Logique & Analyse **233** (2016), 1-35

THE SQUARE OF OPPOSITION IN CATHOLIC HANDS: A CHAPTER IN THE HISTORY OF 20TH-CENTURY LOGIC

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Abstract

The present study describes how three now almost forgotten mid-20th-century logicians, the American Paul Jacoby and the Frenchmen Augustin Sesmat and Robert Blanché, all three ardent Catholics, tried to restore traditional predicate logic to a position of respectability by expanding the classic Square of Opposition to a hexagon of logical relations, showing the logical and cognitive advantages of such an expansion. The nature of these advantages is discussed in the context of modern research regarding the relations between logic, language, and cognition. It is desirable to call attention to these attempts, as they are, though almost totally forgotten, highly relevant against the backdrop of the clash between modern and traditional logic. It is argued that this clash was and is unnecessary, as both forms of predicate logic are legitimate, each in its own right. The attempts by Jacoby, Sesmat, and Blanché are, moreover, of interest to the history of logic in a cultural context in that, in their own idiosyncratic ways, they fit into the general pattern of the Catholic cultural revival that took place roughly between the years 1840 and 1960. The Catholic Church had put up stiff resistance to modern mathematical logic, considering it dehumanizing and a threat to Catholic doctrine. Both the wider cultural context and the specific implications for logic are described and analyzed, in conjunction with the more general philosophical and doctrinal issues involved.

Keywords: Aristotle, Blanché, Catholicism, cognition, Jacoby, logical hexagon, logical relations, predicate logic, Russell, Sesmat, Square of Opposition, undue existential import.

0. Overview

In Section 1 of the present study, we discuss the logical relations between standard modern predicate logic (SMPL) on the one hand and various other systems of predicate logic on the other from a technical point of view. These 'other systems' include the traditional *Square of Opposition* (henceforth the *Square*), going back to Aristotle and his ancient commentators, the correction of the Square introduced by the medieval French philosopher *Abelard*, the triadic system devised by the nineteenth-century Edinburgh philosopher *Sir William Hamilton*, and the expansion of the Square to a

doi: 10.2143/LEA.233.0.3149529.

logical Hexagon by the little-known mid-20th-century logicians Jacoby, Sesmat and Blanché.

In Section 2, we discuss the wider cultural context of SMPL, against the backdrop of philosophical positivism and psychological behaviourism which dominated Western intellectual life, especially in America, during the first half of the 20th century, showing how this intellectual climate left no room for an interest in questions of the natural logic of human cognition and language. The Catholic Church, which was a force to be reckoned with during the period at issue, strongly opposed both positivism and behaviourism, also in the area of logic. The prominent French Catholic philosopher Jacques Maritain was the spearhead of this Catholic resistance movement. In his logical writings he rejected SMPL *in toto* and clung to the traditional Square, complete with all the metaphysical and psychological-spiritual implications it had been caught up in during centuries of Catholic philosophy. The Church, however, was unable to stop these 'modernist' ways of thinking. This was achieved by the new development of *cognitive science*, which came about after 1950 and promoted the view that the human mind is far richer and deeper than suspected by the positivist-behaviourists, developing the computational view of the mind which is now the dominant paradigm. We show how the logical Hexagon, developed by the three Catholic logicians Jacoby, Sesmat and Blanché especially with a view to the logical aspects of the human mind, fits into this pattern of rebellion against the domination of positivism and behaviourism, though not from a cognitive-science point of view but inspired by a basically Catholic ideology.

Section 3 is of a more general cultural-historical nature, discussing the important social, cultural and intellectual revival of Catholicism during, roughly, the period from 1840 till 1960, during which period the Church established many hundreds of new institutions of middle and higher education throughout the Western world and became a cultural and political factor of great significance. This is relevant because the three logicians Jacoby, Sesmat and Blanché were not only Catholics themselves but were part and parcel of the Catholic revival movement. That this is so is shown in Section 4, which describes the life and works of these three men, all three fully integrated into the newly expanded Catholic system of higher education.

1. Standard modern logic and the tradition[1]

Traditional predicate logic is a still unexhausted source of insight into both the foundations of logic and the workings of the human mind and of language.

[1] The logic we discuss here is predicate logic, the logic of quantification, in its various traditional and modern forms. This has to be emphasized straightaway, because many readers

It would have been wise, a century ago, not to reject the over two millennia old logical tradition out of hand but to reconsider it in the context of the new mathematicization of logic initiated by George Boole during the 1840s and brought to its peak by Gottlob Frege and Bertrand Russell from the 1870s till the 1910s. It would then have become clear that traditional predicate logic is a highly interesting and revealing part of modern logic and worthy of serious attention.

Why precisely the traditional Square was banned from the logic court is not entirely clear. Nowhere does one find a clear statement of the reasons, nor is there any published discussion on the matter. What one picks up from the literature is a variety of reasons, none of which, however, cuts wood. One reason gleaned from the literature lies in the metaphysical, psychological and spiritual overtones of the traditional study of logic, which were not shared by the new mathematically minded logicians. This reason, however, is not an argument pertaining to the *logic* of the Square. Other (Eastern) cultures have developed their logical systems, which, though imbued with religion, are, on the whole, treated with benign respect. Another much vaunted reason is the invention of a new logical language with quantifiers and variables, which enabled logicians to deal with syntactically more complex forms of quantification, as in sentences like *On each occasion, all applicants had to produce at least four letters of recommendation*, which contains three quantifiers. While one will immediately acknowledge the great value of the logical languages developed during the 20th century, they do not constitute an argument against traditional logic, since the very same languages can be used for the traditional Square and its elaborations, or indeed for any other variety of predicate logic. This is, in fact, what the present authors do in their own work. What counts is the *meanings* assigned to the logical operators of the language used: when these are changed, different logical systems arise.

Then, standard modern logic correctly claims that the semantics of its universal and existential quantifiers corresponds directly with the set-theoretic relations of inclusion and nonnull intersection, respectively. This,

might think primarily of syllogistic or some other form of (non-monotonic) reasoning. There is a substantial amount of literature (e.g. Beth & Piaget 1961, Johnson-Laird 1983, Brewka 1991, Stenning & Van Lambalgen 2008) on the relation between formal logic on the one hand and psychological reasoning processes on the other. While there can be no doubt that this branch of research is of great value and indispensable for an adequate insight into the relation of language and cognition, it fails to address the questions focussed on in the present study or other studies by the present authors and their few predecessors discussed here on the natural predicate logic of mankind. Yet there is a common philosophy, in that it is agreed that (a) human logic can be formalized and is not a matter of mere associations or stimulus-driven reinforcement and (b) the natural human logical faculty is innate and universal for the species and does not vary from culture to culture. To the extent that the work presented and discussed here differs from the literature mentioned, it stands on its own and has, to the best of our knowledge, no predecessors other than the three discussed in the present study.

however, though no doubt of great value in a mathematical context, is less relevant when one's interest is in the logical workings of the human mind, since it is fairly certain that the set theory implicitly at work in natural human cognition deviates in important respects from that developed in a mathematical context. For example, human cognition does not seem to have a notion of 'null set', in the sense in which it has been developed in standard (Zermelo-Fraenkel) set theory, as appears from the fact that beginning students have great difficulty with this notion and tend to go on for some time making mistakes when operating with it in in their mathematics course (see Seuren 2010, Ch. 3 for extensive discussion). Furthermore, the impressive and sophisticated technical and formal elaborations of modern predicate logic, such as the theory of (non-)monotonicity, are often held up as a sign of victory. Again, this is not a valid argument, since the very same formal prowess could be put to the advantage of the old Square. In fact, an early start was made by the three authors Jacoby, Sesmat and Blanché, who are the protagonists in the present study.

The main reason, however, was and still is, in the eyes of modern logicians, the Square's defect of so-called *Undue Existential Import* (UEI), which means that the system is valid only for domains where the set of entities quantified over—the *restrictor set* or *R-set*—is nonnull. Thus, when I say *All centenarians in the town of Canterbury have received a medal*, this sentence can only be processed in the terms of the Square of Opposition if there are centenarians in the town of Canterbury (the R-set in this example). This is a defect because logic, any logic, is or should be, in its pure form, a system of entailments based on analytical necessity, that is, necessity grounded in the *meanings* of the operators used and thus independent of any contingent circumstances such as the nullness or nonnullness of any given class of entities. Since the Square is not valid for cases where the R-set happens to be null, it is, *prima facie*, defective as a logical system.

This in itself is serious but it does not mean that, therefore, the Square is, logically or otherwise, uninteresting. On the contrary, logically it is of great interest to see what happens to a logical system when the unrestricted set of possible situations forming its universe of discourse **Un** is restricted one way or another, for example by filtering out situations where the R-set happens to be null. When the unrestricted **Un** is restricted, contrariety may become contradictoriness, one-way entailment may become equivalence, subcontrariety may arise where there was logical independence before, etc. In general, **Un** restriction leads to a greater and more diversified supply of logical relations compared with the mere equivalence and contradictoriness found in standard modern logic.

This is, or should be, of interest to the logician, first because the dimensions of logical space have hardly been studied and what has been found so far reveals an intriguing hierarchy of logical systems, and secondly because it is this particular approach that has proved fruitful and relevant in the study of how cognition, and thus language, deals with logic. In this sense, further study of traditional logic, using up-to-date formal methods, opens a window on cognition and language. While it is recognized that the new mathematical logic is a safe guide to the scientific study of the physical world, this does not mean that the human mind makes use of that logic to deal with the mundane details of everyday life. In fact, we contend that, for good functional reasons, the human mind imposes certain well-defined restrictions on the universally valid predicate logic of our modern age, thus making it more suitable for the restricted purposes of daily life. Natural human logic is, in principle, sound and reliable, though not made for the purpose of science. The traditional Square is to be seen as an important stage in the historical process of undoing the restrictions imposed by natural logic on the unrestricted, universally valid and mathematics-based modern system of predicate logic. Mathematical predicate logic is thus not rejected. On the contrary, its achievements are of supreme importance. But it is a mistake to think that the human mind covers the full scope of that logic. The human mind has found its own, highly functional, niche in logical space, developing its own (restricted) variety of predicate logic as an emergent property of the lexical meanings of the quantifiers. Further comment on this important question is provided in Section 2 of the present study.

It is this insight that creates the context of the present study, which is about an underground current of highly original logical explorations carried out during the twentieth century by a small number of philosopher-logicians, in particular the American *Paul Joseph Jacoby* (1915–1993) and the Frenchmen *Augustin Sesmat* (1885–1957) and *Robert Blanché* (1898–1975). They were unhappy with Russell's summary dismissal of the tradition and were convinced that this tradition contained hidden treasures that needed to be unearthed. Though they themselves were unable to forge the breakthrough to language and cognition, and also to a general theory of logic, which we think we have laid the foundations for (Jaspers 2005; Seuren 2010, 2014; Seuren & Jaspers, 2014), they were restlessly searching for it and we, the authors of the present study, consider them to be our forerunners in this respect. Apart from that, however, it seems to us that this aspect of twentieth-century cultural history is significant and interesting in its own right and should, therefore, not go unnoticed.

Let us take a quick look to see in concrete terms what is at issue. To begin with, it has been found by the second author progressively in Seuren (2002, 2006, 2010, 2013) that the traditional Square is one in a group Σ of logical systems, whose properties differ according to whether they are valid for all cases, including those where the R-set is null, or only for those cases where the R-set is nonnull.

The notation used for the sentence types at issue is as follows (“R” and “M” are first-order extensional predicate variables):

A	for	All R is M	**A***	for	All R is not M
I	for	Some, perhaps all, R is M	**I***	for	Some, perhaps all, R is not M
Y	for	Some, but not all, R is M	**Y***	for	Some, but not all, R is not M

The postposed asterisk thus stands for what is known as the *internal negation*. The *external* or *sentence negation* is indicated by the standard symbol ¬ preceding the sentence-type symbol. This gives a total of twelve sentence types, the six specified above plus their external negations. Some of the logical systems discussed below only use eight of these sentence types, leaving out **Y** and its internal and/or external negations, others use all twelve.

In the diagrams, “>” stands for one-way *entailment* (“P entails Q” means that whenever P is true, Q is also true on grounds of analytical—i.e. semantic—necessity but not necessarily also vice versa); “C” stands for *contrariety* (“P and Q are contraries” means that P and Q cannot both be true, though they may be false, at the same time); “SC” stands for *subcontrariety* (“P and Q are subcontraries” means that P and Q cannot both be false, though they may be true, at the same time); a cross or star in the centre, or “CD”, stands for *contradictoriness* (“P and Q are contradictories” means that P and Q can be neither both true nor both false at the same time: the truth of the one implies the falsity of the other, and vice versa); and “=” stands for *equivalence* (“P and Q are equivalent” means that whenever P is true, so is Q, and whenever Q is true, so is P, for analytical (semantic) reasons).[2]

Each pair of vertices in the diagrams is or is not connected by a metalogical relation (entailment, contrariety, subcontrariety, contradictoriness, equivalence). Absence of a metalogical relation (*logical independence*: both propositions can be true together, or false together, or the one can be true while the other is false) is shown as absence of a connecting line. The number of vertices used in a system is indicated by means of a superscript

[2] The diagrams are convenient (but not necessarily minimal) representations defining the logical systems at issue. They are not identical with the systems concerned, whose ontological status is best seen as part of mathematical reality (itself a controversial notion). The systems themselves are defined by the totality of metalogical relations said to hold between any two of the semantically well-defined sentence types (vertices) the logic is predicated upon, including their external and internal negations. The diagrams display those metalogical relations in full. The theorems derivable from the system thus represented are not incorporated into the diagrams. If, in the following, we sometimes create the impression of identifying the systems with the diagrams that define them, this should not be seen as a notional confusion. As regards the term *the Square*, this is used as the name of the system, not of the diagram, on a par with names such as SMPL, AAPL, etc. In other cases, the terminology used is merely a convenient and harmless shorthand: Magritte’s “Ceci n’est pas une pipe” is a mildly amusing truism.

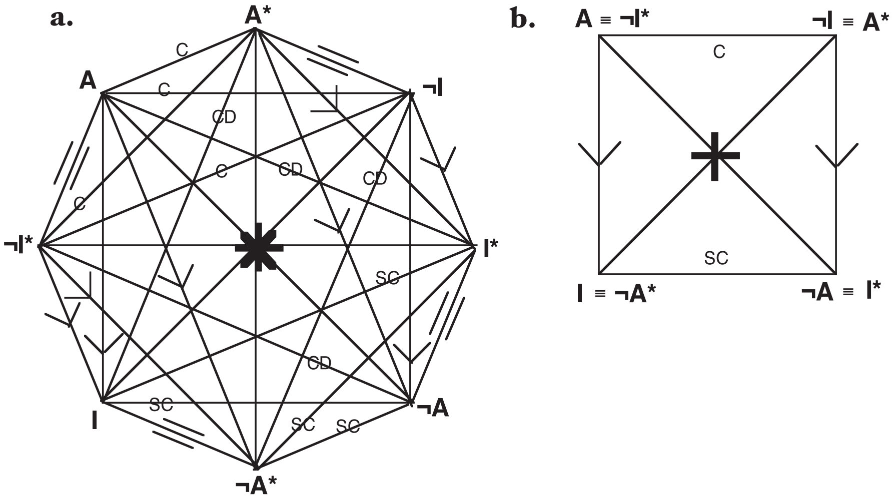


Figure 1. System Σ^{8R} (the Square) (a) as a (complete) octagon, (b) reduced to a square.

digit after the symbol Σ. The restricted variety (valid only for cases where the R-set is nonnull) is characterized by a superscript ^{R} after the digit; the nonrestricted variety by a superscript ^{NR}. The nonrestricted varieties, moreover, are to be distinguished according to whether they assign truth or falsity to **A**-type sentences when the R-set is null. This is symbolized as a superscript + (for truth) or – (for falsity).

The traditional Square and standard modern predicate logic (SMPL) only use eight sentence types: **A**, **I**, **A***, and **I*** plus their external negations. Since the Square is restricted to nonnull R-sets, it represents Σ^{8R}, shown in Fig. 1–a. When the pairs of equivalent types are reduced to one single vertex, the result is Fig. 1–b, that is, the traditional Square representation. One notes that both the octagon and the square are 'complete' in the technical sense that every pair of vertices is connected by a metalogical relation: there are no missing lines.

Figure 2 represents Σ^{8NR-}, that is, the Square extended to cover cases with a null R-set but with falsity assigned to **A**-type sentences when the R-set is null. Seuren calls this system *Aristotelian-Abelardian predicate logic* or AAPL.[3] In Σ^{8NR-} there are no equivalences, so that no reduction is possible.

[3] AAPL represents Aristotle's original concept of predicate logic as reconstructed on the basis of his *On Interpretation*. Aristotle was almost certainly aware of the danger of UEI and took proper care to avoid it. The still widespread belief that the classic Square, with the error of UEI, was Aristotle's product is incorrect. It was his commentators Apuleius, Ammonius and Boethius who, during the first centuries CE, explicitly introduced the Conversions and thus, implicitly, UEI. Much later, the French philosopher-logician Peter Abelard (1079–1142),

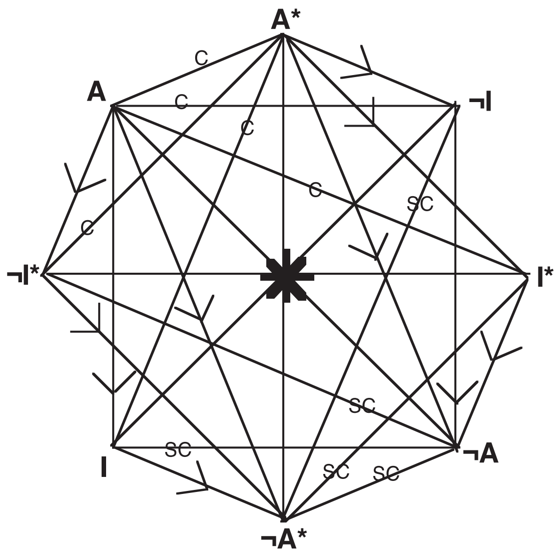


Figure 2. System Σ^{8NR-} (AAPL), not reducible.

The equivalences of Fig. 1–a have been reduced to the one-way entailments from **A** to **¬I*** and from **I** to **¬A***, and thus from **I*** to **¬A** and from **A*** to **¬I**. Σ^{8NR-} (AAPL) is logically sound in the sense that no inconsistencies arise and the system is valid for all possible situations. Σ^{8NR-} is less powerful than Σ^{8R} (the Square), in that some vertex pairs are logically independent.[4]

When the same is done for Σ^{8NR+} or SMPL, often referred to as Russellian logic, a dramatic change occurs, merely due to the fact that **A**-type sentences are considered true in this system. Here, the equivalences and the contradictoriness relations of Fig. 1–a have been preserved and all other relations are lost, in the sense that no pair of vertices in this system instantiates any of the metalogical relations lost (which means that these relations play no role in the definition of SMPL). When all equivalences are pooled, the result is Fig. 3–b. Figure 3 shows that the loss of metalogical relations compared with Figs. 1 and 2 is dramatic. All that remains of the Square are

who read Aristotle carefully and with a good eye for logic, discovered that Aristotle's *On Interpretation* had been misread by the commentators and restored Aristotle's logic according to the original text. Hence the name *Aristotelian-Abelardian predicate logic* (AAPL). The 'tradition', however, preferred to ignore—or failed to understand—Abelard's discovery. See Seuren (2010, pp. 149–156) for a full account.

[4] The technical proofs of the theorems regarding the various logical systems discussed are effortlessly provided by the method of *Valuation Space Analysis*, developed and presented in Seuren (2010, 2013, 2014) and other publications by the same author. The interested reader is referred to those publications.

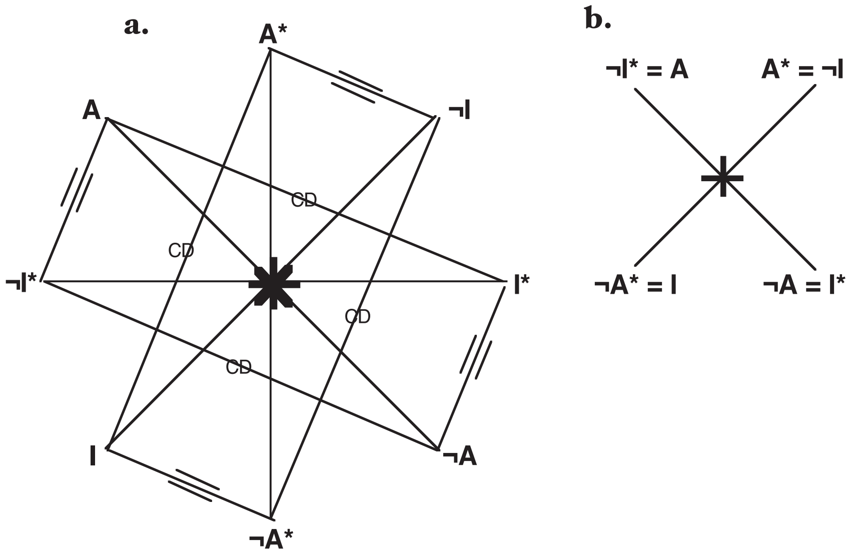


Figure 3. System Σ^{8NR+} (SMPL) (a) as an octagon, (b) reduced to two diagonals.

the two diagonals. This shows that the Russellian system SMPL is drastically impoverished when compared with the classic Square.

The main significance of Jacoby, Sesmat, and Blanché is that they extended predicate logic from the eight sentence types dealt with so far to all twelve sentence types, adding the **Y**-type (plus its negations) with the quantifier ‘some but not all’, thereby incorporating the Hamiltonian Triad of Contraries <**A**,**Y**,¬**I**>, as proposed by the Edinburgh philosopher Sir William Hamilton (1788–1856) in Hamilton (1866), but generally dismissed by modern logicians. This extension shows up some surprising things, only some of which were seen by the three authors mentioned.

Consider first the restricted version S^{12R}, adopted by Jacoby, Sesmat, and Blanché. S^{12R} gives the forbidding (complete) dodecagon of Fig. 4–a, which can be reduced to the (complete) hexagon of Fig. 4–b (as regards the proofs of all the metalogical relations stated, see note 4). This hexagon has meanwhile proved to be the central element in the present authors’ developing theory of natural logic and of lexicalization barriers. To conform to a usage already established, we call it the *Blanché Hexagon*, even though Blanché did not invent it. It was first devised by Paul Jacoby (1950), then by Augustin Sesmat (1951, 1955), and finally, perhaps independently, by Robert Blanché (1953, 1957, 1966).[5] These authors, however, arrived at the hexagon not by

[5] Blanché’s relation to Jacoby and Sesmat is not free from ambiguity. On page 44, note 7, in his (1966), Blanché refers to Jacoby (1950), mentioning, however, only Jacoby’s (Hamiltonian)

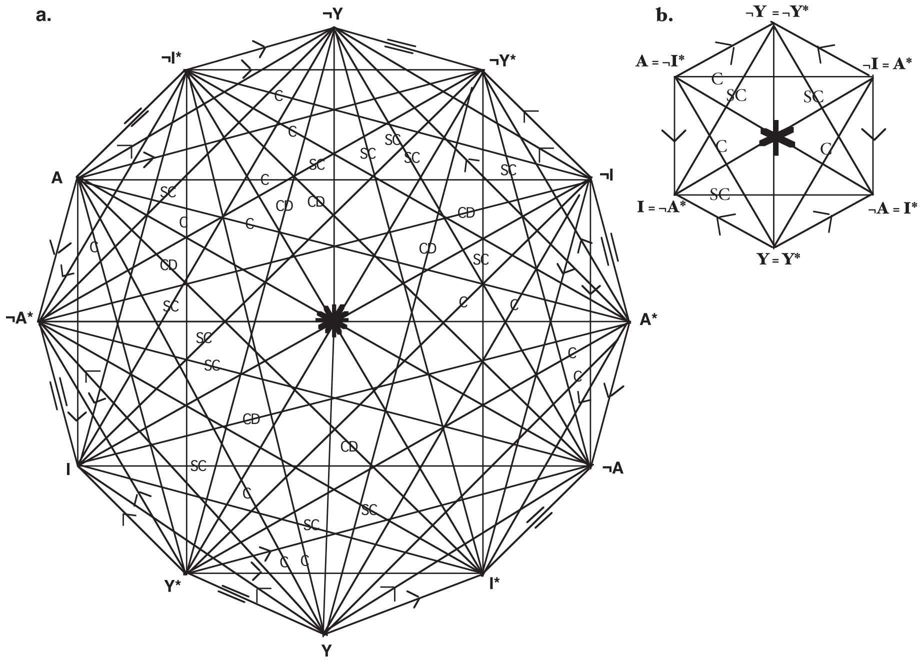


Figure 4. System S^{12R} (a) as a (complete) dodecagon, (b) reduced to the Blanché hexagon.

a reduction of the dodecagon, as is done here, but by extending the classic Square with two new vertices, **Y** for **I** ∧ **I***, expressing the quantifier 'some but not all', and **¬Y** (called **U** by Blanché) for **A** ∧ **¬I**, expressing the contradictory of **Y**.

As it stands, system S^{12R} suffers from UEI, from which it can be redeemed by declaring **A**-type sentences either false or true when the R-set is null. If the former, we have the system S^{12NR-}, shown in Figure 5, which can be reduced to the decagon of Fig. 5–b, there being only two equivalence relations. If the latter, we get system S^{12NR+}, shown in Fig. 6.

The picture arising from all this is that the restricted systems are the richest, always representable by means of a complete polygon, whereas the

Triangle of Contraries, without referring to the fact that Jacoby, in the same (1950, p. 44), had added the contradictories of the three contraries, thus creating what he called the "double triangle"—that is, the hexagon. As regards Sesmat, in Blanché (1953, p. 130, note 24), repeated in Blanché (1966, p. 51, note 2), Blanché apologizes for not mentioning Sesmat (1951), saying that when Sesmat proposed his hexagon, he himself had already invented it independently. That this was indeed the case is at least doubtful, as Sesmat's hexagon of arithmetical relations (≥, =, <, ≥, ≤, ≠), in Sesmat (1951, p. 412) is found, without attribution, in identical form in Blanché (1953, p. 111), and again in Blanché (1966, p. 64)—hardly a plausible coincidence.

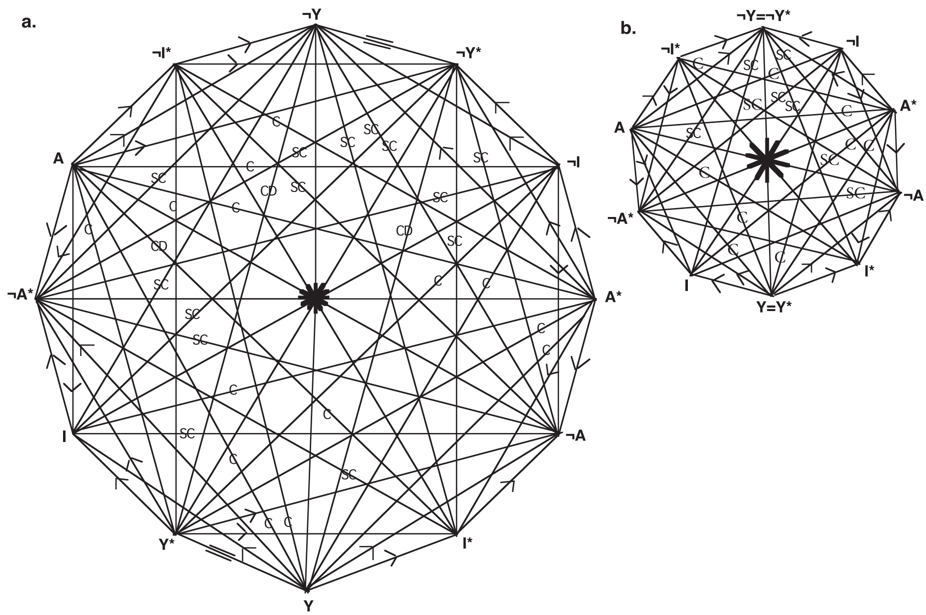


Figure 5. System S^{12NR-} (a) as a (non-complete) dodecagon, (b) reduced to a decagon.

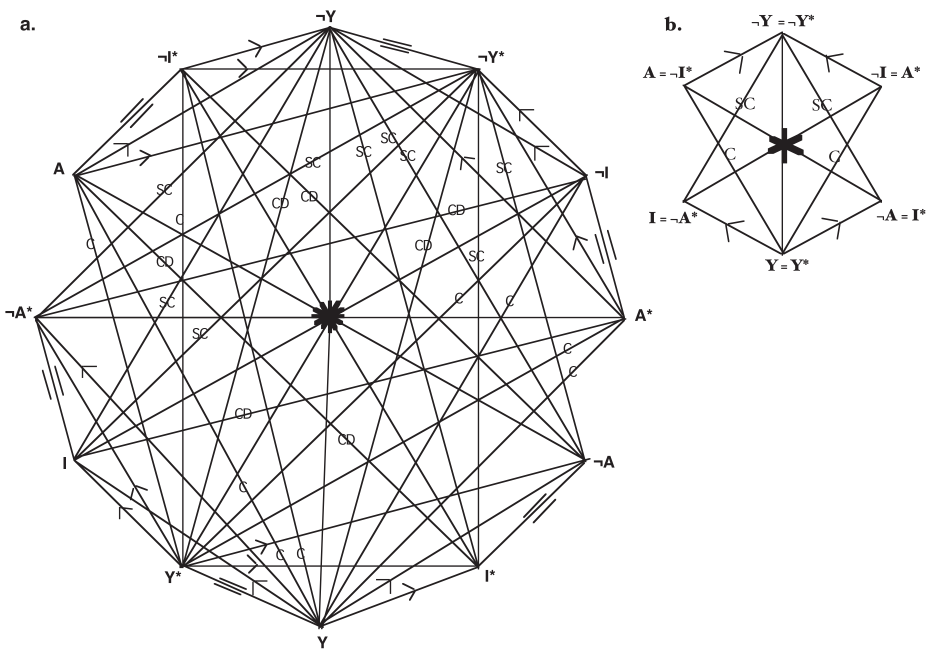


Figure 6. System S^{12NR+} (a) as a (non-complete) dodecagon, (b) reduced to a hexagon.

non-restricted systems all lose logical power, the ones declaring **A**-sentences true when the R-set is null (much) more so than the ones declaring them false. Since falsity for cases where the R-set is null agrees (much) better with natural intuitions, it seems that, if it is our aim to discover the natural logic we take mankind to be innately equipped with, we must look among the restricted logics, based on an analogously restricted natural human set theory (Seuren 2010). In this regard, the Blanché Hexagon is of great importance, since (a) it unites the Square with the Hamiltonian Triad of Contraries <**A**,**Y**,¬**I**> (Hamilton 1866), much despised by modern logicians but a highly intuitive, and sound, logic, (b) it provides a better approximation to natural logical intuitions than SMPL. Yet it is still too rich, as it contains relations of subcontrariety, which are just about the most highly counterintuitive. These are eliminated if both the ¬**Y** ≡ ¬**Y*** and the **I*** ≡ ¬**A** vertices are kept out of business, which, as appears from the following, is precisely what cognition appears to do.

There is an old question, going back to Thomas Aquinas, (Horn 1972; 1989, p. 253; Levinson 2000, pp. 69–71; Jaspers 2005, pp. 1, 213; Seuren 2010, p. 114), regarding the well-nigh universal fact that the **I*** ≡ ¬**A** vertex in the Square (the **O** vertex in the traditional notation) lacks a simple, monomorphemic lexicalization. To explain this systematic lexical gap, Jaspers (2005) proposed a non-logical, cognitive principle in virtue of which natural-language negation selects its complement first in a Universe of Discourse **Un** defined by the opposition between **I** and **not-I**, and then in a more restricted $\mathbf{Un^R}$ defined by the opposition between **A** and **not-A**, in which **I** is taken for granted (presupposed). This hypothesis puts the **O** vertex out of cognitive business and thus makes it an unsuitable candidate for lexicalization. It is confirmed by the fact that similar lexicalization gaps are found systematically all over the lexicons of languages for groups of lexical items that stand in analogous logical relationships (Seuren & Jaspers, 2014). Having finished his (2005), Jaspers noted that the same holds for the ¬**Y** ≡ ¬**Y*** (Blanché's **U**) vertex. This further confirmed the hypothesis, since the complement of **Y** is thereby restricted to the opposition between **A** and **not-A** (now equivalent with **Y**: 'some but not all') (see Seuren, 2014). Cutting out the **I*** ≡ ¬**A** (the old **O**) vertex and the ¬**Y** ≡ ¬**Y*** (Blanché's **U**) vertex from basic, non-sophisticated cognitive processes leaves the quadrilateral *kite structure* shown in Figure 7 as the remaining part of the Blanché Hexagon, taken to be operative in natural cognition. One notes that, in this analysis, the contrariety (C) of **A** and **Y** is contradictoriness (CD) in the restricted subuniverse defined by **I**, **A**, and **Y**, which explains the otherwise inexplicable fact that *not all* is naturally felt to be equivalent to *some but not all*. In this subuniverse, **I** is taken for granted (presupposed), since both **A** and **not-A** entail **I**.

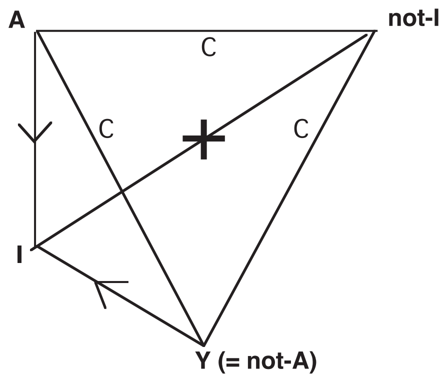


Figure 7. The logical *kite structure* thought to be basic to human cognition.

The intellectual significance of these new insights is, in our view, considerable. Not only is there an intrinsic logical interest in this area of research, the work is also directly relevant to the study of cognition and to the field of language studies as it is today, in particular with regard to semantics and pragmatics. It has been uncontested during the past century that modern mathematical logic does not fit natural logical intuitions at all well. The 'official' view nowadays is that pragmatic principles, in particular the conversational maxims devised by the British-American philosopher Paul H. Grice (1913–1988), suffice to bridge this gap, which allows philosophers, linguists and pragmaticists to continue without bothering about logic, and logicians to continue without bothering about the mundane contingencies of language and cognition. If logicians do bother about the latter, it is because they consider them a challenge for their formalization prowess—a concern one might consider marginal.

One might feel, as the present authors do, that this division of labour is inadequate and that logic itself should be taken by the horns, in conjunction with a general theory of grammar. In the footsteps of Jacoby, Sesmat, and especially Blanché, one might look for a more adequate answer in the logical faculty that the human species is cognitively endowed with and which is intimately connected with the way the faculty of language is organized. A formal system of natural logic would thus take the place of what is now standardly believed to be the answer: a 'pragmatic' play of mutual expectations in linguistic interaction. To the extent that the logical alternative, explored by Jacoby, Sesmat, and Blanché and further developed by the present authors, is valid, it will have an impact on the overall structure and content of current linguistic theory and open a new perspective on the relation between logic, language, and cognition. Seen from this angle, it seems fully justified to provide some historical context.

2. Modern logic in a wider historical context

No-one will deny that the 20th century brought unimagined scientific and technological advance, as it took us from atoms to particles, from the steam engine to jet-propelled aircraft, from the exploration of Africa to that of outer space, from the dip pen to the computer, from stamped envelopes to email, from the reign of disease to public health and longevity, and also from Aristotelian to modern logic. Yet only few have realized that this enormous scientific and technological progress went hand in hand with a dramatic loss of appreciation of the complexities and depths of the human mind and of human nature generally. As the secrets of physical matter were being unveiled, those of the mind were covered up. The psychological doctrine of behaviourism, as we know, even went so far as to deny the existence of the mind, or at the least to exclude it from the remit of science.

Yet this impoverished view of humanity was perhaps a necessary stage on the road to a scientific study of the mind. As from the 17th century, the physical sciences had developed a highly successful view of the physical world, including the human body, as a system of interconnected 'machines', the well-known 'mechanization of our world picture' (Dijksterhuis 1961). From the late 19th century on—in the wake of Hippolyte Taine's (1828–1893) masterpiece *De l'intelligence*, which appeared in 1870, with multiple editions till well into the 20th century but now almost forgotten—the same 'mechanization' view began to be applied to the mind, which came to be seen as a complex 'machine', to be unearthed by hypothesis and experimental testing.[6] This was the origin of structuralism in the human sciences (see Seuren 1998, pp. 141–44). The 'mental machinery', however, could only be called that in a metaphorical sense, as it lacked a direct mapping onto hardware neurological structures and processes.[7] All that could be achieved

[6] One of Taine's main themes was the parallelism of mental occurrences with, or their reduction to, physiological processes, assuming mental mechanisms that are inaccessible to consciousness or awareness. See, for example, Taine (1878, vol. I, p. 188):

> There thus comes about in us an infinite underground work, of which only the products come to our conscience, and that only in rough outline. As regards the elements and the elements of the elements, these are out of reach for consciousness; we come to them by reason; they are to sensations what the secondary molecules and the primitive atoms are to bodies; we only have an abstract idea of them and what represents them for us is not an image but a notation.

Others, in particular Taine's contemporary, the German Wilhelm Dilthey (1833–1911), felt that the study of the mind would become scientific not by taking over the 'machine' view but by a deep 'verstehen', an introspectively felt understanding or recognition of what goes on in the mind. This methodology, which led to an enormous amount of Continental European philosophy and, partly doubtful, psychology, including Freudianism, has now been generally discredited as being potentially therapeutical but in principle unscientific.

[7] Taine, in his *De l'intelligence*, did all he could to establish such a mapping, but one has to admit that this problem has remained unsolved till the present day, despite our powerful new brain scanning machines.

was a 'software' realism, whose reduction to hardware realism has so far proved elusive (see Seuren 2009, pp. 8–18).

In this perspective, it is easily understood, perhaps even justifiable, that in behaviourist psychology one wished to start with the simplest possible hypothesis for the explanation of observable human behaviour, a mechanism of stimulus-response associations—that is, behaviourism. Until that time, psychology had not quite managed to rid itself of the 'soul' as a metaphysical entity of some kind explaining behaviour and it was obvious that the 'soul' would not satisfy positivist criteria of science. Due to the general admiration for, and overestimation of, anything 'scientific' in a positivist sense, behaviourism quickly became a belief and an ideology, which spread through all the human sciences and even through the worlds of advertizing and politics, leaving in its wake a record of mass manipulation, cruelty and crime perpetrated in its name on individuals and societies.[8] If only it had not been turned into an ideology and had been recognized for what it was, a mere scientific hypothesis set up to explain a restricted set of behavioural phenomena!

An important correction came during the 1950s and 1960s, with the birth of cognitive science, when it was realized that the minimalist hypothesis of behaviourism had to be given up in favour of a richer hypothesis admitting of specialized computational processes in the mind and specific autonomous levels of functioning. Concurrently, the hubris of behaviourism with regard to human nature gave way to a wiser, more appropriate and humbler attitude of awe and caution and a recognition of the vastness, the richness, and the mysteries of the human mind.[9]

Logic followed this pattern with a precision that befits the discipline. During exactly the years that the study of the mind was striving to become 'scientific' (in a positivist sense), logic underwent a process of mathematicization, whereby the discipline moved out of the humanities and into the realm of mathematics. Traditional predicate logic, that is, the Square of Opposition, was thrown overboard, much in the way traditional psychology was rejected by behaviourism, and a new, mathematical logic took its

[8] It should be realized that behaviourism had its sinister sides too, especially as regards mass manipulation techniques, including politics. Modern techniques of mass manipulation, euphemistically called "public relations", were developed mainly on the strength of a behaviourist philosophy of human nature, opportunistically mixed with Freudian elements. The biographies of the early developers of mass manipulation techniques speak volumes. Vivid examples are Edward Bernays (1891–1995), nephew of both Sigmund Freud and Freud's wife Martha Bernays, whose writings were, ironically, admired and avidly studied by the Nazi propaganda leader Joseph Goebbels, or Walter Lippmann (1889–1974), who held that the general public should be (mis)led because it is too ignorant and fickle to be taken into account, who invented the term *manufacture of consent*, and who advised several US presidents.

[9] See Levelt's monumental *History of Psycholinguistics* (2013) for ample discussions and a wealth of historical data.

place—a process in which, of course, the figure of Bertrand Russell (1872–1970) loomed large. Yet compared with behaviourism, the new logic had the distinct advantage of being correct, as the simple and metaphysically necessary theorems of the new mathematical set theory had been grafted one-to-one onto the semantics of the corresponding logical language. Unlike behaviourism, therefore, it has survived, but, with the advantage of hindsight, we must recognize that its status of unassailability produced an inordinate degree of hubris and imperiousness in its practitioners.

This attitude left no room for a dispassionate comparison of the new logic with its traditional predecessor, the Square of Opposition, in the light of an overall general theory of logic. There is little awareness in present-day logical circles of the extreme poverty of modern predicate logic, which is defined in terms of only the metalogical relations of equivalence and contradiction (see Fig. 3), no one-way entailments and no contrarieties or subcontrarieties. In particular, no account was taken of the possibility that the human mind, as part of a supervenient level of mental reality, might well work with a logic subject to less simple, metaphysically contingent (that is, empirical) restrictions and principles serving the mundane interests of daily life more than those of metaphysics or the study of physical matter. Again, as in psychology, the mind became the victim of an extreme and uncompromising reductionism, as it was reduced to a general, non-autonomous mechanism grounded merely in the mathematics of physical matter. Among professionals, and also in the eyes of the world at large, the new Russellian logic became so inviolate that one could not even imagine the human mind as a practically functional logic processor adaptable to an infinite variety of limited domains and specific purposes. Consequently, one had no idea of the richness and structural elegance of, and the new horizons opened by, such variably restrictable logical systems. The impoverished view of the human mind, current in behaviourism, matched the impoverishment of Russellian logic compared with its much richer traditional predecessor and the as yet hidden treasures contained in it. It was this barrier that Jacoby, Sesmat and Blanché tried to break through.

So did, in a way, Ludwig Wittgenstein, who, at the breaking point between his 'old' (Tractatus) and his 'new' (Investigations) self—that is, around 1930—reflected on the fact that lexical predicates have their own 'spaces' in which logical relations hold. His examples mainly center around colours and sounds. Bertrand Russell allows us a revealing look into the kitchen in a letter he wrote to the Cambridge philosopher George Edward Moore on May 5th, 1930 (Russell 1968, p. 198):

> I had a second visit from Wittgenstein, but it only lasted thirty-six hours, and it did not by any means suffice for him to give me a synopsis of all that he has done. […] I think, however, that in the course of conversation with him I got a fairly good idea of what he is at. He uses the words 'space' and 'grammar'

> in peculiar senses, which are more or less connected with each other. He holds that if it is significant to say 'This is red', it cannot be significant to say 'This is loud'. There is one 'space' of colours and another 'space' of sounds. These 'spaces' are apparently given a priori in the Kantian sense, or at least not perhaps exactly that, but something not so very different. Mistakes of grammar result from confusing 'spaces'. […] *His theories are certainly important and certainly very original. Whether they are true, I do not know; I devoutly hope they are not, as they make mathematics and logic almost incredibly difficult.* One might define a 'space', as he uses the word, as a complete set of possibilities of a given kind. If you can say 'This is blue', there are a number of other things you can say significantly, namely, all the other colours. (italics ours; DJ/PAMS)

We, the present authors, read this as telling us that Wittgenstein was intrigued by the fact that human cognition operates in terms of restricted universes of discourse ($\mathbf{Un}^{\mathbf{R}}$s)—his 'spaces'—but was unable to cast his thoughts into anything resembling a fully coherent, let alone a formal, system. Russell accepted the validity of Wittgenstein's point of view, at least privately, recognizing that this would be a serious complication for logical theory, and did not see his way through either.

Our three heroes were on to the same set of issues. Their attempts were made during the 1950s and 1960s, the very same period in which cognitive science came into being, though the latter had incomparably greater force and prestige. Unlike the angry protests by the founders of cognitive science, such as Karl Lashley (1890–1958), Jerome Bruner (b. 1915), George A. Miller (1920–2012), or Noam Chomsky (b. 1928), the protests by the 'angry logicians' were not heard. When Jacoby, Sesmat, and Blanché, and who knows others still to be discovered, put up their proposals, urging the world to look again, with new eyes, at the old traditional logic, their feeble voices went unheard and their names were forgotten. Nobody knew about them until their rediscovery by Horn (1990)[10] and others, notably Vernant (2009) and Moretti (2009) (both of whom mention Sesmat and Blanché, but not Jacoby). None of our three innovators has had a following.[11] Even finding out elementary biographical details turned out to be a major undertaking.

[10] Blanché is first mentioned in Horn (1989, p. 254), but only in passing and without reference to his efforts to uphold and expand traditional logic. Jacoby and Sesmat are first introduced in Horn (1990). The most recent and fullest treatment is Horn (2012).

[11] Perhaps an exception should be made for the French Canadian Pierre Sauriol, who published two articles in *Dialogue*, the journal of the Canadian Philosophical Association, on the Blanché Hexagon (Sauriol 1968, 1976). We refrain from commenting on Sauriol's work, interesting though it may be, as it aims at further possible extensions of the Blanché Hexagon, encompassing trivalent varieties represented by three-dimensional diagrams. This research programme does not seem relevant to ours, which aims at a reconstruction-by-hypothesis of natural human predicate logic and results in a restriction, rather than an extension, of the Blanché Hexagon. It should be noted, however, that Sauriol's publications show that he taught at the École Normale Jacques-Cartier in Montreal, founded in 1857 by the

3. Catholicism between 1840 and 1960

All three, Jacoby, Sesmat, and Blanché, were ardent members of the Catholic Church—a highly significant element in the unfolding story. The Catholic Church was the only institution that did not go along with the new Russellian logic and kept teaching traditional logic in its schools and seminaries until the 1970s, much to Russell's dismay, who wrote, during the early 1940s (Russell 1961, p. 206):

> Even at the present day, all Catholic teachers of philosophy and many others still obstinately reject the discoveries of modern logic, and adhere with a strange tenacity to a system which is as definitely antiquated as Ptolemaic astronomy.

Russell's annoyance will have been enhanced by the fact that, by the time he wrote, Catholicism in Europe and the United States was at the apogee of a century of revival that had started in the 1840s. He could not know that this revival was to come to an abrupt end during the 1960s.

It is worth expanding a little on this, even if the connection with logic may seem remote. After the democratic revolutions of the 1790s, and again of 1848, the Church was in a bad state. Having traditionally supported the *ancien régimes* of the aristocracy, with a merely charitable interest in the well-being of ordinary people but no regard for their legal rights or economic well-being, the Church was rapidly losing credibility, and thus popular support. To regain lost ground, it gradually changed its policies, drastically reducing, or covering up, its identification with the aristocracy and concentrating more on winning the hearts and minds of the masses of ordinary citizens. And it did so with great cunning and psychological insight, using a large variety of means. The number of bishop seats and seminaries was drastically increased, especially in non-Catholic countries and their colonies. Sanctifications and beatifications of 'ordinary' people not belonging to the aristocracy and often not even priests were the order of the day. Many new 'congregations' (the new word for religious orders) were founded. Thousands of new churches were built. A large array of semi-ritualized forms of personal piety and devotion was created or recreated. Catholic participation in the political processes of democracy was stimulated and, concurrently, Catholics were put under heavy pressure to have large families so that Catholics would gain a larger share of the votes. Generally, a sense of identity was created, built on the Catholic Middle Ages.[12] But above all,

Sisters of the Congregation Notre Dame, and later at the University of Montreal, founded in 1878 as an offshoot of the Catholic Université Laval in Quebec (see note 14).

[12] The Church strongly aligned itself with the widespread Romantic revival of the Middle Ages taking place all over Europe, where new nationalisms needed, and forged, a historical base not provided by Greek and Roman antiquity (Raedts 2011). One finds the 19th-century medieval revival monumentally expressed in the often magnificent specimens of neo-Gothic

large numbers of Catholic schools, colleges and universities were established, both in Europe and in America, so as to create a Catholic élite and thus to exert maximal control over the education of those who would occupy leading positions in society. In sum, the Church was aiming, if not at a restored absolute dominion, at least at the building of strong Catholic bastions all over the world.

The policy worked, at least for the time being. During the first half of the 20th century, Catholicism was expanding and prominently present in all spheres of life, not least in the *haute culture* of literature and the humanities, especially philosophy, though less in the natural sciences. Around 1950, Catholicism was a force to be reckoned with. After 1960, this Catholic revival proved self-destructive in that the moral and intellectual emancipation brought about by it turned back on it, leading to a sudden and massive wave of disaffection and defection, especially in Europe and especially among the educated classes.

The Frenchman Jacques Maritain (1882–1973) (now being considered for beatification) was the intellectual hero of the day, teaching intensively in both France and the United States (mainly at Princeton from 1949 till 1960) and producing many volumes on Catholic philosophy. His philosophy was Thomistic in principle, with an admixture of modern French existentialism. His logic was nothing but late 19th-century traditional logic, without any original idea and without any serious discussion of the new mathematics-based logic, which he despised.[13]

architecture, found side-by-side with equally magnificent neo-classical buildings of the same period (such as the monumental Palace of Justice in Brussels, built between 1866 and 1883). The British Houses of Parliament, erected between 1840 and 1870, are an impressive example of neo-Gothic architecture, as are many British schools, colleges and hospitals of the period. In Amsterdam, the Catholic architect Pierre Cuypers (1827–1921) built, besides some 70 churches all over the country, the Rijksmuseum (1876–1885) and the Central (Railway) Station (1881–1889), all in his own version of the neo-Gothic style. The Rijksmuseum was commissioned and built against stiff resistance of many prominent Dutchmen (including the protestant king Willem III), who objected to a piece of 'Catholic' architecture in the service of the state (Raedts 2011, pp. 227, 258).

[13] We read, for example (Maritain & Maritain 1987, vol. II, p. 651) (translation by the authors):

> [W]hat is called nowadays the *algebra of Logic* refers to a certain art by means of which the work based on reason is replaced with the regulated manipulation of ideographical signs (Logistics), a discipline whose foundations are alien to the true Logic, or the art of rational thinking, and are, in fact, for most Logisticians, derived from a general concept (the "Logic of Relations"), which destroys any sound philosophy of reasoning. Yet nothing stands in the way of an entirely different—and much more modest—*logical algebra*, which conforms to the principles of traditional Logic and which places at the logician's disposal a system of artificial symbols especially adapted to the reflective analysis of actual reasoning.

He then goes on, for the next three pages, to give a demonstration of such a 'logical algebra' for the theory of syllogisms, not for predicate logic.

Yet "at the Catholic universities in the United States in the mid-1930s, logic was completely dominated by the writings of Jacques Maritain and the philosophical school of Laval University,"[14] which was "quite opposed to mathematical ('merely formal') logic" (Menger 1994, p. 216). These doctrines were taught in all Catholic seminaries and in the philosophy courses for aspiring 'regular' priests living under the rule of an ecclesiastical order, society or congregation. They were also taught in the many newly founded Catholic colleges and universities for the laity, about which more below, though more room was left there for other, more up to date, approaches and points of view.[15] Russell's exasperation is understandable.

The main centers of the new Catholic cultural power were France and the United States, with Great Britain as a strong third. The link between France and the United States went back to the very foundation of the latter as an independent republic in 1776, its Constitution being based on 18th-century French political thought. A special connection, however, consisted in the fact that France had colonized, and at one time possessed, the entire territory from Louisiana in the south fanning out to the northern states of Montana, North and South Dakota, Minnesota, Wisconsin, Illinois, Indiana, and Michigan, and further into Canada all the way up to Quebec—a vast French rainbow across the middle of North America. By the well-known Louisiana purchase of 1803, those territories that still belonged to France, up to the Canadian border, came under the sovereignty of the United States, but many traces of the French past are still there today. They are found not only in the many French place names and French spellings of Indian names (*Chicago*, *Michigan*), but also in the relatively large proportion of Catholics in that area.[16]

[14] Laval University was founded (given a charter) in Quebec in 1852, at the instigation of the Church, as a continuation of the *Séminaire de Québec*, which dated back to 1663 and had been founded by the first bishop of 'Nouvelle France'. During the early 20th century, its philosophy department was a center of staunch opposition to modern forms of philosophy, including logic.

[15] Thus, the University of Notre Dame, founded in 1842 by the Congregation of the Holy Cross and situated in South Bend, Indiana, east of Chicago, showed its independence *inter alia* by inviting, in 1938/9, at the behest of the Austrian mathematician Karl Menger, the non-Catholic, but equally Austrian, mathematician Kurt Gödel (1906–1978) to a professorship in logic and mathematics, a position he kept for one year before returning to Austria to be reunited with his wife. Soon after, in 1940, Gödel and his wife emigrated for good to the United States, where he took up a position at Princeton (see Dawson 1997).

[16] For the statistics see:
http://en.wikipedia.org/wiki/Catholic_Church_in_the_United_States#Catholicism_by_state
Catholicism is the largest single denomination in the US, forming roughly one quarter of the total population. The relatively strong presence of Catholicism in the North-Eastern states is accounted for by the 19th-century Irish, German, Polish and Italian immigration. In Texas and California it is, of course, by the former Spanish presence and the many immigrants from Mexico.

As has been said, part of the Church's strategy was the massive founding and funding of Catholic schools, colleges and universities, especially in France and the United States.[17] In France, the passing of a law in 1875 guaranteeing freedom of education was immediately, in the same year, followed by the foundation of no less than five Catholic universities, in Lille, Rennes, Angers, Lyon, and Paris, and two years later, in 1877, by the foundation of the Catholic University of Toulouse (Institut Catholique de Toulouse). Some of these were new foundations, some were revivals of existing ones or ones that had been abolished earlier. Some were established by the French bishops, some by private individuals, often Catholic businessmen. In the US, we see the foundation of over 200 Catholic colleges and universities during the period concerned, roughly one quarter of which in the French-colonized territories.[18]

It was the teachers and professors of philosophy in these institutions that Russell referred to when he spoke of the "Catholic teachers of philosophy" still "obstinately reject[ing] the discoveries of modern logic." And it was in these institutions that Jacoby and Sesmat were working—Jacoby at Seton Hill College in Pennsylvania, now Seton Hill University, founded in 1885 as a Catholic college for women students, Sesmat at the Catholic University of Paris, founded in 1875 as mentioned above. Blanché was philosophy professor at the University of Toulouse, founded in 1229 by a collection of French bishops and the second university in France after the Sorbonne in Paris, but he entertained close ties with the Catholic University of Toulouse, founded in 1877 and referred to in the previous paragraph.

The lack of recognition, however, that befell Jacoby, Sesmat, and Blanché did not mean that there was no interest in traditional logic. On the contrary, one corollary of the Catholic revival was a widespread interest in medieval philosophy and logic, mostly in the Catholic universities. But that interest was largely historical and those attempts that were made at extension and innovation, were, on the whole, modest and not very creative.

In Europe, medieval logic began being studied intensively during the second half of the 19th century and even more after the two world wars. This led to a massive labour of text editions of medieval philosophical and logical texts, making the arcane world of medieval philosophy accessible to wider circles of modern scholars. (Catholic) Poland was an important center. An obvious name, in this regard, is that of Józef Maria Bocheński (1902–1995), a Polish Dominican priest, who served not only as a chaplain in the Polish army during World War II, taking part in the battle of Monte Cassino in Italy in 1944, but also as consultant on matters of Russian communism

[17] Somewhat later, during the first decades of the 20th century, this policy was extended to South America, Japan, the Philippines, Indonesia, and other parts of the world.

[18] Information garnered from the internet.

to several Western governments after that war. Professor of philosophy at the University of Fribourg in Switzerland, he lectured all over the world and published widely on politics and on modern and traditional logic and its history. But, as far as we know, he did no original, innovative work in traditional logic.

In England, we encounter not only the Jesuit priest Frederick C. Copleston (1907–1994), philosopher of international fame, especially known for his nine-volume *History of Philosophy*, in which much attention is paid to the Middle Ages, but also, more to the point, Peter Thomas Geach (1916–2013), who had a Polish mother and converted to Catholicism as a young man. Geach developed a then uncommon interest in medieval logic, trying, without much success, to bring it in line with the new Russellian logic. Likewise in France, where we see a sudden enormous increase in historical studies of medieval philosophy and logic, but hardly any innovations (except for Sesmat and Blanché). Maritain, as we have seen, was prototypical in this regard.

In the United States, again, historical studies flourished, as they still do today. But apart from the isolated figure of Paul Jacoby, whose light was kept hidden under a bushel, the only attempt at an innovation of the system came from Ernest A. Moody (1903–1975), again a Catholic, whose father (the founder of Moody's rating agency) had converted to Catholicism. Moody's concern, in the study of logic, was to save the Square of Opposition from the disaster of UEI, mentioned at the outset of Section 1. His answer, soon followed up by large sections of the US world of medieval philosophical studies but not or hardly by the professional logicians, consisted in postulating that what is traditionally called the **O**-corner of the Square (as exemplified by *Some flags are not green* or *Not all flags are green*) lacks existential import in that it is taken not to entail the existence of any flags—the so-called 'leaking **O**-corner' analysis (see Seuren 2010, pp. 158–70; 2012). At first sight, this has the appearance of saving the Square, which, on this interpretation, looks as if it has become logically sound while remaining intact. That it stridently violates the natural reading of such sentences was admitted by Moody and his many followers, but considered to be of no great relevance: a sound logic was taken to be more important than natural semantic intuitions (that the latter require an explanation too was sadly overlooked).

In addition, Moody argued, on false historical grounds, that the main currents in the ancient and medieval history of logic sustained his interpretation of the Square (see Seuren 2010, pp. 158–70 for a refutation of this totally false historical claim). Moreover, as argued in Seuren (2010, 2012), the Moody solution does not work because it makes the existential quantifier *some* logically ambiguous, in that *some* is taken to induce existential import (the non-nullness of the R-set) when used with a non-negated predicate while it would have this existential import taken away from it when used to quantify over a set of entities defined by means of the explicit use of the negation word *not*. Thus, *Some boys like motorcycles* and *Some boys dislike motorcycles* would both entail the existence of boys, while *Some boys do not like motorcycles* would not.

The question is, of course, why Moody received such a large hearing, at least in the American world of medieval logic and to some extent also among American philosophers of language, while Jacoby was completely ignored. We have studied Jacoby's few publications and also his private notes, kindly made available to us by his son Leo Jacoby, and we found no logical mistakes (as we did in Moody's work). What we did find was a continuous searching and trying out but a lack of solid, generalizing conclusions. So perhaps the reason is to be found in Jacoby's failure to take a stand and make himself heard. But we believe that there was a deeper reason. Whereas Moody's solution was, on the face of it, easy to understand and had the effect of making everyone believe that all was well, Jacoby's work, far from having an appeasing effect, was innovative and thus required intellectual effort and a rethinking of basics. His work implied no correction or re-interpretation of history but rather opened new doors to logical theory, which was of less concern to the historians but should have made the professional logicians prick up their ears. These, however, were (and to some extent still are) just not open to this kind of innovation which would have the effect of dethroning Russell, with all the consequences thereof.

A similar answer holds with regard to the ignoring of Sesmat and Blanché in France. They too failed to have an impact on the world of logic.[19] Again, the numerous French medieval historians were hardly interested in the innovative work of their two logical colleagues, while the French formal logicians, if they had heeded the two dissident logicians—which they did not—would themselves have had difficulty to be heard outside France. Their attitude, moreover, with regard to traditional logic was extremely hostile, if we may go by Louis Couturat's arrogant and condescending reaction

[19] An isolated exception is Van Heijenoort's short appreciative comment in *The Journal of Symbolic Logic* of 1959 on a 1957 short article by Blanché in the same journal. (Van Heijenoort's equally short comment on Sesmat (1951) in the 1960 volume of the same journal was not at all appreciative, clearly expressing the ideology-driven mutual animosity between the mathematical logicians and those who defended the old paradigm.) Jean van Heijenoort (1912–1986), whose Dutch father died in 1914 as a result of conditions during World War I, was brought up by his French mother in France and did not move to America until the mid-1930s, first to Mexico as Leon Trotsky's private secretary and bodyguard (having earlier joined Trotsky in Turkey), then, just before Trotsky's assassination in 1940, to the United States as a student, and later a professor, of logic and mathematics. He was himself assassinated by his estranged wife in Mexico City in 1986 (see Feferman 2001 for Van Heijenoort's unusual life story).

(Couturat 1913) to Ginzberg (1913).[20] In sum, our three protagonists were ignored both by the establishment of formal logicians and by the historians of logic, by the former because they were undermining the ideology and the absolute dominance of the Russellian tradition, by the latter because their work, being innovative, hardly had any historical interest. For each of the three, moreover, there are special explanations for their not having sought wider publicity. Jacoby was a timid and modest man who felt unequal to the great names; Sesmat was not primarily a logician but a religiously inspired philosopher with an interest in the physical sciences, without worldly aspirations; Blanché was neither timid nor unworldly, but proved unable to reach the Anglo-Saxon world—a necessary condition for having an impact.

4. Who were Jacoby, Sesmat, and Blanché?

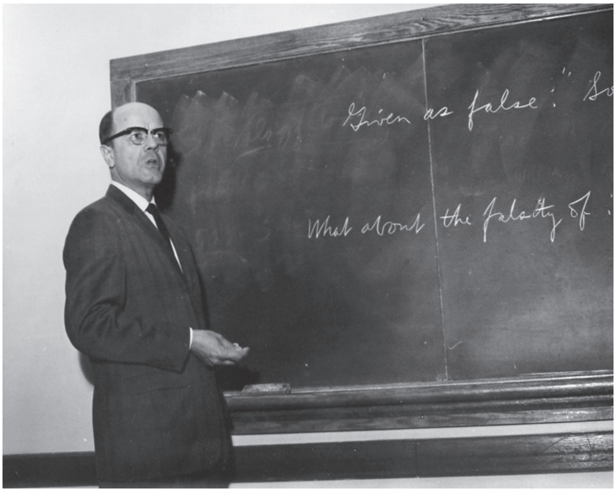
4.1. *Paul J. Jacoby (1915–1993)*

Paul Joseph Jacoby was born in Hannibal, a port town along the Mississippi River in Missouri, on August 31, 1915, first child of Henrietta Glass (1884–1980) and Joseph George Jacoby (1887–1973), whose father, George Jacoby (1847–1930) was born in southern Germany and emigrated to America in 1868, where he settled in the town of Hannibal and set up a bakery, later taken over by his son, Paul's father, Joseph (the slogan of the "Jacoby and Son Bakery" was "Jacoby's peerless bread"). Paul had two

[20] S. Ginzberg is a man we have not been able to trace. Even his full first name could not be detected (unless he is identical—which we doubt—with the Salomon or Simon Ginzberg who acted as Albert Einstein's ad hoc secretary during the latter's trip through the United States in the spring of 1921). In any case, this S. Ginzberg had proposed to reinstate Hamilton's existential quantifier 'some but not all' as a logical operator alongside the standard 'some perhaps all'—a central element in the explorations of Jacoby, Sesmat, and Blanché. Louis Couturat, the well-known philosopher-logician and follower of Bertrand Russell, reacted acerbically in (Couturat 1913), the opening sentence of which runs as follows: "The problem raised by Mr Ginzberg […] was solved a long time ago by formal logicians in a way that seems to me to be satisfactory and decisive." In his rejoinder (Ginzberg 1914), Ginzberg, quite rightly, pointed out that no logical principle prohibits the introduction of a quantifier meaning 'some but not all' and that such a quantifier, being more precise than 'some perhaps all', has certain functional advantages. This left Couturat with no other argument than to say, at the end of his short final repartee (translation by the authors):

> [O]ne steps out of the framework of classical logic the moment one tries to make propositions 'quantitatively' precise. One may well elaborate alternative legitimate and consistent systems, but one cannot make the classical system more 'perfect' by introducing greater 'precision', which is alien to its spirit.

This last-resort appeal to the "spirit" of "classical logic" shows the weakness, and indeed the untenability, of Couturat's position in this debate. Between the first sentence of his (1913) and the last sentence of his (1914), he had to climb down from his high horse. In fact, he lost the discussion.



younger sisters, Marietta (1917–1965), who never married and became an elementary school teacher, and Margery (1922–2009), who worked as a secretary in the Navy Department and for the Red Cross during World War II and later as the executive secretary for the director of the Federal Civil Defense Administration in Battle Creek, Michigan, during which time she witnessed a testing of the atomic bomb. She did not marry until 1965.

Having taken his final exams at the McCooey Memorial High School at Hannibal, which he attended from 1929 till 1933, Paul Jacoby passed on to Quincy College and Seminary at Quincy, Illinois. This school, renamed Quincy University in 1970, had been founded in 1860 by the Franciscans. There he took an A.B. degree in chemistry in 1937. In a letter addressed to the first author of the present study, dated February 4th, 2012, Leo Jacoby, Paul's second son, writes: "Perhaps diagramming organic chemistry problems planted seeds for visualizing logic issues in similar fashion." He then spent a year privately reading philosophical and logical works.

In September 1938, he enrolled at Notre Dame University as a PhD-student in philosophy, where he obtained his doctoral title in May 1942. His dissertation was entitled *Common Sense in Epistemology*. His supervisor there was Yves René Marie Simon (1903–1961), student and friend of Maritain at the Institut Catholique de Paris. Among his other mentors was the Holy Cross Rev. Leo R. Ward (1893–1984), professor at Notre Dame, a family friend and specialist in ethics and in the philosophy of Jacques Maritain. Jacoby's second son Leo was named after this Leo Ward, as he told the first author in the letter quoted from above.

At Notre Dame, Jacoby will have met also the Austrian mathematician Karl Menger (1902–1985), mentioned above, a personal friend of Kurt Gödel (see note 15) and professor of mathematics at Notre Dame from 1937 till 1946, before moving on to the Illinois Institute of Technology. There is a published discussion between Yves Simon and Karl Menger, chaired by Leo Ward, on the topic of "Aristotelian demonstration and postulational method" (Simon and Menger 1948), where the two discussed questions of scientific epistemology or, if you like, faith and reason. Though the discussion was not in any way adversarial, the two discussants were totally at cross purposes, the one defending a somewhat mystical Aristotelian-Thomistic point of view, the other giving a lecture on formalization in mathematics and physics—which shows the enormous intellectual and ideological gap between the two, unbridgeable even in the friendliest of atmospheres.

Having finished his PhD, Jacoby worked in two companies as an 'Administrative Supervisor' and 'Chief Chemist', respectively. Then, in 1945, he obtained a post as philosophy instructor at Fontbonne College in Clayton, Missouri (founded in 1923 as a women's only college by the French Sisters of the Congregation of St. Joseph and converted into a co-educational university in 1970), where he stayed for one year. In 1946 he moved to St. Louis University in St. Louis, Missouri (founded in 1818 by the Jesuit order), as a philosophy instructor and lecturer. His closest colleague there was James D. Collins (1917–1985), a well-known historian of philosophy.

In 1950 he accepted a post as assistant professor (full professor in 1959) and permanent chairman of the department of philosophy at Seton Hill College in Greensburgh, Pennsylvania. He retired from there in 1984. In the same letter quoted from above, Leo Jacoby writes:

> Serving at a small Catholic college, the pressure to "publish or perish" was perhaps not so severe, and probably lessened after he obtained tenure. We knew our father's specialty was symbolic logic. Who knows what he might have developed, if he had been able to concentrate primarily on this first love. He was often the only professor in the department and responsible for teaching the whole gamut of undergraduate courses. I have always admired my father for putting family first, and staying his whole career at the same college, which provided his children great stability growing up.

On February 14, 1942 he married Virginia Maas-Harbison (1916-2001), whom he had met at Quincy College. They had one daughter, Karen Cote, and three sons, John, Leo and Paul, in that order. After his retirement in 1984, both he and his wife became members of the Third Order Secular Franciscans. He died on November 9, 1993, in Greensburg, Pennsylvania.

In the same letter quoted from above, Leo Jacoby writes, again giving us a feel of the stable and traditional, emotional and intellectual atmosphere in the Jacoby home:

> All four children knew about their father's triangles. We often noticed him musing over and developing them on any scrap of paper—out of his own thinking rather than researching published resources. […] Maritain was likely my father's favorite philosopher, since he mentioned Maritain more than any

> other. I understand Maritain was a leading light in the neo-Thomistic movement, once prevalent in all Catholic universities, including Notre Dame University when my father studied for his doctorate in the early '40's.

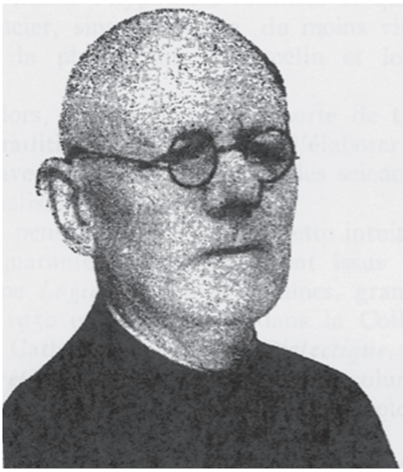
This was not a man to go out and conquer the world. In a profound sense, he was unworldly, putting his family and his intellectual work before himself and his career.

Jacoby was the first to develop what we have called the Blanché hexagon but what he called the "double triangle" (Jacoby 1950, pp. 44,46). His motivation for expanding the Square to a hexagon lay in his desire to come to a complete *theory of oppositions* in traditional logic. In this respect, he found the Square deficient, as it expresses only one relation of contrariety, between **A** and **A*** (or ¬**I**) (in our notation), whereas, in fact, two further such relations can be found: between **A** and **I** ∧ **I*** (= **Y**), and between **Y** and ¬**I**. Moreover, the contradictory of the new **Y** (Blanché's **U**) had to be inserted as a third relation of contradictoriness. The result of incorporating these two additional relations of contrariety and one additional relation of contradictoriness is a beautifully 'harmonic' hexagon containing three relations of contrariety, three relations of contradiction, and, as a final bonus, three relations of subcontrariety (see Fig. 4b). Strictly speaking, this is the measure of his progress. In hindsight, we may feel that this is not all that spectacular, but one should realize that nobody else, until then, had made that progress. Jacoby's thinking about logic was still heavily bound up with (Thomistic) ontology, metaphysics and epistemology, but less so than that of most others still holding on to traditional logic at the time. What he still lacked was the detached mathematical frame of mind we are accustomed to in our day. Moreover, like Sesmat, to be discussed in the following Section, he failed to see the asymmetrical selections made by cognition in the logical hexagon, leading to general constraints on lexicalization processes and thus to a direct link with cognition (see Seuren and Jaspers, 2014).

4.2. *Augustin Sesmat (1885–1957)*

About Sesmat we know a great deal less than about Jacoby, our main source of information being his publications, a note about his publications by Pierre Colin (1923–2009), the then dean of the philosophy faculty of the Institut Catholique de Paris, and an obituary (with photo) (Anonymous–1 1957), the latter two having been sent to us by Dr. Hubert Faes, again of the Institut Catholique de Paris.

Augustin Sesmat was born on March 7, 1885 in Dieulouard, not far from Nancy in north-eastern France. Seen from the outside, his life was not spectacular. On July 9th 1911, he was ordained a secular priest (not belonging to a religious order but under the direct authority of his bishop), to be sent as a vicar to the parish of Saint-Jacques in nearby Lunéville, where he



stayed for one year. From 1912 to 1921 he was a teacher at the École de St. Sigisbert in Nancy, after which he became a professor at the Grand Seminary attached to the Institut Catholique de Paris. He stayed there in that capacity till 1925, when he was appointed to a chair in "History and Critique of the Sciences" at the Institut Catholique de Paris, retiring from that institution in 1955, two years before his death in 1957. In 1931 he obtained his doctorate in philosophy at the same Institut Catholique with a dissertation entitled *Mouvements réels et mouvements apparents* (Real and Apparent Movements), and in 1937 his Doctorat d'État with two dissertations named *Le système absolu classique et les mouvements réels* (The Absolute Classical System and real movements) and *Les systèmes privilégiés de la physique relativiste* (The Privileged Systems of Relativist Physics), for which, he was awarded, in 1939, the "Prix Binoux d'Histoire des Sciences" by the French Academy of Sciences.

Sesmat was untypical for French Catholic philosophers of the time, in that he concentrated principally on the natural sciences. His record shows that he was a specialist in the philosophy of the natural sciences, in particular the theory of relativity and related matters. To the extent that this might cause a conflict with official Catholic doctrine, his reply was, according to the obituary mentioned above, that God had given the human race the faculty of thought not to leave it unused but, instead, to make the best possible use of it.

His work in logic was a sideline, but not an unimportant one. In his (1951, pp. 411–2), he designs a hexagon for the three 'positive' arithmetical relations >, =, and <, and their contradictory counterparts ('contrapositives') ≤, ≠, and ≥, respectively (copied—without attribution—by Blanché in his 1950, p. 111 and his 1966, p. 64; see note 5). Then (Sesmat 1951, pp. 449–51), he applies the arithmetical hexagon to predicate logic, giving the logical (Blanché) hexagon shown in Fig. 4–b above, using the symbol "U" for Blanché's later "Y" and the symbol "Y" for Blanché's later "U". Finally, on p. 461, he presents diagrammatically the five possible relations of identity, proper inclusion (both ways), mutual exclusion and partial mutual intersection between the nonnull R-set denoted by the subject term and the nonnull Matrix set or M-set denoted by the predicate, thus anticipating the theory of generalized quantification, which treats quantifiers as binary higher-order predicates over pairs of sets. In doing so, he omitted the two possibilities of both sets being mutually exclusive but jointly forming the universe of discourse **Un** (contradiction), and of the two sets having a nonnull mutually partial intersection and again jointly forming **Un** (subcontrariety).

The point of this exercise, in Sesmat (1951), was the formalization of traditional logic in terms of an algebra of relations. In this respect, he is more in agreement with modern mathematical logic than with Blanché, who aimed at reconstructing the logic of cognition. A few years later, however, he shows himself more open to Blanché's perspective, presenting his hexagon of logical relations as a formalization of the logic of natural and scientific thought: "the human method, as a tool of concrete science," and characterizing modern mathematical logic as "the abstract science of the general types of our rational operations" (Sesmat 1955, p. 153). Unfortunately, there is no mention at all of the problem of undue existential import plaguing both the Square and the Hexagon.

Like Paul Jacoby, Augustin Sesmat was a profoundly unworldly man. Both sacrificed everything, in total integrity, in the service of their academic work, part of their service to God. Not having a family to care for, Sesmat lived a life of contemplative solitude, renouncing any unnecessary comforts or diversions, despite a precarious health. The obituary mentioned above describes him as a man of great modesty and genuine self-effacement, again, like Paul Jacoby, not a media-driven world conqueror but preferring quiet reflection to the spotlights of publicity. He died on December 12th, 1957.

4.3. *Robert Blanché (1898–1975)*

As regards Robert Blanché, biographical information proved, again, extremely difficult to retrieve and has, therefore, remained fragmentary and incomplete. All we have is four obituary notes: an anonymous very brief note (Anonymous–2, 2005), a longer and more informative note by Alain Guy (Guy 1976), a note by F. Deschamps (Deschamps 1976) containing a short autobiographical sketch by Blanché himself, and a short note by Pierre Maxime Schuhl (Schuhl 1975).

From these sources, we learned that Robert Louis Blanché was born in Sauzé-Vaussais (Deux-Sèvres), about 150 km south-east of Nantes, on November 6, 1898. In 1923, he married a young lady called Suzanne (email from one of our informants, who could give no further details). There is no mention of any children anywhere. Though known as a devout Catholic, he was clearly less closely associated with the Catholic faith and Catholic organizations than either Jacoby or Sesmat. He died of cancer in Toulouse on December 6, 1975. There is a "Prix Robert Blanché" of 1500 EURO, awarded every four years for an excellent work in philosophy by the Académie des Sciences Morales et Politiques, of which he became a corresponding member in 1961.

In 1919 he was accepted as a student at the École Normale Supérieure, where he studied philosophy till 1922. From 1923 till 1937, he was a philosophy teacher at two French 'lycées', the second at Caen in Normandy. In 1935, he obtained his PhD ('doctorat'). From 1937 till 1941, he taught philosophy as Maître de Conférences at the University of Caen, from where he transferred to the University of Toulouse (in Vichy France), first, till 1947, as Maître de Conférences, then, till his retirement in 1969, as a tenured 'associate professor' (agrégé of higher education) of philosophy.

After this last appointment in 1947, he started publishing a number of introductory textbooks on a variety of topics, including physics and its philosophy, general philosophy, axiomatics, modern logic, the history of logic, epistemology, reasoning, natural law. In his autobiographical sketch, Blanché makes it clear that he vehemently opposed the new 'existentialist' and 'deconstructivist' movements in French philosophy and adjacent areas. He also abhorred the French revolutionary student movement of 1968. He wrote, in his autobiographical note in Deschamps (1976): "[J]ai profondément déploré la tournure partisane que certains d'entre eux, en 1968, ont fait prendre aux événements" ('I deeply deplored the partisan turn that, in 1968, some among them [the students] caused the events to take').

What interests us most, of course, is his work in predicate logic, as published in Blanché (1953, 1957, 1966, 1967). We understand from his publications that he was relatively well versed in modern mathematical logic and did not suffer from the kind of obscurantist prejudices found in, for example, Maritain's work. His motivation and point of departure was, on the one hand, his wish to develop a complete theory of logical oppositions—a wish he shared with Jacoby—and, on the other, his philosophically more profound conviction that a bridge should be forged between purely formal, mathematical logic and the natural logic of human cognition, which, in his view, is only partially formalizable and not mathematical.

This latter conviction led him inevitably into the maze of old, all-encompassing philosophical problems regarding the nature and powers of the cognitive mind and its relation with reality, and the nature and genesis of knowledge. In this context, he distinguished between formal, mathematical logic on the one hand and a natural 'operational' logic on the other. Their mutual relation is described as follows (Blanché 1967, p. 122; translation ours):

> Thus, a reflection on natural operational logic, though allowing for a partial utilization of formalisms, will never be amenable to a total rigorous formalization. […] For if total formalization were possible, natural operational logic

> would, in the end, merge with scholarly logic, or at least with one of its parts, differing only in the direction taken—a useless duplication. By contrast, it is because the two kinds of study cover each other only in part that the one can be seen as the complement of the other.

Apart from the curious notion of two partially overlapping fields being each other's complement—a confusion, though unexpected from a logician, one may perhaps let pass—one detects here another confusion relating to the nature of logic as a system of *formally* establishing relations of entailment, (sub)contrariety, contradiction or equivalence between sentences. It is only when thinking is confused with logic that one can speak of partial formalization, in that thinking has, so far, not been amenable to full formalization while logic, any logic, whether natural or mathematical, must be formal by definition.

Leaving aside such questions, one sees that Blanché, in order to keep matters under control, restricted himself, in principle and in practice, to an investigation of the logical relations between concepts, treating these as specific instances of general logical relations (Blanché 1966). Like the present authors, therefore, he was looking for logical structure in coherent groups of concepts and their expression as lexical predicates. Unlike the present authors, however, he failed to distinguish between the natural logical relations of entailment, contrariety, equivalence and contradiction, and the highly nonnatural relation of subcontrariety, whose elimination from the hexagonal system gives rise to the fully formalized kite structure of Fig. 7. Nor did he see the internal asymmetry within this kite structure as regards the complement-selecting function of the negation. The empirical shortcomings of his still deficient analysis he tried to cover up by an appeal to an ultimately ungraspable and mysterious faculty of thought, "a more fundamental logic" (Blanché 1967, p. 84), which is left unclear and can only be seen as part of what Catholics take to be the human soul, a moral 'self' destined to an eternity of heaven or hell.

5. Conclusion

We have tried to show, in the preceding pages, that the summary dismissal of traditional logic by Russell and company in the early 20th century was inconsiderate and counterproductive, as it stymied research into deeper questions of logic and metalogic needed to clarify both the logical properties of the various systems involved and the position of logic with respect to language and cognition. Those who objected were mainly Catholics, who felt that the new mathematical turn detached logic from traditional Aristotelian-Thomistic epistemology and metaphysics, central elements in Catholic doctrine. There was also a general feeling of unhappiness because one needed specialized knowledge of mathematics and its foundations to counter the modernists with relevant arguments—a kind of knowledge and expertise the traditionalists did not have. Logic was moved from philosophy to the sciences, much to the displeasure of the traditionalists, who felt that this had dehumanized logic and made it superficial. To a large extent, this was indeed so, as the new logic had severed all links with human cognition and many of the old links with human language, thereby depriving itself from any relevance with regard to the study of cognitive structures and processes and from most of its relevance with respect to human language.

To restore the lost 'depth', the Catholic objectors had little more to offer than largely emotional and rhetorical appeals to 'deeper' values attributed to human beings, as they lacked the necessary expertise to attack the modernists on their own ground. Though hardly ever mentioned in explicit terms, the concern of the Catholic traditionalists centered around the fact that the modernists had excised cognition from logic. Since cognition was, vaguely, taken to be a part of the human soul as an essential constituent of human beings, this was seen as a direct attack on the validity of the notion 'soul' and thus on Catholic doctrine. Much of the resistance to modern logic was thus based on doctrinal grounds.

In hindsight, we now see that there was a basic mutual lack of proper understanding. On the one hand, the banning of cognition by the modernists was far too radical, in that the study of cognitive logic is rewarding in many ways to do with logic itself as well as with human nature. On the other hand, the traditionalists' fear that the banning of cognition meant the banning of the soul was unfounded, as the soul, vaguely defined as it was, could be taken to encompass a great deal more than just cognition. Any conflict could, and can, be avoided when it is agreed not only that a removal of the mind from mathematical logic has no doctrinal implications but also that cognitive logic is a valid branch of logic besides mathematical logic. The former point of view has, it seems, now been generally accepted by Catholics and others. The latter, however, still remains to be vindicated.

We have shown how, in this doctrinal warfare, three Catholic logicians of the mid-twentieth century, Jacoby, Sesmat, and Blanché, were trying to resolve the conflict mainly in two different ways. They tried to get the better of modern logic by showing the potential richness of traditional logic (forgetting to address the basic issue of undue existential import). And they tried to widen the bridge between logic and cognition by generalizing their system of logical relations to the world of concepts as a whole and to the corresponding lexicons of the languages of the world. For a variety of reasons, their attempts failed to acquire the momentum necessary for a flourishing branch of research. The present authors revive these well-nigh forgotten attempts, adding new analyses and new results.

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Acknowledgements

The authors are grateful to Mr. Leo Jacoby and Mr. Paul Jacoby Junior, sons of Paul Jacoby, for providing detailed information and documentation on Paul Jacoby's life and work and for allowing us to print his picture. We also thank Dr. Hubert Faes of the Institut Catholique de Paris for information about Augustin Sesmat and for permission to print Sesmat's picture. The Presses Universitaires de France granted permission to print the hand-drawn picture of Robert Blanché, which appeared on the cover of his book *Axiomatique* (PUF 1955). Finally, we thank Prof. Hans Smessaert of the University of Louvain for references and for help in drawing the diagrams.

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