Bouncing cosmologies from quantum gravity condensates

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We show how the large-scale cosmological dynamics can be obtained from the hydrodynamics of isotropic group field theory condensate states in the Gross-Pitaevskii approximation. The correct Friedmann equations are recovered in the semi-classical limit for some choices of the parameters in the action for the group field theory, and quantum gravity corrections arise in the high-curvature regime causing a bounce which generically resolves the big-bang and big-crunch singularities.

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Introduction — A major challenge for any theory of quantum gravity is to extract its large-scale physics at cosmological scales in order to make predictions about the early universe.

We will consider this problem within the scope of loop quantum gravity (LQG) [1] and group field theory (GFT) [2], in which the fundamental building blocks of space-time are quanta of geometry, elementary excitations above a fully degenerate 'no-space' vacuum. The recovery of semi-classical gravity requires then the manipulation of states involving a large number of these elementary quanta, whose collective behaviour will determine the effective theory via coarse-graining of the microscopic dynamics. For the case of the simplest example, the spatially flat, homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) space-time, the calculation of the effective dynamics as determined by the particular quantum gravity model will require: the identification of a family of cosmological states, controlling the relevant global degrees of freedom (e.g., Hubble rate, matter energy density), and then a coarse-graining of the dynamics to extract the large-scale effective Friedmann equations.

Interestingly, even though loop quantum cosmology (LQC) —where symmetry-reduced space-times are quantized mimicking the procedures of LQG [3]— starts from a different perspective, it gives further insight into the type of states that correspond to the cosmological sector of LQG. While the exact relation between LQG and LQC remains an open question, a heuristic relation between the two theories has been proposed [4–7], suggesting that the LQG states corresponding to the cosmological sector are spin network states with a large number of nodes, all labeled by the same quantum numbers, and where all links are coloured by the same $SU(2)$ representation label (typically taken to be $j = 1/2$), and the connectivity information of the nodes is disregarded. Due to homogeneity, it is usually further assumed that all of the nodes in the spin network have the same valence (here, for simplicity we assume all nodes are four-valent). In short, LQC suggests to consider states in which all the (many) quanta of geometry are in the same state, i.e., a condensate of quantum geometry.

All these features are in fact naturally encoded in a specific family of states: the GFT condensates [8–10] that have been recently used to extract cosmology from the full GFT formalism. GFTs can be seen as a second-quantized reformulation of LQG [11], embedding nicely in a single quantum field theory formalism the tools and results of LQG and spin foams. We refer to [12] and references therein for a detailed presentation of the conceptual and technical aspects related to GFT condensates. The key point here is that these states, belonging to the Hilbert space of the fundamental theory, offer a unique opportunity to tackle the problem of the emergence of cosmological dynamics directly at the level of a candidate quantum gravity theory, taking into account the effects of all of the degrees of freedom.

Group Field Theory — In GFT models for geometry coupled to a (massless) scalar field, the elementary GFT field operators, in the spin representation, are

$$\hat{\varphi}^{j_v,\iota}_{m_v}(\phi) = \hat{\varphi}^{j_1,j_2,j_3,j_4;\iota}_{m_1,m_2,m_3,m_4}(\phi) \tag{1}$$

and its conjugate $\hat{\varphi}^{\dagger j_v,\iota}_{m_v}(\phi)$ [11, 12]. They can be taken to be bosonic ladder operators acting on a Fock space. The creation operator $\hat{\varphi}^{\dagger j_v,\iota}_{m_v}(\phi)$ creates a quanta of geometry: a four-valent spin network node, each link coloured by j_i and m_i, and with the intertwiner ι associated to the usual gauge invariance of spin network nodes, for a given value ϕ of the scalar field.

The familiar geometric operators of LQG can be imported to the second-quantized language; generic operators can be constructed in terms of strings of ladder operators. The presence of an additional argument, the matter field ϕ, also permits the definition of relational observables, i.e., operators evaluated at a specific value of ϕ, and these are crucial for describing physical evolution in a fully diffeomorphism invariant language. For example, the number operator counting the number of quanta above the Fock vacuum $|0\rangle$ when the matter field assumes the value ϕ is given by

$$\hat{N}(\phi) = \sum_{j,m,\iota} (\hat{\varphi}^\dagger)^{j_v,\iota}_{m_v}(\phi)\, \hat{\varphi}^{j_v,\iota}_{m_v}(\phi). \tag{2}$$

The microscopic dynamics is controlled by equations of motion that also determine the Feynman expansion. The various monomials in the field operators are chosen to allow for a natural mapping between Feynman graphs and four-dimensional (simplicial) complexes decorated with geometric data, with amplitudes encoding suitably discretized gravitational actions evaluated on the given discrete configurations. In particular, one can define GFT models where these amplitudes exactly match any desired spin foam amplitudes (e.g., those directly motivated from LQG). These are easily generated starting from simple action functionals, that we split into linear and nonlinear parts as $S[\varphi,\bar{\varphi}] = K[\varphi,\bar{\varphi}] + V[\varphi,\bar{\varphi}]$, with the kinetic term encoding the edge amplitude of the spin foam model and having the form (with a minimally coupled massless scalar field)

$$K = \sum_{j,m,\iota} \int \mathrm{d}\phi_1 \mathrm{d}\phi_2 \Big[\bar{\varphi}^{j_{v_1},\iota_1}_{m_{v_1}}(\phi_1)\, \varphi^{j_{v_2},\iota_2}_{m_{v_2}}(\phi_2) \times K^{j_{v_1},j_{v_2},\iota_1,\iota_2}_{m_{v_1},m_{v_2}}((\phi_1-\phi_2)^2)\Big], \quad (3)$$

while the potential $V[\varphi,\bar{\varphi}]$ encodes the vertex amplitude, is of fifth order in the field variables φ and $\bar{\varphi}$ (for simplicial GFT models) and is local in the scalar field ϕ.

It is convenient to rewrite the kinetic term as a derivative expansion in ϕ in the field variable $\varphi^{j_{v_2},\iota_2}_{m_{v_2}}(\phi_2)$ around $\phi_2 = \phi_1 = \phi$, giving

$$K = \sum_{n=0}^{\infty} \sum_{j,m,\iota} \int \mathrm{d}\phi\, \bar{\varphi}^{j_{v_1},\iota_1}_{m_{v_1}}(\phi) \varphi^{j_{v_2},\iota_2}_{m_{v_2}}(\phi) (K^{(2n)})^{j,\iota}_{m}, \quad (4)$$

where the notation on $K^{j,\iota}_m$ has been compressed, and

$$(K^{(2n)})^{j,\iota}_m = \int \mathrm{d}u\, \frac{u^{2n}}{(2n)!} K^{j,\iota}_m(u^2). \quad (5)$$

In cases where the difference between ϕ_1 and ϕ_2 in (3) is small compared to the Planck mass (i.e., a slowly changing scalar field), a good approximation to the full kinetic term can be provided by a truncation of the derivative expansion. This is the case we will consider here, keeping only the first two non-trivial terms $n = 0$ and $n = 1$.

Finally, for a GFT model with the action $S[\varphi,\bar{\varphi}]$, the quantum equations of motion for a state $|\Psi\rangle$ are simply

$$\widehat{\frac{\delta S}{\delta \bar{\varphi}}} |\Psi\rangle = 0, \quad (6)$$

together with the conjugate of this equation.

As with any interacting field theory, it is not possible to obtain the general solution of these equations. The particular formulation given by GFT, however, allows us to make use of ideas and methods that are used in analogous problems in condensed matter physics. We will seek some state that approximates a full solution state $|\Psi\rangle$, at least for a restricted set of observables. The restriction to the case of homogeneous cosmologies suggests that these states should be modeled with a wave function homogeneity principle [8–10, 14], i.e., by condensate states in which the wave functions associated to the each of the quanta are the same.

Isotropic Condensates — The simplest way to model such cosmological states, including an arbitrary large number of quanta, is to use the field coherent states

$$|\sigma\rangle = e^{-\|\sigma\|^2/2} \exp\left(\sum_{j,m,\iota} \sigma^{j_v,\iota}_{m_v}(\phi) (\hat{\varphi}^\dagger)^{j_v,\iota}_{m_v}(\phi) \right) |0\rangle, \quad (7)$$

where $\sigma^{j_v,\iota}_{m_v}(\phi)$ is the condensate wave function and $\|\sigma\|^2 = \int \mathrm{d}\phi\, \|\sigma(\phi)\|^2$. An important point here is that the condensate wave function is not normalized: rather the norm of $\sigma^{j_v,\iota}_{m_v}(\phi)$,

$$\|\sigma(\phi)\|^2 = \sum_{j,m,\iota} |\sigma^{j_v,\iota}_{m_v}(\phi)|^2, \quad (8)$$

is the expectation value of the number operator $\hat{N}(\phi)$ on the condensate state $|\sigma\rangle$ at the relational time ϕ.

These states have been extensively studied in the GFT context [8–10] as approximate solutions of the quantum equations of motion. As they neglect correlations between different quanta (and thus the connectivity of the spin network nodes), these are approximate solutions only in regimes in which the interaction term in (6) is subdominant.

Since we are only interested in the homogeneous and isotropic degrees of freedom, it is possible to choose a particularly simple form of the condensate wave function by imposing that the condensate wave function be isotropic, i.e., that all of the spin labels be equal, and that the other geometric indices be uniquely defined by j. Hence, for an isotropic condensate wave function,

$$\sigma^{j_v,\iota}_{m_v}(\phi) = C^{j_v,\iota}_{m_v} \cdot \sigma_j(\phi), \quad (9)$$

where the $C^{j_v,\iota}_{m_v}$ are uniquely determined by the value of j (in particular, the intertwiner is chosen so that it is an eigenvalue of the LQG volume operator and that its eigenvalue is the largest possible for a spin network node with four links all coloured by j, see [12] for details). Therefore, the coarse-grained degrees of freedom of isotropic GFT condensate states are entirely captured by the functions $\sigma_j(\phi)$, one for each spin.

The effective dynamics are obtained by asking that the condensate states (7) approximately solve the quantum equations of motion (6). To be specific, we assume a simple Gross–Pitaevskii form of the dynamics, obtained by taking the expectation value of the equations of motion:

$$\langle\sigma| \widehat{\frac{\delta S}{\delta \bar{\varphi}}} |\sigma\rangle = 0, \quad (10)$$

which is clearly a weaker condition than (6).

For the isotropic GFT condensate states (7), and for a GFT model with a minimally coupled massless scalar field whose geometric contribution is based on the Engle–Livine–Pereira–Rovelli spin foam model [13] (the most developed one for 4D Lorentzian quantum gravity), (10) gives the equation of motion for the $\sigma_j(\phi)$

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \bar{\sigma}_j(\phi)^4 = 0. \quad (11)$$

It is clear that the scalar field ϕ is acting as a relational clock here and can be interpreted as 'time'. This will be important when extracting the coarse-grained cosmological dynamics from this condensate state. Here A_j

depends on a combination of $(K^{(2)})^{j,\iota}_m$ and $C^{j,\iota}_m$, B_j depends on $(K^{(0)})^{j,\iota}_m$ and $C^{j,\iota}_m$, while w_j (and the form of the fourth order term in the condensate wave function) depends on the specific form of $V[\varphi, \bar\varphi]$ as well as $C^{j,\iota}_m$. We refer to [12] for the exact relations.

This Gross–Pitaevskii approximation of the dynamics, based on the ansatz (7) for the quantum state, is expected to hold in a weakly interacting regime. Indeed, when the last term in (11) will become important for sufficiently large $|\sigma_j(\phi)|$, this approximation will no longer be valid [12]. In the 'strong interaction' regime, correlations between quanta (encoding connectivity information) will play a prominent role, and simple coherent states will have to be replaced by more elaborated condensate states (perhaps of the type introduced in [14]). Therefore, in the following we will only consider the mesoscopic regime in which there are a sufficient number of quanta for a hydrodynamical picture to exist, but small enough for the non-linear term in (11) to be subdominant.

In this mesoscopic regime, rewriting $\sigma_j(\phi) = \rho_j e^{i\theta_j}$ in terms of its modulus and phase, and denoting derivatives with respect to ϕ by primes, (11) gives the two equations of motion

$$\rho_j'' - \Big(m_j^2 + (\theta_j')^2\Big)\rho_j \approx 0, \tag{12}$$

$$\rho_j \theta_j'' + 2\rho_j'\theta_j' \approx 0, \tag{13}$$

with $m_j^2 = B_j/A_j$. In this approximation, the quantities

$$E_j = (\rho_j')^2 + \rho_j^2(\theta_j')^2 - m_j^2\rho_j^2, \tag{14}$$

$$Q_j = \rho_j^2\theta_j', \tag{15}$$

are conserved with respect to the relational time ϕ. Then, using Q_j, (12) becomes

$$\rho_j'' - m_j^2\rho_j - \frac{Q_j^2}{\rho_j^3} \approx 0. \tag{16}$$

Emergent Friedmann Equations — To extract the dynamics of the large-scale coarse-grained cosmological observables, these observables must be related to the microscopic GFT degrees of freedom. In this case, this procedure is straightforward since the quantities of interest are the total volume and the momentum of the massless scalar field, evaluated at a (relational) time ϕ.

These operators, evaluated on the isotropic GFT condensate states, are respectively

$$V(\phi) = \sum_j V_j\,\rho_j(\phi)^2, \tag{17}$$

where, given the definition of the isotropic condensate states, $V_j \sim j^{3/2}\ell_{\rm Pl}^3$ is the largest eigenvalue of the LQG volume operator for a spin network node with four links labeled by j, and

$$\pi_\phi(\phi) = \frac{\hbar}{2i}\sum_j \Big(\bar\sigma_j(\phi)\sigma_j(\phi)' - \bar\sigma_j(\phi)'\sigma_j(\phi)\Big). \tag{18}$$

It is easy to check that $\pi_\phi = \hbar\sum_j Q_j$ and that therefore the momentum of the massless scalar field is a constant of the motion in this context. This is exactly the continuity equation: for an FLRW space-time with a massless scalar field, the continuity equation reduces to $\pi_\phi' = 0$.

Therefore, the only other equation of motion to be recovered in order to determine the coarse-grained cosmological dynamics is the Friedmann equation relating the Hubble rate to the energy density of the matter field. This can be calculated, using $V' = 2\sum_j V_j\,\rho_j\rho_j'$, to be

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\rho_j\sqrt{E_j - Q_j^2/\rho_j^2 + m_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2. \tag{19}$$

This and $\pi_\phi' = 0$ are the emergent (generalized) Friedmann dynamics coming from the microscopic GFT quantum equations of motion for isotropic condensate states.

In order to check that the correct semi-classical limit is obtained, we take ρ_j to be sufficiently large so that $m_j^2\rho_j^2$ is the dominant term in the square root in the numerator. (Note that strictly speaking, the semi-classical limit corresponds to the low curvature limit, not the large volume or large ρ_j limit. However, in FLRW space-times the terms that dominate in the Friedmann equation at low curvatures are the same as those that dominate at large volumes, simply because as the FLRW space-time expands the space-time curvature decreases.) Then, in this semi-classical limit

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j m_j\rho_j^2}{3\sum_j V_j\rho_j^2}\right)^2. \tag{20}$$

It is clear that in order to reproduce the classical Friedmann equation $(V'/3V)^2 = 4\pi G/3$ (expressed in terms of the relational time ϕ), we must have $m_j^2 = 3\pi G$ for all j. Therefore, we have shown that a rather large class of GFT models (those giving the correct value of m_j) have a coarse-grained, large-scale dynamics of isotropic condensate states with modified Friedmann equations that have the correct semi-classical limit, and with quantum gravity corrections when the space-time curvature is sufficiently large. This is our first main result.

Note that this result also suggests that the classical Friedmann dynamics is a rather universal emergent feature of GFT quantum gravity (provided a discrete geometric interpretation of the kinematical data is possible), since most differences between specific microscopic models can be encoded in the interaction terms, which are here subdominant. This is consistent with the hydrodynamics perspective on cosmology we have adopted.

The quantum gravity corrections play an important role when the space-time curvature becomes large and give rise to a striking feature of the emergent cosmological dynamics. As is obvious from (16), the ρ_j can never become zero but will instead necessarily reach a turning point due to the repulsive and divergent 'potential' $-Q_j^2/\rho_j^3$, assuming $Q_j \neq 0$. Note that in an FLRW space-time, $\pi_\phi \neq 0$ and so at least one Q_j must be non-zero. Therefore, it immediately follows that at least one $\rho_j > 0$ for all ϕ, and clearly the volume V will never be zero. As a result, the big-bang and big-crunch singularities of classical general relativity are generically resolved here, due to quantum corrections, within a full quantum gravity formalism. Furthermore, there will be a bounce since there is only one turning point for each ρ_j in (16). This is our second main result.

Finally, it is possible to consider the case where $\sigma_j(\phi)$ only has support on a single $j = j_o$ — these are GFT states that closely match the heuristic relation between LQG and LQC. In that case, the sums in the Friedmann equation (19) trivialize, and (setting $m_{j_o}^2 = 3\pi G$)

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o} E_{j_o}}{9V}, \qquad (21)$$

where $\rho = \pi_\phi^2/2V^2$ is the energy density of the massless scalar field and the critical energy density is $\rho_c = 6\pi G\hbar^2/V_{j_o}^2$, with $V_{j_o} \sim j_o^{3/2}\ell_{\rm Pl}^3$. Interestingly, this equation is almost identical to the LQC effective equations, with the only difference being the last term. For states with $E_{j_o} = 0$, the resulting effective Friedmann dynamics will then be exactly that of LQC. While the geometric interpretation of the 'energy' E_{j_o} is not clear, its effect on the dynamics is: if $E_{j_o} > 0$, the cosmological bounce will occur at a higher space-time curvature, while if $E_{j_o} < 0$ then the bounce will occur at a lower space-time curvature. Even the effective LQC dynamics, then, can be obtained from a complete quantum gravity formalism. This is our third main result.

Discussion — In this paper, we have argued that cosmological states in GFT (for models with a direct LQG interpretation of quantum states and dynamics) correspond to highly-excited (with respect to the degenerate vacuum) condensate states. We have shown how to extract the hydrodynamical equations for the coarse-grained cosmological observables from the microscopic quantum dynamics. The correct Friedmann equations are recovered in the semi-classical limit for a large class of GFT models (i.e., those whose action gives $m_j^2 = 3\pi G$).

Strikingly, the cosmological singularity is generically resolved, due to quantum gravity corrections, and is replaced by a bounce. Finally, a specific choice of condensate states actually reproduces very closely the effective LQC dynamics. These results open a very promising route for studying cosmology from within a complete quantum gravity formalism, and show that the macroscopic cosmological consequences of the underlying microscopic quantum geometry can be analysed in detail.

These results are based on a number of necessary assumptions: (i) we assumed that, in the cosmological sector of GFT and LQG, the massless scalar field does not evolve rapidly compared to the Planck mass, keeping only the first two terms of the derivative expansion of the GFT kinetic term, (ii) we neglected the connectivity of the spin networks since it is not needed in order to extract isotropic and homogeneous observables, (iii) we only imposed that the expectation value of the quantum equations be zero, and (iv) we worked in the regime in which the contribution of interactions to the equations of motion are subdominant, this being necessary for the approximation (ii) to be valid.

Despite the above limitations, these results provide important insights on the cosmological sector of GFT and LQG, and are also in strong qualitative agreement with those obtained in the (homogeneous) LQC context.

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