The effect of entanglement in gravitational photon-photon scattering

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The differential cross section for gravitational photon-photon scattering calculated in perturbative quantum gravity is shown to be larger for polarization entangled photons than for not entangled photons. PACS numbers: 13.88.+e, 14.70.Bh, 14.70.Kv, 42.50.-p, 03.67.-a, 04.60.Bc, 04.60.-m

Introduction- Entanglement is an inherently quantum mechanical property. It is the basis of tests of non-classicality like the renowned Bell-tests [1, 2]. Hence, the question if there is an effect of entanglement of a physical system on its gravitational interactions is in the overlap of quantum mechanics and gravity which makes it a question of general physical interest.

In this article, we will investigate the gravitational effect of entanglement in perturbative quantum gravity by calculating the differential cross section of gravitational photon-photon scattering for polarization entangled photons and not entangled photons. First, we will shortly review the derivation of the polarization averaged differential cross section for gravitational photon-photon scattering in perturbative quantum gravity. Then, we will consider polarization entangled photons.

Photon-photon scattering- In perturbative quantum gravity, the metric is written as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ and $h_{\mu\nu}$ becomes a quantum field of spin two. The coupling to the electromagnetic field is given via the interaction Lagrangian

$$\mathcal{L}_I := -\kappa\, h_{\mu\nu} T^{\mu\nu} \,. \tag{1}$$

with $\kappa := \sqrt{8\pi G/c^3}$ [3, 4]. It was shown by N. Grillo

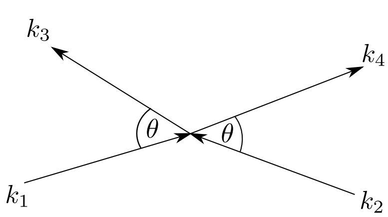

FIG. 1. Photon-photon scattering by the angle θ - in-going momenta k_1, k_2 and out-going momenta k_3, k_4.

that $\mathcal{L}_I$ gives rise to a finite perturbation theory even in the first loop order (see remark and reference in [5]). Here, only the second order tree-level will be considered.

As a consequence of the peculiar split of the metric into an a priori background metric $\eta_{\mu\nu}$ and a quantum-perturbation $h_{\mu\nu}$ we can avoid the conceptual problems of the standard interpretation of quantum mechanics applied to quantum gravity: the classical observers doing preparations and measurements live themselves in the spacetime that they prepare and measure. Our classical observers live on the classical, flat spacetime given

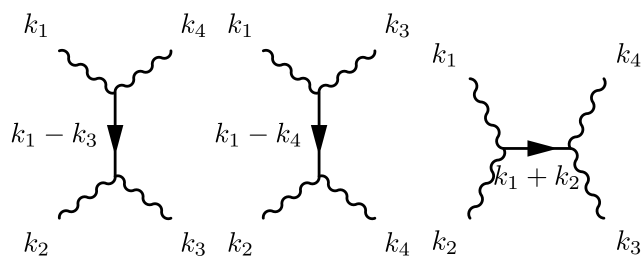

FIG. 2. Contributing diagrams

by $\eta_{\mu\nu}$. We assume that their detectors are infinitely far separated, and we assume that preparation and detection are performed at the temporal infinities. Furthermore, we assume that the gravitational interaction of the photons is the significant gravitational process beyond the effect of the classical metric background $\eta_{\mu\nu}$. Then, it seems safe to separate preparation and detection from the gravitational interaction process of the two photons, and the quantum properties of the metric do only appear in our considerations via virtual gravitons mediating the interaction between the photons.

Transition matrix elements and differential cross sections for the graviton mediated interaction of photons on second order tree-level were calculated in [6–8]. In [6] by Boccaletti et.al., the following expression is found for the nonzero transition matrix elements for the in-going momenta $k_1\,, k_2$ and the out-going momenta k_3, k_4 (see Fig. 1) in the center of momentum frame where $k_1 = -k_2$ and $k_3 = -k_4$ by evaluating the three contributing diagrams illustrated in Fig. 2:

$$M_{ijkl} = \delta^{(4)}(k_3 + k_4 - k_1 - k_2)\tilde{M}_{ijkl}\,, \tag{2}$$

with

$$\tilde{M}_{1111} = \tilde{M}_{2222} = \frac{\kappa^2}{4(2\pi)^2\sin^2\theta}[-9 - 6\cos^2\theta - \ cos^4\theta]$$

$$\tilde{M}_{1122} = \tilde{M}_{2211} = \frac{\kappa^2}{4(2\pi)^2\sin^2\theta}[\ \ 7 - 6\cos^2\theta - \ cos^4\theta]$$

$$\tilde{M}_{1212} = \tilde{M}_{2121} = \frac{\kappa^2}{4(2\pi)^2\sin^2\theta}[-8 - 4\cos\theta - 4cos^3\theta]$$

$$\tilde{M}_{1221} = \tilde{M}_{2112} = \frac{\kappa^2}{4(2\pi)^2\sin^2\theta}[-8 + 4\cos\theta + 4cos^3\theta]\,,$$

where θ is the angle between k_1 and k_3 (and equivalently k_2 and k_4) . The indices of the matrix elements refer to the polarizations of the photons with momentum k_1, k_2 and k_3, k_4 in that order. The index 1 refers to the polarization perpendicular to the plane of collision and the index 2 refers to the polarization parallel to that plane. In [6], the polarization averaged differential cross section is then given as

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2 E^2}{4} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=1}^{2} |\tilde{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}|^2 = \frac{128\pi^2\, l_P^4}{\lambda^2} \frac{\left[1+\cos^{16}\frac{\theta}{2}+\sin^{16}\frac{\theta}{2}\right]}{\sin^4\theta}\,, \tag{3}$$

where $l_P = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6162\times 10^{-35}m$ is the Planck length and E and λ are the common energy and wavelength of the photons, respectively, in the center of momentum frame.

The values of the differential cross section (3) are very small. E.g. for $\lambda = 500$nm and for $\lambda = 10$nm, we find for the factor $\frac{256\pi^2\, l_P^4}{\lambda^2}$ the order 10^{-124}m^2 and 10^{-120}m^2, respectively. This is extremely small even in comparison to the very small QED photon-photon scattering cross section [7] which is of the order 10^{-68}m^2 and 10^{-58}m^2 for $\lambda = 500$nm and $\lambda = 10$nm, respectively. And even the QED photon-photon scattering is far from being directly observable in experiments, and, thus, there is no chance to detect the gravitational photon-photon scattering in the near future directly. However, it is of conceptual interest.

In [9, 10], it was shown that the differential cross section in equation (3) coincides with the classical differential cross section for the scattering of two light pulses for small scattering angles. In contrast to the classical differential cross section, however, equation (3) does depend on the polarization of the photons. We will show below that this leads to a dependence of the differential cross section on the degree of polarization entanglement between the two photons.

Entangled photons- We denote one of the polarization directions as $|1\rangle$ and the other one as $|2\rangle$. The photons are distinguishable since they have different wave vectors. We set $k_1 = k = E/c$ and $k_2 = -k = -E/c$ without loss of generality. We consider the state

$$|\psi\rangle_e = \frac{1}{\sqrt{2(1+\iota^2)}} \left((1+\iota)|1\rangle_k|2\rangle_{-k} + (1-\iota)|2\rangle_k|1\rangle_{-k}\right)\,, \tag{4}$$

where ι parameterizes the entanglement of the state: for $\iota = 1$ and $\iota = -1$ the state is not entangled and for $\iota = 0$ it is maximally entangled. Then, the differential cross section can be read off directly from the matrix elements in (2) as

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2 E^2}{2(1+\iota^2)} \Big(|(1+\iota)\tilde{M}_{1212} + (1-\iota)\tilde{M}_{2112}|^2 + |(1+\iota)\tilde{M}_{1221} + (1-\iota)\tilde{M}_{2121}|^2\Big) = \frac{128\pi^2\, l_P^4}{\lambda^2} \frac{\left[4+\iota^2\left(cos\theta+\cos^3\theta\right)^2\right]}{2(1+\iota^2)\sin^4\theta}\,. \tag{5}$$

Hence, the differential cross section for the maximally entangled state and the one for the not entangled states coincide for the scattering angle $\theta \to 0$ and the former is larger than the latter for $\theta > 0$ (see Fig. 3): polarization entangled photons gravitate more than not entangled photons.

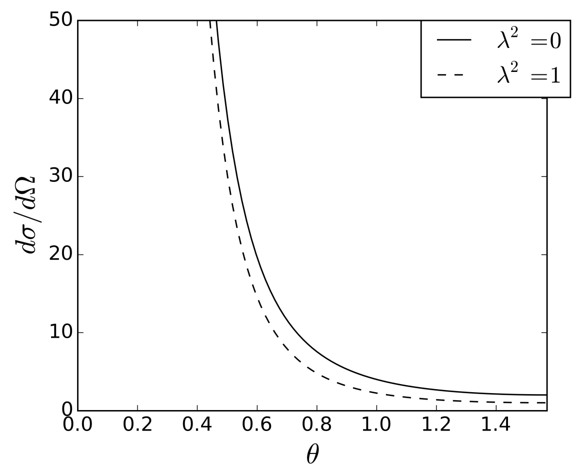

FIG. 3. The differential cross section for not entangled photons ($\lambda^2 = 1$) and maximally entangled photons ($\lambda^2 = 0$) in units of $128\pi^2 l_p^4/\lambda^2$.

Conclusions- In the framework of perturbative quantum gravity, the differential cross section for the scattering of two photons was derived in [8] and [6]. We used the result of [6] to show that polarization entangled photons gravitate more. Since entanglement is an inherently quantum mechanical property, the effect we found can be only consistently investigated in a framework of quantum gravity. But even if we ignore the conceptual problems, it is not possible to derive the results presented here with semi-classical gravity. The effect we found is based on the dependence of the gravitational interaction of photons on their polarization direction, but the classical gravitational field of light is independent of its polarization direction. In a gravity theory with torsion like Einstein-Cartan-Theory and the Poincaré-Gauge-Theory of gravity [11] the gravitational field of light depends on the polarization direction. However, in these theories, the electromagnetic field cannot be coupled minimally to the gravitational field without loosing its gauge invariance [12]. One has to resort on non-minimal coupling, and the constitutive tensor is modified. A way to deal with such modifications was developed in [13] and [14]. This program could be used to investigate the polarization dependent gravitational interaction of photons in a semi-classical approach and compare the results with the findings presented here.

It could be worthwhile to apply our approach to the interaction of photon wave packets. This would enable the derivation of the gravitational interaction of photons entangled in position or momentum degrees of freedom which was already considered by D. E. Bruschi using semi-classical gravity with all its inherent conceptual issues (for more about the conceptual issues of semi-classical gravity see [15–19]). For the sake of conceptual rigor, it could be also interesting to consider the gravitational interaction of entangled photons in the general boundary formulation of quantum field theory [20, 21], where the scattering process could be restricted to a compact spacetime region. It would be also very interesting to investigate the gravitational effect of entanglement in a background independent framework of quantum gravity like Loop Quantum Gravity.

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