Relativistic runaway electrons in a near-threshold electric field

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Runaway electrons (RE) generated via the avalanche mechanism [1] during disruption events in tokamaks have to be mitigated to minimize the detrimental impact from their losses to the wall, especially in future tokamaks such as ITER. Even if the inductive electric field is initially very large, it is expected to drop to the avalanche threshold level after a transient phase of fast RE generation. The subsequent longer-lasting behaviour of RE is governed by the near-threshold electric field. This regime is of particular interest for the mitigation of the RE current and energy.

In this work we first discuss a kinetic theory for relativistic runaway electrons in the near-threshold regime [2]. In a uniform magnetic field B and constant electric field E along the magnetic field lines the RE distribution function F satisfies the relativistic Fokker-Planck equation

$$\begin{aligned}&\frac{\partial F}{\partial s}+\frac{\partial}{\partial p}\left[E\cos\theta-1-\frac{1}{p^2}-\frac{p\sqrt{1+p^2}}{\bar{\tau}_{rad}}\sin^2\theta\right]F=\\&\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\left[E\frac{\sin\theta}{p}F+\frac{(Z+1)}{2}\frac{\sqrt{p^2+1}}{p^3}\frac{\partial F}{\partial\theta}+\frac{1}{\bar{\tau}_{rad}}\frac{\cos\theta\sin\theta}{\sqrt{1+p^2}}F\right],\end{aligned}\tag{1}$$

where p is the particle momentum (normalized to mc), θ is the pitch-angle, s is the time variable (normalized to $\tau\equiv\frac{4\pi\varepsilon_0^2m_0^2c^3}{e^4n_e\ln\Lambda}$), E is the electric field (normalized to $\frac{mc}{e\tau}\equiv E_c$), and $\bar{\tau}_{rad}\equiv\frac{\tau_{rad}}{\tau}=\frac{6\pi\varepsilon_0m_0^3c^3}{e^4B^2}\frac{1}{\tau}$ is the normalized time of synchrotron losses. The normalization of the distribution function is given by $\int Fdp\sin\theta d\theta=1$. In fully ionized plasmas Z is the ion charge, whereas in cold post-disruption plasmas with impurities, Z should be adjusted to capture the effects of fast-electron scattering on impurity ions and atomic nuclei. Also, the expression for τ needs to be generalised to take into account collisions with bound electrons (as pointed out in [1]).

In the near-threshold regime the pitch-angular relaxation timescale is shorter than the momentum evolution timescale in (1). In this case, a small parameter $\varepsilon=\frac{E-E_a}{E_a}$ (where E_a is the avalanche threshold as introduced in [2]) enters the equation allowing us to find the angular distribution to the lowest order. The average pitch angle of this distribution is

$$\cos\theta_{av}=\frac{1}{\tanh\left[A(p)\right]}-\frac{1}{A(p)},\tag{2}$$

where

$$A(p)\equiv\frac{2E}{(Z+1)}\frac{p^2}{\sqrt{p^2+1}}.\tag{3}$$

We use this angular distribution to integrate Eq. (1) over all pitch-angles, which gives a one-dimensional continuity equation,

$$\frac{\partial G}{\partial s}+\frac{\partial}{\partial p}U(p)G=0, \tag{4}$$

for the momentum distribution function $G(s;p)$ with the "flow velocity"

$$U(p)\equiv E\cos\theta_{av}-1-\frac{1}{p^2}-\frac{Z+1}{E\bar{\tau}_{rad}}\frac{p^2+1}{p}\cos\theta_{av}. \tag{5}$$

The profile of the flow velocity $U(p)$ (shown in Fig. 1) predicts peaking of the runaway distribution at $p = p_{max}$ in phase space. This feature is consistent with the prediction obtained from a truncated dynamical model in Ref. [3]. Note that it is straightforward to generalize this solution to include bremsstrahlung effects. This will preserve the trend for peaking of the runaway distribution [4].

To verify this analytical result we solve Eq. (1) numerically (using FiPy Finite Volume PDE Solver [5]). Fig. 2 shows an example of calculation for the parameters of Fig. 1. The initial distribution in Fig. 2 consists of two separated RE populations (left plot), one with the momentum between p_{min} and p_{max} and the other with the momentum higher than p_{max}. The right plot in Fig. 2 shows the distribution function F at the time $s = 10$. We note that both populations have gathered at the anticipated $p_{max} \approx 8$ location. The angular part of the equilibrated distribution function F also agrees reasonably with Eq. (2).

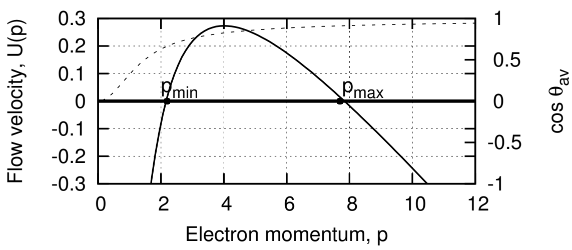

Figure 1. The flow velocity $U(p)$ defined by Eq. (5) for $Z = 2$, $E = 2.2$ and $\bar{\tau}_{rad} = 10$ (solid line) and $\cos\theta_{av}$ defined by Eq. (2) (dashed line).

Note that Eq. (4) predicts contraction of the momentum distribution into a delta-function. This is not observed in the numerical calculations, where such contraction eventually stops and the established distribution has a finite width. This width reflects the limitation of accuracy of the lowest order solution obtained in [2].

With the lowering of the electric field the root p_{min} increases and p_{max} decreases. They merge when the electric field has a certain value E_0, which is referred to as the "sustainment threshold" in [2]. If the electric field is less than E_0, then all electrons decelerate, which precludes sustainment of the RE population. The equality $p_{min} = p_{max}$ serves as a formal condition for determining E_0. A convenient analytic fit for E_0 is given by Eq. (8) in [2]:

$$E_0\approx 1+\frac{(Z+1)}{\sqrt{\bar{\tau}_{\rm rad}}}\left(\sqrt[6]{\frac{1}{8}+\frac{(Z+1)^2}{\bar{\tau}_{\rm rad}}}\right)^{-1}. \tag{6}$$

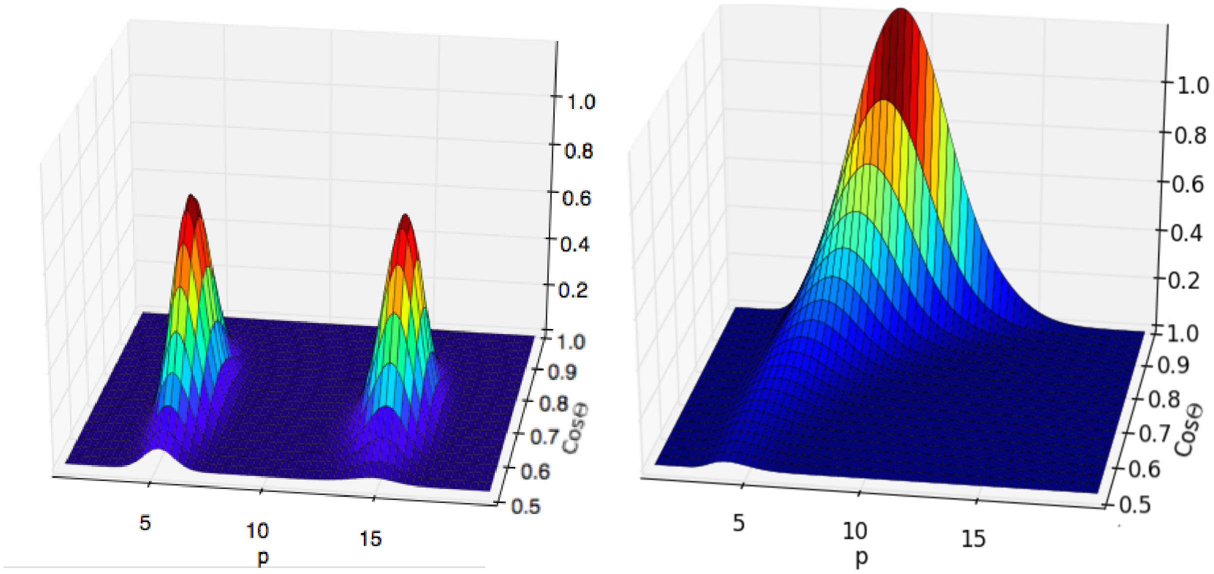

Figure 2. Numerical calculations of the distribution function F. Left figure - initial condition. Right figure - equilibrated distribution function F at the time $s = 10$.

This value of the sustainment threshold E_0 agrees with the experimental observations of the fast loss of the RE population [6]. For the cases shown in Fig. 10 of [6], Eq. (6) predicts $E_0 \approx 2-2.5$, which is in a reasonable agreement with the position of the fast drop of the HXR signal in Fig. 10(c) of [6] (i.e. $E_\phi/E_c \approx 2.7$). Such effect of fast loss of the RE population is miscalculated in a recent work [7] due to a simplified description of the secondary electron source, which assumes extremely high energies of the primary electrons.

Note that E_0 is close to unity when $\bar{\tau}_{rad}$ is high, and that the critical momentum $p_{min}(E_0) \equiv p_{max}(E_0)$ has relatively weak dependence on $\bar{\tau}_{rad}$, as shown in Fig. 3.

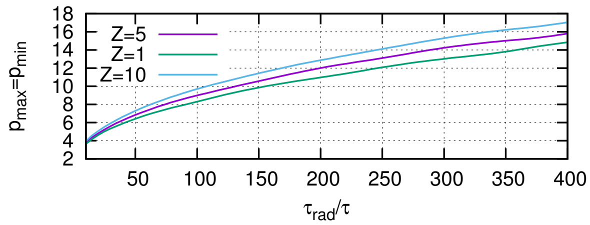

Figure 3. Dependance of the stable point $p_{min} = p_{max}$ at $E = E_0$ on the plasma parameters.

The near-threshold regime for runaway electrons determines the decay time of the toroidal current during runaway mitigation in tokamaks. To verify that the electric field does drop to the E_0 value during the current decay we compare our estimates with the self-consistent Monte Carlo modelling of the RE and the plasma current evolution [8]. In these calculations, a high-Z impurity (Ar) is introduced at the "plateau" stage of RE evolution, to mimic the Massive Gas Injection. Thus instantly changes E_c, Z and $\bar{\tau}_{rad}$ values.

Figure 4 shows an example of such comparison. In this example, a prescribed seed RE current initiates the avalanche, which reaches saturation by $\approx 15ms$. The injection of Argon with $n_{Ar} = 3 \cdot 10^{20} m^{-3}$ takes place at $\approx 30ms$. The subsequent evolution of the RE current is apparently linear. This is explained by the hysteresis phenomenon introduced in [2], which involves the avalanche threshold field $E_a > E_0$, thus bounding the inductive electric field to $E_a > E > E_0$ (E_a is only somewhat higher than E_0).

The green curves in Fig. 4 show the values expected from the analytical solution of the Eq. 1. The inductive electric field, self-consistently calculated from the Monte Carlo modelling of the RE kinetic equation with the current channel evolution, indeed, saturates at a value of $E_0 = 5V/m$, as predicted by Eq. (6). The average RE energy is also in a reasonable agreement with the p_{max} value, given that a certain time is required for equilibration of the distribution function. This timescale (the time required for the bump formation) can be estimated from the value of the flow velocity between p_{min} and p_{max} and the width of this interval.

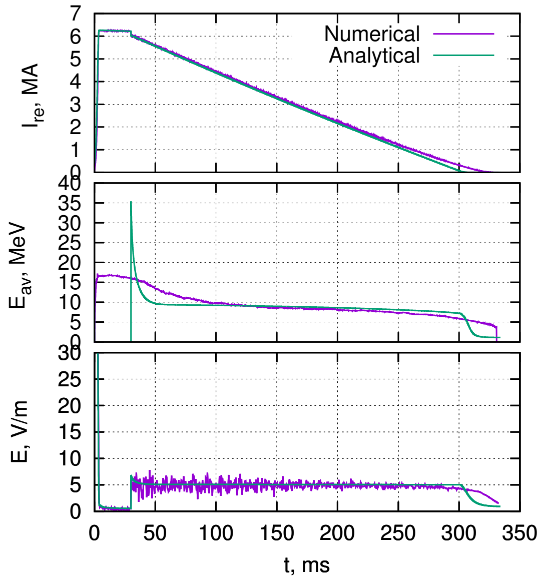

Figure 4. The comparison of RE current evolution (top), RE average energy (p_{max}) (middle) and inductive electric field (E_0) (bottom) between self-consistent calculations [8] and reduced model [2].

In summary, we have compared our approximate analytical solution of the kinetic equation [2] with the numerical results. This comparison confirms the peaking of the RE distribution function at the phase-space attractor near the expected p_{max} value. Self-consistent Monte Carlo modelling of the RE and the plasma current gives the electric field that provides sustainment of the RE. This electric field agrees with the theoretical predictions of the runaway sustainment threshold E_0. The sustainment threshold governs the passive RE current decay (RE mitigation timescale), which appears to be consistent with the numerical calculations as well as with what is seen in the mitigation experiments [9].

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References

[1] M. N. Rosenbluth, S. V. Putvinski, Nucl. Fusion **37**, 1355 (1997)

[2] P. Aleynikov, B. Breizman, Phys. Rev. Lett., **114**, 155001 (2015)

[3] J. R. Martin-Solis et. al., Phys. Plasmas **5**, 2370 (1998)

[4] M. Bakhtiari, et al. Phys. Rev. Lett., **94**, 215003 (2005)

[5] J. E. Guyer, D. Wheeler, J. A. Warren, Computing in Science & Engineering **11**(3) (2009)

[6] C. Paz-Soldan et al., Phys. Plasmas **21**, 022514 (2014)

[7] A. Stahl, E. Hirvijoki, J. Decker, O. Embreus, and T. Fülöp PRL **114**, 115002 (2015)

[8] P. Aleynikov, et al. 25th IAEA Fusion Energy Conference,TH/P3-38 (2014)

[9] E. Hollmann et al., Nucl. Fusion **53**, 083004 (2013).