Supplemental Material: Dynamic Response of a Single Interface in a Biocomposite Structure

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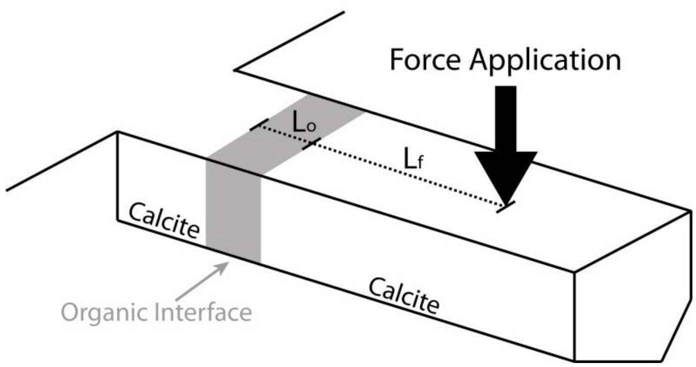
SCHEMATIC REPRESENTATION OF THE EXPERIMENTAL SET-UP



CALCULATION OF THE DISPLACEMENT AMPLITUDE:

The displacement, *U*, under the periodic force $F\exp(i\omega t)$, is described by the standard differential equation:

$$m\ddot{U} + C\dot{U} + KU = Fe^{i\omega t}, \tag{S1}$$

where *m* is the effective mass, *C* and *K* are the effective damping and stiffness of the specific tip/microcantilever configuration, respectively. The solution of Eq. (S1) is the periodic displacement, $U = U_0\exp(i\omega t)$, with an amplitude:

$$U_0 = \frac{F}{K - m\omega^2 - i\omega C}\,. \tag{S2}$$

Therefore, the measured amplitude, $|U_0|$, equals:

$$|U_0| = \frac{F}{|K-m\omega^2-i\omega C|} = \frac{F}{\sqrt{(K-m\omega^2)^2+(\omega C)^2}}\,. \tag{S3}$$

Whereas the phase, φ, is defined by:

$$tan\varphi = \frac{\omega C}{K-m\omega^2} \tag{S4}$$

CALCULATION OF THE STATIC MICROCANTILEVER DEFLECTION:

Organic edge deflection and slope angle as a result of the force F_K (normal stresses):

$$\delta_1 = \frac{F_K l^3}{3EI} \quad ; \quad \theta_1 = \frac{F_K l^2}{2EI}. \tag{S5}$$

Organic edge deflection and slope angle as results of the edge moment M_K (normal stresses):

$$\delta_2 = \frac{M_K l^2}{2EI} \quad ; \quad \theta_2 = \frac{M_K l}{EI}. \tag{S6}$$

Organic edge deflection as a result of the force F_K (shear stresses):

$$\delta_3 = \kappa\frac{F_K l}{GA}. \tag{S7}$$

Considering $l = L_o$ and applying the superposition principle, the total microcantilever deflection at the point of force application is:

$$\delta = [\delta_1 + \delta_2 + \delta_3] + [\theta_1 + \theta_2]L_f = \left[\frac{F_K L_o^3}{3EI} + \frac{M_K L_o^2}{2EI} + \kappa\frac{F_K L_o}{GA}\right] + \left[\frac{F_K L_o^2}{2EI} + \frac{M_K L_o}{EI}\right]L_f. \tag{S8}$$

Introducing $M_K = F_K L_f$ yields:

$$\delta = F_K\left[\frac{L_o^3}{3EI} + \frac{L_f L_o^2}{2EI} + \kappa\frac{L_o}{GA}\right] + F_K\left[\frac{L_o^2}{2EI} + \frac{L_f L_o}{EI}\right]L_f. \tag{S9}$$

After rearranging:

$$\delta = F_K\left[\frac{L_o^3}{3EI} + \kappa\frac{L_o}{GA}\right] + F_K\left[\frac{1}{2EI} + \frac{1}{2EI}\right]\left(L_o^2 L_f\right) + F_K\frac{1}{EI}\left(L_o L_f^2\right)\,. \tag{S10}$$

Dividing by F_K/EI and introducing the microcantilever stiffness as $K_b = F_K/\delta$ yields:

$$\frac{1}{K_b}IE = \left[\frac{L_o^3}{3} + \kappa\frac{E}{G}\frac{I}{A}L_o\right] + \left(L_o^2 L_f\right) + \left(L_o L_f^2\right). \tag{S11}$$

CALCULATION OF THE DYNAMIC MICROCANTILEVER DEFLECTION:

Due to the linearity of the Kelvin-Voigt solid model, the general derivation for the microcantilever velocity is similar to that one for the microcantilever deflection (the viscoelastic beam theory). Organic edge deflection velocity and slope angle rate as a result of the force F_C (normal stresses):

$$\dot{\delta} = \frac{F_C l^3}{3DI} \quad ; \quad \dot{\theta} = \frac{F_C l^2}{2DI}. \tag{S12}$$

Organic edge deflection velocity and slope angle rate as results of the edge moment M_C (normal stresses):

$$\dot{\delta} = \frac{M_C l^2}{2DI} \quad ; \quad \dot{\theta} = \frac{M_C l}{DI}. \tag{S13}$$

Organic edge deflection velocity as a result of the force F_C (shear stresses):

$$\dot{\delta} = \kappa \frac{F_C l}{SA}\,, \tag{S14}$$

The total velocity of the microcantilever at force application point is therefore:

$$\dot{\delta} = \left[\frac{F_C L_O^3}{3DI} + \frac{M_C L_O^2}{2DI} + \kappa \frac{F_C L_o}{SA}\right] + \left[\frac{F_C L_O^2}{2DI} + \frac{M_C L_o}{DI}\right] L_f. \tag{S15}$$

Applying $M_C = F_C L_f$, dividing by F_C/DI and introducing the microcantilever damping as $C_b = F_C/\dot{\delta}$ yields:

$$\frac{1}{C_b} ID = \left[\frac{L_O^3}{3} + \kappa \frac{D}{S}\frac{I}{A} L_o\right] + \left(L_o^2 L_f\right) + \left(L_o L_f^2\right)\,. \tag{S16}$$