we assumed indistinguishability of the vertices, consistently with the condensate hypothesis and the form of our states. If we now include the degeneracy $\Delta(a)$ of the single vertex Hilbert space, the number of *states* to be counted is $\tilde{\mathcal{N}}(n,a) = \mathcal{N}(n)\Delta(a)$.

A point requires attention. We are implicitly assuming that the only structure that is left in the state is the horizon shell, while one would expect the computation of the full entanglement entropy to require the computation of the reduced density matrix of the part of the bulk up to the horizon shell. Performing the same calculation above, one would expect to obtain the number of graphs in the bulk times the single vertex Hilbert space degeneracy associated to all the bulk shells. However, the construction of our states makes this extra counting not necessary. In fact the refinement operators are applied on the whole state, and act in such a way that every new vertex on a shell is matched to a new vertex in a neighboring shell. Consequently, the actions of the refinement operators on different shells, and hence the number of graphs to be counted, are perfectly coordinated. The counting of the graphs on a single shell, then, exhausts the number of states. Moreover, in order for the action of the refinement operators to be correctly defined on each part of every shell, the form of the condensate wave-function and its functional dependence on the different colours have to be the same for the whole graph (as it can be seen from the commutation relations (2)). This implies that the degeneracy factor $\Delta(a)$ covers the dimension of the space of allowed wave-functions also when the whole bulk is included in the calculation.

Therefore, *the weak holographic principle is not assumed in our analysis, but it follows from the condensate hypothesis and the features of our construction.*

We conclude that the Boltzmann entropy is

$$S(n,a) = \log(\tilde{\mathcal{N}}) = 2n\log(2) + \log(2n+1) + \log(\Delta(a))\,. \tag{13}$$

In (13), the central result of our analysis, we recognize an area law, as the first term is an extensive quantity proportional to the total number of plaquettes composing the horizon, and thus, for given average area for a single-plaquette a, to the total area $\mathcal{A} = a\,n$ (and the degeneracy factor $\Delta(a)$ only contributes a constant shift). It should be stressed that the structure of the result holds for any spherically symmetric state, as we have not yet discussed the extra conditions characterising an horizon. This also implies that there is no reason, yet, to require matching with the Bekenstein-Hawking entropy, i.e. requiring our states to give a specific value for a. Notice that area laws for the entanglement entropy for any smooth closed codimension two surface emerge in various situations [12]. In this sense, as anticipated, the commutation relations (2) acquire a physical meaning, ensuring consistency between the quantum features of our GFT condensates and expected properties of classical smooth geometries, confirming their interpretation.

To proceed beyond this point one should use the equations of motion to determine $n, a, \Delta(a)$, not fixed by the defining properties of the condensate states alone.

Even without the exact dynamics, we can make significant progress by imposing horizon boundary conditions.

As pointed out above, there are two possibilities to do that. Using the isolated horizon boundary condition would a priori introduce an extra dependence of the degeneracy Δ on the total value of the horizon area, since this enters the resulting constraint on the vertex wave-function σ. The area law, then, is not guaranteed and one needs a detailed analysis of the space of constrained wave-functions. This would be a highly non-trivial task.

We use instead a maximum entropy principle, and we determine the values of $a, n, \Delta(a)$ for the most generic state compatible with a fixed macroscopic value of the total horizon area $\mathcal{A}_H$. Further investigations are needed to show that this is a good characterization of quantum horizons, compatible with the quantum dynamics.

Compatibly with the semiclassicality conditions stated above, for fixed $\mathcal{A}_H$, we are considering condensate states such that n is a large pure number and, consequently, a is small. Introducing the constraint on the area, we look for extrema of $\Sigma(n,a,\lambda) = S(n,a) + \lambda\,(\mathcal{A}_H - 2an)$, when varying with respect to a, n, λ. Let us point out that, if $\Delta(a)$ was known explicitly, then the system of equations would fully determine the free parameters a, n, λ as functions of $\mathcal{A}_H$ and the microscopic parameters of the theory. This not being the case, we use one of the equations to determine $\Delta(a)$, thus leaving the final result dependent on the Lagrange multiplier λ. More precisely, we obtain $a = \log(2)/\lambda, \Delta = c_0 \exp(\lambda\mathcal{A}_H)$, where c_0 is an irrelevant integration constant. As a result, the entropy is:

$$S(\mathcal{A}_H, \lambda) \sim 2\lambda\mathcal{A}_H + \log(\mathcal{A}_H/a)\;. \tag{14}$$

We obtained the desired area law from first principles.

From the entropy result (14) we recover the semiclassical Bekenstein–Hawking formula by setting the Lagrange multiplier $\lambda = 1/8\ell_P^2$. Within our working assumption about the compatibility of the classical dynamics with our hydrodynamical approximation of GFT, this last step can be interpreted as a thermodynamical consistency condition. More precisely, exploiting the continuum (and semiclassical) geometric interpretation of our states, the value of λ above yielding the factor of 1/4 in front of the area law is obtained from the compatibility with the thermodynamic relation $\beta = \frac{\partial S}{\partial E}$, where β is the horizon temperature and E its energy, which implies convergence between macroscopic GR dynamics and effective equations of motion derived from the GFT dynamics.

Let us clarify an important aspect of this final result. The value of λ yielding the correct semiclassical result implies $a = \log(2)8\ell_P^2$, which is also consistent with our semiclassicality condition of a small, i.e. large n limit. The (average) area a for a single vertex can be computed for each specific choice of our microscopic GFT condensate states. The agreement with this precise value is then a constraint selecting those states, among those solving also the dynamics of the theory, which admit a good semiclassical interpretation. In this way, the (implicit) dependence of a on the Immirzi parameter does *not* imply that the Bekenstein–Hawking formula is recovered only for a specific choice of γ. On the contrary, the leading term in the semiclassical entropy result remains explicitly *independent on* γ. This is a striking consequence of the GFT formalism. More precisely, the availability of a number operator (a purely GFT observable), and the possibility to construct and control condensate states incorporating a large (possibly infinite) superposition of graphs, rather than simple area eigenstates, represent key improvements over similar calculations in canonical LQG. The standard LQG calculation (with its dependence on γ) would be recovered for very special condensate states which are eigenstates of the horizon area.

Remarks. — We notice that ℓ_P, appearing in λ (and thus in the entropy formula), is going to be a function of the microscopic parameters of the theory, notably its dynamical coupling constants. These, in turn, are subject to renormalisation in going from the microscopic definition of the theory to the effective continuum (and semiclassical) regime. To determine the flow of such parameters, for realistic models, is an active direction of current developments in the GFT approach [13].

Finally, let us point out that the coefficient in front of the logarithmic correction, equal one in our case, depends directly on the form chosen for the refinement operators in the microscopic definition of our condensate states, which dictates the counting of graphs. Thus it can also be computed explicitly. Moreover, it is possible to work with a more general mixed density matrix, $\hat{\rho}_{red} = \sum_n w(n)\hat{\rho}^{(n)}$, containing a mixture of states with different number of vertices, coming from the trace of a generic state as in (3). For semiclassical mixtures (thus peaked around some value n_0) the dominant area law contribution for these states is robust and independent from any detail of the mixture of graphs; on the other hand, the numerical coefficient of the logarithmic correction takes a different value, still of order unit. It remains independent on γ, due to its purely combinatorial origin, unless one modifies the construction by using γ-dependent weights $w(n)$ for the mixture of states with different n. Also, an additional entropy term appears, the Shannon entropy of the weights w_n.

Acknowledgments. We thank A. Perez and D. Benincasa for useful comments, which led to significant improvements in the presentation of our results. LS has been supported by the Templeton Foundation through the grant number PS-GRAV/1401.

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