Scattering of electromagnetic wave beams from density fluctuations in tokamak plasmas

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Well collimated electron cyclotron (EC) wave beams are routinely used in present-day tokamaks for heating, current drive, and for diagnostic purposes. In several applications, turbulent fluctuations of the density can potentially affect the desired result, broadening the power deposition profile or altering the spectrum of the probing beam. The theoretical treatment of the problem is made difficult by the fact that, in general, turbulent fluctuations do not satisfy the WKB ordering on which most of the tools usually employed for the description of short-wavelength waves in fusion plasmas rely. On the other hand, an increasing number of theoretical estimates indicates that beam scattering can be important in particular on reactor-size tokamaks, where small deviations of the beam trajectory can propagate over large distances and have thus sizeable effects. In this paper, an approach to this problem based on the solution of the wave kinetic equation, for which a rigorous treatment of the scattering term has been developed, is presented, and the corresponding numerical implementation in the new code WKBeam is described. First results for a model distribution of the turbulence are discussed.

Theoretical background

We look for a solution of the wave equation for the wave field $E(\omega,x)$,

$$\nabla \times \nabla \times E(\omega,x) - \kappa^2 \hat{\varepsilon} E(\omega,x) = 0, \tag{1}$$

in the short-wavelength (WKB) limit $\kappa \gg 1$, where $\kappa = Lk_0$, with L the inhomogeneity scale of the medium and $k_0 = \omega/c$ the vacuum wavevector of a wave of fixed frequency ω. In the previous equation, the coordinate x is normalized to L. The wave equation can be written in the symbolic form $DE = 0$, where D is the dispersion tensor. The wave kinetic equation is constructed starting from the equation for the field correlation $DEE^\dagger = 0$ by applying the Weyl symbol map. In the case of the field correlation function, the result of the Weyl symbol map is the Wigner function

$$W(x,N) = \int e^{-i\kappa N\cdot s} E\left(\omega, x+s/2\right) E^\dagger\left(\omega, x-s/2\right) \mathrm{d}s$$

(properly speaking, one should talk about a Wigner matrix, due to the vector nature of the electric field, but this distinction will be ignored here, as in the following we will treat different modes of propagation separately; see [1] for a complete discussion).

The Wigner function satisfies an equation equivalent to Eq. (1), which in the limit $\kappa \to \infty$ leads to:

$$\begin{aligned}
&\mathscr{O}(1): && D(x,N)W(x,N) = 0,\\
&\mathscr{O}(\kappa^{-1}): && -\frac{i}{2\kappa}D(x,N)\left(\overleftarrow{\partial_x}\cdot\overrightarrow{\partial_N} - \overleftarrow{\partial_N}\cdot\overrightarrow{\partial_x}\right)W(x,N) \equiv \frac{i}{2\kappa}\{D,W\} = 0,
\end{aligned}$$

where in the last step the Poisson brackets have been introduced. Imposing the condition of non-trivial solutions for W on the first equation leads to the dispersion relation. The second equation has the form of a steady state kinetic equation and is used to determined the evolution of the Wigner function, starting from the antenna plate. If fluctuating and dissipative media are considered, adopting the ordering $D \to D_0 + \kappa^{-1/2}D_{\mathrm{fluct}} + \kappa^{-1}D_{\mathrm{abs}}$ (where D_0 is the unperturbed, cold-plasma dispersion operator) leads to the form of the wave kinetic equation considered in this work:

$$\{D_0, W\} = -2\gamma W + S(\Gamma, W), \tag{2}$$

where $\gamma = e^*\varepsilon^a e$ is the absorption coefficient and S the scattering term, which is described in some more detail below and depends on the density correlation upon its Wigner transform Γ. The choice of the ordering of the fluctuating term simply reflects the fact that the fluctuation term appears quadratically in the final result.

The derivation of the scattering operator follows McDonald [2] and relies on the approach of Ref. [3] which is briefly sketched here, since it represents the most important approximation of the present approach. Write the wave equation in the compact form $(D_0 + D_1)E = 0$, where the ensemble average of the fluctuations part vanishes, $\langle D_1\rangle = 0$. If E_0 is a solution of $D_0E_0 = 0$, the solution of the wave equation can be written as $E = E_0 - (D_0^{-1}D_1)E$. We look for an iterative solution, $E = E_0 - (D_0^{-1}D_1)E_0 + (D_0^{-1}D_1D_0^{-1}D_1)E_0 + \cdots$. Truncating at the lowest significant order (*Born approximation*) and applying ensemble average yields

$$D_0\langle E\rangle = \langle D_1D_0^{-1}D_1\rangle E_0$$

where to lowest order one can replace E_0 with $\langle E\rangle$. Applying the same procedure to $(D_0 + D_1)EE^\dagger(D_0^\dagger + D_1) = 0$ yields

$$D_0\langle W\rangle = \langle D_1W_0^{-1}D_1\rangle\,(D_0)^{-1} + \langle D_1D_0^{-1}D_1\rangle W_0,$$

where the right-hand side leads (applying the Weyl symbol map and the WKB limit) to a scattering term in the form

$$S = \int \left[\sigma(x,N,N')W(x,N') - \sigma(x,N',N)W(x,N)\right]\mathrm{d}N'$$

where the scattering cross section σ is

$$\sigma(x,N,N') = \frac{\kappa^4}{(2\pi)^3}\left|e^\dagger(x,N)(\mathbf{I}-\varepsilon)e(x,N')\right|^2 \delta(H(x,N'))\Gamma(x,N-N').$$

Note that Γ varies on the scale length of the equilibrium scale (typically the scale of the gradients driving the turbulence), so that the application of the WKB ordering is justified.

Benchmarks

The previous equations are implemented in the code WKBeam [4], which evolves the Wigner function along Hamiltonian orbits, including scattering and absorption. Diffraction effects are accounted for by initializing the rays at the antenna plane according to the Wigner function. Scattering events are generated along each ray according to the scattering amplitude σ. Relevant quantities (e.g. power deposition profiles) are reconstructed through the corresponding Monte Carlo estimators (binning in phase space). A detailed code verification in the absence of turbulence has been performed, i.e. against analytic solutions (free space, lens-like medium, linear-layer medium) and numerical solutions in tokamak geometry (beam tracing code TORBEAM [5]). See Ref. [1] for some examples. Here we present the first results of a verification exercise that includes turbulence and relies on the two-dimensional model of Sysoeva *et al.* [6], which is based on the same assumptions (Born approximation) as employed in the derivation of the scattering term implemented in WKBeam.

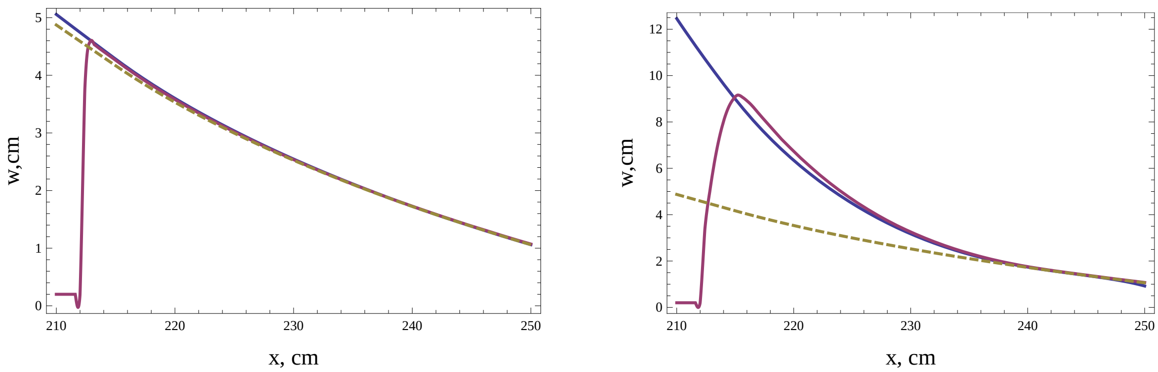

Fig. 1. *Benchmark of the beam width as obtained from WKBeam (red) against Eq.(11) of Ref. [6] (blue). The green dashed line shows the result in the absence of turbulence. The fluctuation level δn_e is 0.5% of the cutoff density in the left panel, 5% in the right panel.*

It is indeed possible to show that the equation (Eq.(11) of Ref. [6]) for the Fourier-transformed amplitude $|a_{k_y}|^2$ of the electric field, which is connected to the Wigner function through the relation

$$|a_{k_y}|^2 = \frac{L^2\sqrt{k^2-k_y^2}}{2\pi}\int W(x,N)\mathrm{d}y\mathrm{d}k_x,$$

can be derived exactly from the wave kinetic equation (2). Fig. 1 shows a very good agreement between the beam width resulting from a numerical solution of the equation for $|a_{k_y}|^2$ and that obtained from WKBeam in slab geometry.

Results

Preliminary results obtained with the WKBeam code have been presented in [7]. For ITER, it has been found that a turbulence layer with a fluctuation level $\delta n_e/n_e \approx 10\%$ located around the plasma edge can lead to a broadening of the absorption profile by about a factor of two for parameters typical of the upper EC antenna, confirming previous estimates.

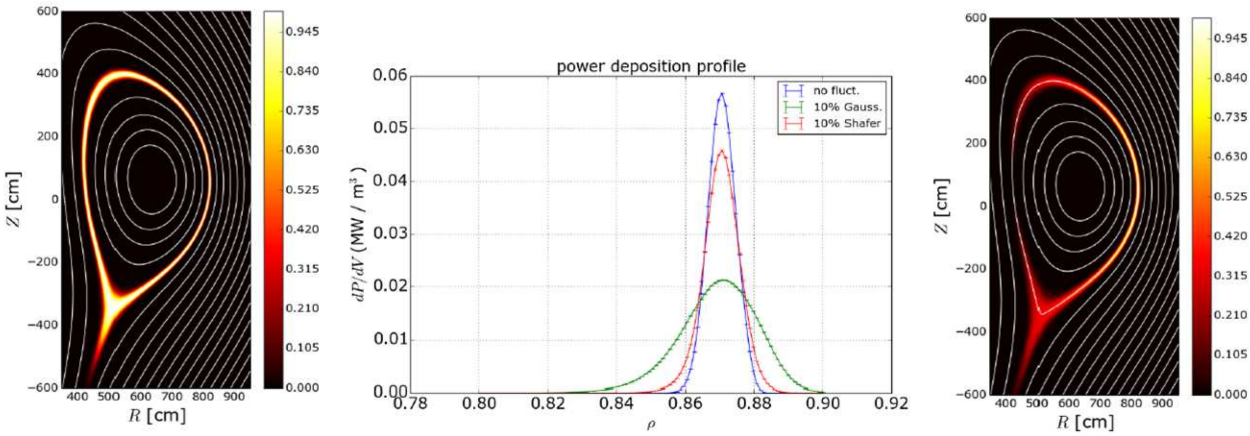

Fig. 2. *Power profile broadening in ITER, homogeneous poloidal distribution of the turbulence (left) vs. the model of Ref. [8] (right).*

Here, we present as an example a comparison of the power deposition profile for ITER mentioned above (homogeneous poloidal distribution of the turbulence) with a model which takes into account the poloidally inhomogenous distribution of the turbulence [8]). For the ITER upper launcher, which is located well above the midplane, the effect of turbulence “ballooning” reduces significantly the impact of the fluctuation on the deposition profile. This result stresses the importance of a realistic desription of the fluctuations for this analysis.

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