Representation of finite amplitude ideal magnetohydrodynamic modes

D. Meshcheriakov[1], V. Igochine[1], H. Zohm[1], A. Gude[1], M. Maraschek[1], D. Moseev[1], D. Vezinet[1], M. Dunne[1] and the ASDEX Upgrade Team

[1]*Max-Planck-Institut fur Plasmaphysik Boltzmannstrasse 2, 85748, Garching bei Muenchen, Germany*

Introduction

The ideal modes are described by the equations of ideal magnetohydrodynamics (MHD). One of the most important properties of ideal MHD is that there can be no change in topology of magnetic flux surfaces. The perturbation of the equilibrium field through $\delta\vec{B} = \nabla \times (\vec{\xi} \times \vec{B})$ is the conventional way to represent ideal MHD modes. It was shown [1] that the original topology is only conserved if the perturbed field is calculated to all orders in displacement $\vec{\xi}$. The simple representation of the MHD code results as a superposition of equilibrium and linearized perturbed field will result in an island structure which is unphysical for the ideal instabilities [2]. Our method is shown to conserve the magnetic field topology even at experiment relevant perturbation amplitude. In order to illustrate possible practical applications, the developed approach is applied to a real ASDEX Upgrade equilibrium.

Analytical expression for the covariant component of the Lundquist identity

The analytical expression for the change of the magnetic field connected with a finite displacement of the ideally conducting incompressible liquid was derived by Lundquist [3] in 1951. It's derivation is based on the consideration of the magnetic flux tube deformation by an arbitrary perturbation and the conservation of the magnetic flux through any element of a surface wholly composed of lines of force. Finally this expression casts into the so-called Lundquist identity:

$$\vec{B}(\vec{r}_0,t) = \vec{B}(\vec{r}_0 - \vec{\xi},0) + [\vec{B}(\vec{r}_0 - \vec{\xi},0) \cdot \nabla]\vec{\xi}(\vec{r}_0,t) \quad (1)$$

Advantages of use of this expression over the standard method of perturbing an equilibrium magnetic field through $\delta\vec{B} = \nabla \times (\vec{\xi} \times \vec{B})$ were clearly demonstrated in [1]. In the present work we generalize this approach to arbitrarily shaped plasmas and perturbations.

First we derived a general expression for the covariant representation of the perturbed magnetic field:

$$B(\vec{r}_0,t)_j = B(\vec{r}_0 - \vec{\xi},0)_j + \frac{1}{\sqrt{g_{ii}}} B(\vec{r}_0 - \vec{\xi},0)_i \frac{\partial \xi_j}{\partial x_i} - B(\vec{r}_0 - \vec{\xi},0)_i \xi_k \Gamma^k_{j,i} \tag{2}$$

where g_{ij} is a metric tensor of the coordinate system for which the magnetic field needs to be calculated and $\Gamma^k_{j,i}$ are Christoffel symbols, which are defined in the following way:

$$\Gamma^k_{j,i} = \frac{1}{2} \sum_s g^{ks} \left(\frac{\partial g_{is}}{\partial x^j} + \frac{\partial g_{js}}{\partial x^i} + \frac{\partial g_{ij}}{\partial x^s} \right) \tag{3}$$

Calculation of the perturbed ASDEX Upgrade field

The choice of the most convenient coordinate system for description of the perturbation arises from the variation of the poloidal and toroidal components of magnetic field on the equilibrium flux surface due to toroidicity and shaping of the plasma. As discussed in [4], there is a variation of the pitch angle along the field line in a general toroidal equilibrium and thus MHD modes can no longer be described by a single Fourier harmonic in poloidal angle. The proposed solution of this problem is a mapping of a general equilibrium to a screw pinch like geometry with a so-called straight field line angle θ^* instead of a poloidal angle. Such coordinate system, where coordinates are ρ - normalized poloidal flux, θ^* -straight field line angle and ϕ - toroidal angle, is chosen for the present studies.

In order to demonstrate consistency of the presented method at realistic plasma shapes, the equilibrium reconstruction of ASDEX Upgrade discharge ($AUG28746, t = 1.7s)$) is chosen as a test bed. The equilibrium reconstruction was done with CLISTE interpretative code [5].

A radial component of the plasma perturbation corresponding to a single ideal mode can be written as:

$$\xi(\rho) = \xi_0(\rho) \cdot \cos(m\theta^* - n\phi + \Phi_0) \tag{4}$$

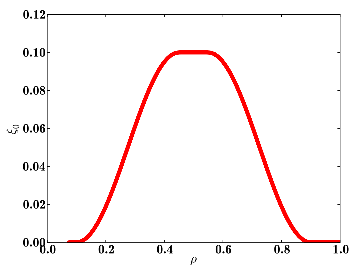

Figure 1: *Normalized test perturbation profile*

where Φ_0 is the initial phase. The poloidal and toroidal components of $\vec{\xi}$ can be determined from the additional constraints of incompressibility of the plasma ($\nabla \cdot \xi = 0$) and the slip motion condition ($\nabla \times (\vec{\xi} \times \vec{B}_\phi) = 0$):

$$\vec{\xi} = \begin{pmatrix} \xi_\rho \\ \xi_{\theta^*} \\ \xi_\phi \end{pmatrix} = \begin{pmatrix} \xi_0(\rho) \cdot \cos(m\theta^* - n\phi + \Phi_0) \\ -\frac{1}{m}(\xi_0(\rho) + \rho \xi_0'(\rho)) \cdot \sin(m\theta^* - n\phi + \Phi_0) \\ 0 \end{pmatrix} \tag{5}$$

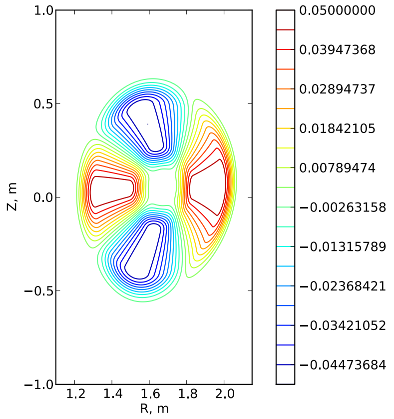

Figure 2: *Contours of the radial component(ξ_ρ) of the test perturbation*

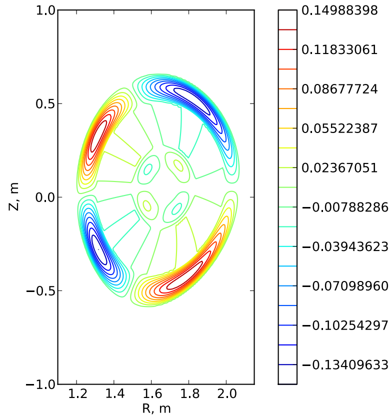

Figure 3: *Contours of the "poloidal"(ξ_{θ^*}) component of the test perturbation*

In order to simulate an impact of a localized mode, the radial mode structure shown in Figure 1 was assumed. It consist of a region with a constant displacement and two gradient regions which represent vanishing of the mode far from the resonant surface. Since the expression for the θ^* component of the perturbation contains the first derivative of $\xi_0(\rho)$, our function should be continuously differentiable. Therefore, the shape of the gradient regions was chosen to be a squared cosine half-wave. The 2D structures of the radial and straight field line angle components of the $m = 2$ perturbation are shown on Figure 2 and Figure 3, respectively.

$$\xi_0(\rho) = \begin{cases} 0 & \rho \in [0, \rho_1[\\ \xi_0 \cos^2\left(\frac{\pi}{2}\frac{\rho-\rho_2}{\rho_2-\rho_1}\right) & \rho \in [\rho_1, \rho_2] \\ \xi_0 & \rho \in]\rho_2, \rho_3] \\ \xi_0 \cos^2\left(\frac{\pi}{2}\frac{\rho-\rho_3}{\rho_4-\rho_3}\right) & \rho \in]\rho_3, \rho_4] \\ 0 & \rho \in]\rho_4, 1] \end{cases} \tag{6}$$

In this expression, ξ_0 defines the perturbation amplitude and the parameters $\rho_1, \rho_2, \rho_3, \rho_4$ the location of the mode, it's width and the width of the gradient regions. It is important to mention here that the perturbation amplitude has to be smaller than the width of the gradient regions. Otherwise, according to Equation 1, flux surfaces can be moved directly to the region with $\xi_0(\rho) = 0$, where the convective part of the Lundquist identity is zero and thus the displacement is not compensated in the gradient regions.

The implementation of our method is performed in three major steps: first, components of the displacement vector are calculated in the straight field line coordinates, then a coordinate transformation is made in order to get it's components in the cylindrical coordinate system (R, Z, ϕ) and finally, the cylindrical version of the Lundquist identity is applied. The special

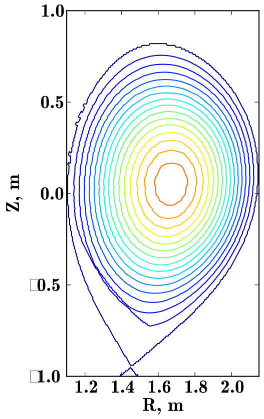

Figure 4: *Flux surfaces of the equilibrium magnetic field*

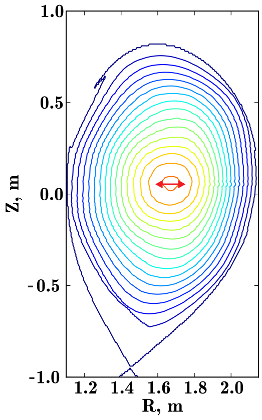

Figure 5: *Flux surfaces of the magnetic field with* $m = 2$ *perturbation.* $\xi_0 = 0.05$

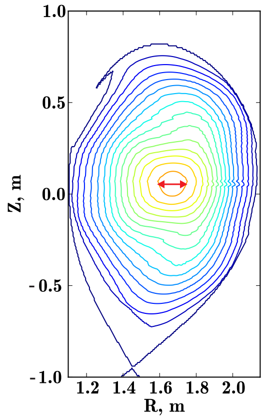

Figure 6: *Flux surfaces of the magnetic field with* $m = 2$ *perturbation.* $\xi_0 = 0.1$

case of an $m = 1$ mode requires additional treatment since it involves a special point $\rho = 0$, where the Jacobian is 0 and thus the coordinate transform is not defined.

Isocontours of the equilibrium and perturbed poloidal flux are shown on Figures 4 5 and 6. When the perturbation with $m = 2$ is applied, one sees the expected compression of the flux surfaces in the horizontal direction and stretching in the vertical one. In the regions with zero perturbation amplitude, flux tubes remain unperturbed in the case with $\xi_0 = 0.05$. However, when the amplitude is increased to $\xi_0 = 0.1$, the width of the gradient regions seems to be insufficient to screen the displacement completely. It is important to point out that no islands are observed and thus the topology is conserved.

Conclusions

An alternative method of calculating the perturbed magnetic field has been presented. It includes both an analytical expression for the covariant component of the perturbed magnetic field and a numerical tool for modification of the ASDEX Upgrade equilibrium field in the presence of a perturbation. The ability to apply finite (up to experiment relevant amplitudes) perturbations without breaking the topology is demonstrated.

Acknowlegments

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] *R. B. White, Physics of Plasmas* **20**, *(2013).*
[2] *A. D. Turnbull et al., Physics of Plasmas* **20**, *(2013).*
[3] *S. Lundquist, Physical Review* **83**, *(1951).*
[4] *H. Zohm,* Magnetohydrodynamic Stability of Tokamaks *(Wiley-VCH, Weinheim, Germany, 2014), pp. 34–37.*
[5] *P. McCarthy and al., Phys. Plasmas* **6**, *3554 (1999).*