The four essays presented in this collection are based on talks given at the Eurosymposium Galileo 2001 in Tenerife. Authored by members of the Institute, they share the same approach while dealing with different aspects of Galileo’s work.¹

We would also like to draw the reader’s attention to Raymond Fredette’s contribution to the same conference, which is already a part of the preprint series.²

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This paper discusses Galileo’s unpublished treatises related to his theory of motion of which we find many traces in his extant manuscripts such as his notes on motion, manuscript 72 of the Galilean collection in Florence. It may thus seem as if this paper focusses on an aspect of the emergence of classical mechanics which is relevant only for those specialists interested in the biographical origins of Galileo’s contributions. We shall argue, however, that a study of unpublished manuscripts from the perspective of a historical epistemology reveals structures of the development of scientific knowledge which tend to be obscured by focusing, as it is common, only on published writings. Such focusing on published writings is, in fact, inherent in interpreting the development of scientific knowledge as resulting from individual contributions that become effective only through publications.

The common concentration on published works is based on two precarious assumptions that will be challenged in the following.

- First, it is usually assumed that the progress of scientific knowledge, in particular the emergence of classical mechanics, is essentially determined by a sequence of events starting with individual discoveries, disseminated by publications, and finally evaluated by the reception of the scientific community.
- Second, it is usually assumed that it were specifically Galileo’s publications that represent the birth of the classical theory of motion, essentially unburdening himself of the millenary tradition of a mistaken natural philosophy.

We do not question the facts which can be put forward in order to justify such assumptions, but the rationale of the argument itself.
THE INVISIBLE HAND OF SHARED KNOWLEDGE

We will illustrate historical circumstances which may raise doubts whether such assumptions can be considered as a sound base of historical research by a thought experiment. Let us hypothetically assume that a scholar contemporary to Galileo pursued experiments with falling bodies and discovered the law of fall as well as the parabolic shape of the projectile trajectory, that he found the law of the inclined plane, directed the newly invented telescope to the heavens and discovered the mountains on the moon, observed the moons of the planet Jupiter and the sunspots, that he calculated the orbits of heavenly bodies using methods and data of Kepler with whom he corresponded, and that he composed extensive notes dealing with all these issues. In short, let us assume that this man made essentially the same discoveries as Galileo and did his research in precisely the same way with only one qualification: He never in his life published a single line of it. Would we deny him credit for any contribution to the history of science just because he did not influence the scientific community with any publication? Would we consider such parallelism of developments as insignificant on account one of the two scholars not influencing the alleged normal chain of events from discovery, via publication to reception?

As a matter of fact, the above description refers to a real person, Thomas Harriot, whose work closely resembles that of Galileo. But how can the striking parallelism between the two scientists be explained? Was there perhaps a secret “influence” which has so far escaped the scrutiny of historians, or is there any other explanation? From the usual perspective, the development of scientific knowledge is considered to be a process involving individual ideas, their reception by a community, and the effect of the ideas of one individual on the other in terms of so-called “influences.” We will argue, on the other hand, that the often striking similarities in the work of Galileo and his contemporaries can be explained by common structures of knowledge, common challenges, and a similar social environment conditioning the communication of scientific information, rather than by the exchange of ideas embodied in publications.

1 For biographical information on Thomas Harriot, see [Shirley, J. W., 1983]. For a comparison between Galileo’s and Harriot’s work on projectile motion, see the contribution by Matthias Schemmel in this preprint.
This becomes particularly evident in the fate of Galileo’s unpublished writings. It turns out that, in many cases, his writings remained unpublished not so much because he failed to succeed in completing his ambitious projects of publication, but for other reasons. We argue that these projects were, in fact, overturned by the development of shared knowledge. To use concepts introduced by Yehuda Elkana, these projects were either superseded by the dynamics of bodies of knowledge which Galileo shared with his contemporaries, or they were hampered by images of knowledge, that is, by ideas about knowledge that are particularly susceptible to political as well as biographical circumstances. Indeed, the development of knowledge proceeded then, as it does today, at a different pace from that of the production of publications.

GALILEO’S UNPUBLISHED WRITINGS

Among the oldest unpublished writings of Galileo, a treatise on a hydrostatic balance has been preserved, entitled *La bilancetta* dating from 1586. A treatise on centers of gravity, entitled *Theoremata circa centrum gravitatis solidorum* from 1587 originally also remained unpublished and was only added as an appendix to the *Discorsi* published half a century later. These treatises show him - like contemporaries such as Guidobaldo del Monte - as an avid follower of Archimedes who had not only assiduously acquired the mathematical techniques of Archimedean mechanics but also shared the Archimedean interest in its practical applications.

When appointed as a professor at the university of Pisa in 1589 Galileo followed an entirely different path. Probably from this time, several unwritten treatises dealing with questions of Aristotle’s natural philosophy and logic are preserved, among them treatises entitled *De mundo, De caelo, De elementis,* and *De demonstratione.* Had these treatises been published they would have shown Galileo as a typical philosopher of his time, diligently studying and disseminating the doctrines of contemporary Aristotelian natural philosophy, and not as the creator of a new science of motion.

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2 See [Elkana, Y., 1988].
3 See [Galilei, G., 1890-1909, I: 215ff.].
5 See [Wohlwill, E., 1993] and [Drake, S., 1978].
Probably the most well known unpublished treatise of Galileo is his manuscript *De motu antiquiora* from around 1590.\(^7\) Actually the manuscript bundle thus labeled comprises several more or less complete versions and notes for even more treatises. They document that he shared the anti-Aristotelian ambitions of many of his contemporaries and that he, just as Benedetti or Guidobaldo before him, grasped the opportunity to revise Aristotle’s theory of motion by reformulating it as a theory of motion in media with the help of Archimedean hydrostatics.\(^8\)

After his move to Padua in 1592, Galileo, aside from his duties as a professor, engaged in quite different activities such as giving private lessons and advising the Venetian republic on technical matters.\(^9\) Such activities were typical of an engineer-scientist of that time. In the context of these activities he wrote or planned several treatises that remained mostly unpublished but circulated among his contemporaries, two treatises on fortification, a treatise on mechanics, entitled *Le meccaniche*, several versions of the manual for the use of his military compass, eventually published only in view of a priority dispute, and finally a treatise on the sphere, entitled *Trattato della sfera ovvero cosmografia*.\(^10\)

The contemporary discussion on Copernicanism confronted Galileo with various occasions to express his changing views on the subject in writings that remained largely unpublished. These writings ranged from anticopernican side-remarks in his treatise on the sphere, via refutations of anticopernican arguments of philosopher colleagues starting as early as 1597, to planned or actually written treatises on mechanical arguments interpreted as evidence in favor of the Copernican system.\(^11\) Among these arguments is an attempt to explain the tides along the model of a swinging pendulum, an attempt which is documented by manuscripts comprising notes dating back as early as the 1590s,\(^12\) and also an unpublished treatise of 1616, entitled *Discorso del flusso e reflusso del mare*.\(^13\) An attempt to explain the constitution of the planetary system along

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8 See [Benedetti, G. B. d., 1585] and [del Monte, G., ca. 1587-1592, p. 41].
9 See the contribution by Matteo Valleriani in this preprint.
10 See [Galilei, G., 1890-1909, II: 7-191, 203-253, 335-510].
12 See [Galilei, G., ca. 1602 - ca. 1637, folio 154 recto]. Galileo’s idea of a tidal theory is also documented in notes from the year 1595 that are found in a notebook of Paolo Sarpi; see [Sarpi, P., 1996, 424-427], notes number 569, 570 and 571; see also the discussion in [Renn, J., et al., 2000].
the model of projectile motion is documented by notes dating from around 1604. In general, most of Galileo’s early treatises on Copernican issues remained unpublished or were merely circulated in the form of letters.

Manuscripts and correspondence show that in the period of his most intense work on problems of motion around 1602, Galileo focused on two key problems of potentially high practical impact, ballistics and the pendulum. Evidently he planned to dedicate treatises to these subjects, which, in spite of the tremendous work invested in them, remained unwritten. Several years later in 1610, he claimed to have almost completed a number of treatises, among them three books offering an entirely new theory of motion which, however, probably did not exist or, at least, remained unpublished.

In his position as a philosopher and mathematician at the Medici Court, which he took on in 1610, Galileo produced numerous scientific writings not intended for publication. Several of these unpublished writings contain expositions of Galileo’s scientific achievements in the context and for the purpose of technical applications. A Galilean treatise has been preserved as an appendix to a diplomatic note written in 1612 by the Grand Duke to the government of Spain concerning free Tuscan access to the East Indies as well as the West Indies. It describes a method for determining longitude on the basis of Galileo’s observations of the Jupiter satellites. With several unpublished writings on hydraulics, among them a lengthy expertise on the regulation of the river Bisenzio written in 1630, Galileo fulfilled his function as an expert advisor to the Tuscan government.

Galileo’s conclusive work of 1638, the Discorsi, actually comprises only a fraction of his writings on motion and mechanics. While books on motion such as the one he had promised in 1610 were actually included in the form of fictive treatises, several other planned or written treatises

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14 See [Galilei, G., ca. 1602 - ca. 1637, folios 134,135 and 146].
15 See [Hill, D. K., 1994].
16 See the letter to Belisario Vinta, May 7, 1610, [Galilei, G., 1890-1909, X: 348-353].
17 See e.g., [Galilei, G., 1890-1909, VIII: 571-587].
19 See [Galilei, G., 1890-1909, VI: 627-647].
dealing with subjects such as percussion or the hanging chain remained unpublished. In spite of unceasing attempts to cover all of this material in his definitive publication by adding ever new chapters to it, the Discorsi themselves were eventually published merely as a torso.\textsuperscript{20}

\section*{The Heritage of Shared Knowledge}

Even this brief survey of Galileo’s unpublished treatises makes it more evident than his few strategically placed publications that his work was conditioned by an array of transmitted knowledge systems which he shared with his contemporaries. They were moreover constrained by controversial interpretations of their social status and their cultural meaning. The most obvious case is represented by the controversial cosmological models developed in the context of the millenary tradition of astronomy, that is, the Ptolemaic, the Tychonic, and the Copernican models of the mechanism of planetary motion. To come to another body of knowledge, Galileo’s deductive theory of motion was evidently not unrelated to mechanics. It was, however, not an immediate continuation of the tradition of deductive mechanical treatises. It rather copied the Archimedean model of constructing deductive theories of physical phenomena. Both approaches were in any case shaped by the tradition of mathematical theory. Obviously the mathematical tradition going back to antiquity did not only provide Galileo’s theory of motion with a model for its deductive form but also with its most powerful instrument of the mathematical analysis of space, Euclidean geometry. Furthermore, Galileo’s new science of motion shared its subject with the dominating doctrines of the natural philosophy of the time, rooted in the ancient tradition of Aristotelian physics, elaborated, questioned, and in various ways revised by generations of commentators of medieval scholastic Aristotelism. Last but not least, let us stress the significance of a further body of knowledge. Most important but often overlooked is the fact that Galileo’s theory of motion draws heavily on the knowledge accumulated in the handicraft and engineering tradition which reaches even further back in history than any theoretical reflection on it.\textsuperscript{21}

\begin{flushright}
\textsuperscript{20} See [Renn, J., et al., 2000].
\textsuperscript{21} See [Renn, J., et al., in preparation].
\end{flushright}
The heritage thus only briefly outlined did obviously not consist of one homogenous body of knowledge but rather of diverse strands of theoretical traditions based on partly incompatible foundations whose mutual relations represented one of the intellectual challenges of the time. This conflict-laden heritage was the basis for any conceptualization of motion in the early modern period and, at the same time, provided numerous obstacles against modifications endeavored in order to meet new challenges.

In the following, we will discuss several examples of shared knowledge. They are taken from Galileo’s unpublished treatises. From the point of view of historical epistemology we will analyze how the emergence and dissemination of his science of motion grew out of the shared knowledge of the time.

**EXAMPLE 1: INTUITIVE PHYSICS AND ARISTOTELISM**

We first turn to Galileo’s relation to Aristotelian physics as it becomes particularly evident in the light of his unpublished treatises. It is a common characteristic of Galileo and his contemporaries that their attitude towards Aristotelian physics was anything but simple. Based on their familiarity with Galileo’s publications only, mid-nineteenth century historians of science have coined an image of Galileo which characterizes him, in contrast to contemporary philosophers, as an ardent defender of experience against Aristotelian dogmatism. When Favaro published most of Galileo’s extant treatises it became clear that this image of Galileo was untenable. Nevertheless this image was never in principle revised. Even today the fact that the young Galileo composed a number of Aristotelian treatises is often considered as merely the excusable lapse of an immature scientist. From our viewpoint, however, Galileo’s unpublished treatises suggest that there is no principal difference between the attitudes of Galileo and of his contemporaries towards Aristotle. In fact they provide a context for his published writings which makes clear that Galileo shared with his contemporaries the adherence to essential assets of Aristotelian physics and that he did so for good reasons.
As a matter of fact certain types of physical knowledge predate any systematic theoretical treatment. The most basic knowledge presupposed by physics is based on experiences acquired almost universally in any culture by human activities such as moving one’s body around or handling objects under normal conditions characteristic of our natural environment on earth.

Although the fundamental notions of such physical knowledge are closely related to concepts of classical mechanics, they do not correspond to its basic categories. Most of them turn out to rather resemble the basic assumptions of Aristotelian physics, including its medieval elaboration, as well as its early modern critique. Solid bodies usually need a force to be moved. This force depends on the amount of motion to be produced; stronger forces can move greater bodies and cause a greater motion. Once bodies are moving they have acquired a certain impetus that makes them continue to move for some time before they come to rest. A moving body can itself exert a force on objects which resist its motion. In order to stop a moving body, some counter-acting force is required depending on the size of the body and the vividness of the motion. All such experiences together constitute a sufficiently reliable basis for the prediction of the behavior of bodies which are subjected to human activities in practical contexts, a system of knowledge and beliefs that may be called, adopting a term from cognitive psychology, “intuitive physics”.

Any attempt to create a theory of nature as general as Aristotelian physics has to start from this basic body of knowledge, even if its goal is to revise the Aristotelian system. This explains why Galileo, as has been pointed out already, in his early attempts to give a systematic account of the physical knowledge of his time in the same way as his contemporaries combined an anti-Aristotelian attitude with an adherence to basic Aristotelian assumptions. Such assumptions anchor, as we have extensively shown elsewhere, the conceptual framework even in what he considered a new science of motion in his definitive publication, the *Discorsi*.

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22 See [Damerow, P., et al., 1992], also for the following.
EXAMPLE 2: THEORETICAL KNOWLEDGE AND SCHOLASTICISM

A second strand of knowledge, not rooted in intuitive physics, was, nevertheless, also part of the shared knowledge of the time because it was transmitted as a canon of theoretical knowledge within the university tradition; it is represented by the sophisticated elaborations of Aristotelian physics in the medieval scholastic tradition. None of the assumptions of this theoretical tradition are as self-evident and irrefutable as the elements of Aristotelian theory rooted in intuitive physics. These assumptions were the issues of debates extending over centuries and were consequently the natural starting points for creating new theories of motion in early modern times.

Historians who attempt to understand the spreading of these new theories in seventeenth century Europe are confronted with a puzzle. The treatises of this time, as they were written by natural philosophers such as Galileo, Descartes, Baliani, or Harriot, show a great variation with regard to the phenomena considered, the basic axioms, or the deductive organization. Nevertheless, these treatises also show a number of peculiar common features that cannot be explained by their shared starting point in the core assumptions of Aristotelian theory rooted in intuitive physics.

For instance, they all conceptualize the phenomenon of acceleration, which plays quite different roles in their individual versions of a new science, in terms of the same odd medieval concepts incompatible with classical physics. In particular, accelerated motion is understood as a quality that has an extension as well as an intension, characterized by changing degrees.

In order to understand this puzzling commonality one can either search for direct influences in individual biographies or search for general structures of knowledge at that time. When historians of science focus on Galileo’s work and attempt to explain what they see as the sudden appearance of the key concept of changing “degrees of velocity” in the course his work on a science of motion, they usually search to identify the sources of this concept in specific influences occurring during his biography, for instance, in the form of books he must have read at a particular time. They have thus, for instance, long been irritated by the fact that the fourteenth century work of Oresme and of the Oxford calculatores, to which the notion of motion as a quality with changing degrees can be traced back, was essentially no longer read in the time of Galileo and could hence not figure as a possible source of his ideas on accelerated motion.
historians of science discuss the general state of ideas in the seventeenth century, they tend to portray medieval Aristotelian scholasticism merely as the counter position against which Galileo’s theory of motion gained its profile as a new science, neglecting the potential of Aristotelianism as a generic knowledge resource available to Galileo and his contemporaries.

Galileo’s unpublished commentaries on Aristotelian physics mentioned above make it not only amply clear that he had thoroughly appropriated the immense knowledge accumulated in the scholastic tradition of elaborating and commenting Aristotle but also that he had thus acquired a resource of knowledge that provided essential assets of the new science of motion, assets such as the conceptualization of acceleration in terms of the changing degrees of a quality. This conceptualization was in fact part of the doctrine of intension and remission transmitted by the lively scholastic tradition of the time, a tradition from which contemporary intellectuals could hardly escape, whether they encountered it in the college of La Fleche, as was the case for Descartes, or in the lecture notes of Jesuit professors of the Collegio Romano, as was the case for Galileo.\footnote{For Descartes see \cite{Gaukroger, S., 1995}, for Galileo see \cite{Wallace, W. A., 1981}.}

**Example 3: Aristotelism and Archimedean Theory as Coexisting Bodies of Knowledge**

Let us turn to our third example of how shared knowledge resources created common framing conditions for Galileo and his contemporaries. This example serves to make evident that they drew on shared bodies of knowledge not only separately from each other by exploiting their individual potentials but also tried to merge incompatible knowledge structures into a new unity. A telling example for this type of theory construction is provided by the attempts of Galileo to combine the Aristotelian theory of motion with Archimedean theory of buoyancy within a deductive framework following the model of Euclid and Archimedes himself. Given that in Aristotelian theory motion is possible only as motion in a medium and that Archimedes gave a precise account of how the medium affects a body submerged in it, any study of the velocity of falling bodies had either to combine both theories in this point or to radically change basic assumptions of either of them. This explains why not only Galileo but also contemporaries such
as Benedetti, Guidobaldo del Monte, Beeckman, or Harriot all advanced new theories of motion of fall different in detail but in agreement on this point. In the case of Galileo we are able to trace the development of such a theory through various stages. He started out by writing a dialogue that gave him the occasion to cautiously revise the Aristotelian theory by introducing proponents holding different positions without identifying himself with one of them. This attempt was followed by a more audacious treatment in the form of a scholastic treatise in which he promoted a radical critique of Aristotle but essentially stuck to Aristotelian assumptions.

Finally he conceived the project of a new science of motion based on this theory and to be formulated following the model of a deductive treatment in the style of the succinct De ponderoso e levi, attributed to Euclid and familiar to Galileo. But he obviously did not realize this project since only an outline survived. The concise style of the planned treatise becomes evident even when considering only a few of the issues covered.

Concerning the ratio of motions of the same mobiles in different media.
Concerning the ratio of the motions of different mobiles in the same medium.
Concerning the cause of the slowness and the speed of motion.

**EXAMPLE 4: THEORETICAL KNOWLEDGE AND MECHANICS**

Our fourth example introduces yet another body of knowledge relevant to the development and reception of Galileo’s science of motion. Besides the uninterrupted Aristotelian tradition one has to take into account that the sixteenth century saw the revival of another ancient tradition namely that of deductive treatises specifically devoted to mechanics. This revival was closely related to the growing practical importance of machine technology since the Renaissance. It was based on a small number of treatises from antiquity and the Middle Ages which use the lever as a mental model for interpreting mechanical phenomena.

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24 See [Benedetti, G. B. d., 1585], [del Monte, G., ca. 1587-1592, p. 41], [Beeckmann, I., 1939] and, for Harriot, BL Add MS 6788 folios 144-148.
25 See [Galilei, G., 1890-1909, I: 367-408].
26 See [Galilei, G., 1890-1909, I: 247-340].
27 See [Galilei, G., 1890-1909, I: 418ff.].
patronage Galileo achieved his positions in Pisa and later in Padua, published a comprehensive compilation of this knowledge thus creating the most influential mechanical treatise of his time. Galileo’s unpublished treatise on mechanics mentioned above which he composed after his move to the Venetian republic, the technological center of that time, similarly represents an appropriation of the ancient tradition of mechanics. At that time Galileo had reoriented his interest from the critique of Aristotelian philosophy to the practical challenges of an engineer-scientist.

Galileo’s unpublished treatise on mechanics is usually studied with the primary intention of finding something innovative. This search has even been successful to a certain extent. From the viewpoint of historical epistemology, however, this treatise deserves attention primarily for another reason. It represents the basic structures of mechanical knowledge which Galileo shared with his contemporaries, and which served them as a universal instrument for their theoretical interpretation of the rapidly developing contemporary machine technology.

**EXAMPLE 5: PRACTICAL KNOWLEDGE AND CHALLENGING OBJECTS**

The opportunities and challenges offered by the new technology represent another shared knowledge area, whose character was, however, much more diverse than that of the bodies of knowledge transmitted by ancient traditions. It does not come as a surprise that Galileo, once he had redirected his activities towards technical problems, became acquainted with technologies such as ballistics, shipbuilding, and fortification and that he prepared a couple of treatises which seem to have nothing to do with the great discoveries he is usually praised for. Precisely these treatises provide us with a key for understanding what type of knowledge triggered the transformation of ancient mechanics into the theory of motion of classical physics. Galileo’s engagement with this type of knowledge becomes particularly clear in the cases of ballistics and of the pendulum. It was only his interest in ballistics which made Galileo realize the theoretical implications of an experiment on projectile motion which he performed years before, as we have shown elsewhere, together with Guidobaldo del Monte. Similarly, as Viviani reports, the

29 See [del Monte, G., 1577] and [del Monte, G., 1581].
30 See [Galilei, G., 1890-1909, II: 146-191].
first reaction of Galileo after having discovered the isochronism of the pendulum was to explore its usefulness for a technical context, that is, for the construction of the pulsilogium for time measurement in medicine.32

In the case of ballistics it has to be pointed out that any theory of projectile motion advanced at that time had to take into account the common knowledge of the practitioners of ballistics as was transmitted not only by participation and oral transmission but also by numerous published as well as unpublished military treatises. The knowledge of the artillerists based on their professional experience included that the speed of a projectile increases with the force the exploding powder exerts on it, that more projectile weight requires more force to reach the same distance, that the distance of the shot depends on the angle, that there is an angle at which this distance reaches a maximum and that there are angles at which flat and steep shots reach the same distance though with different effects.33 This type of knowledge also provided a starting-point for early modern engineer-scientists such as Leonardo, Tartaglia, Aquilone, Puchner, Harriot and last but not least Galileo. Among his papers he left the outline of an unwritten treatise entitled Particular privileges of the artillery with respect to the other mechanical instruments, probably dating back to the early Paduan years.34 It illustrates the role of reflection on practitioner’s knowledge for the creation of a new theory of projectile motion by Galileo. Among the issues Galileo intended to deal with in his planned treatise are topics such as:

If one operates with a greater force in a certain distance or from nearby.
Which line the ball describes in its [course].
...
In which elevation you shoot farthest and why.
That the ball in turning downwards in the vertical returns with the same forces and velocities as those with which it went up

31 See [Renn, J., et al., 2000].
33 See [Büttner, J., et al., in press].
Given that such questions were at the center of interest for early modern engineers it could be only a matter of time before all such questions had been more or less sufficiently embedded into a coherent deductive theory of motion extending theoretical mechanics in the ancient tradition.

In the second case, the case of the pendulum, Galileo was confronted with a quite different type of challenging object. There is ample evidence that Galileo realized early in his Paduan period that theoretical mechanics as it was represented by Guidobaldo’s and his own treatises was unable to explain even such trivial insights as its isochronism not to mention the puzzling parallelism between the law of fall and the dependence of the period of the pendulum on its length, the law of the pendulum. A whole bunch of manuscript pages show that Galileo desperately tried to derive the isochronism from traditional mechanics by approximating the circle described by the descending pendulum by a series of inclined planes with diminishing slope. Galileo outlines the basic idea and describes the obstacles against its realization in the answer to a lost letter of Guidobaldo del Monte, who was obviously skeptical with regard to Galileo’s attempts to use Aristotelian dynamics for understanding phenomena such as the swinging of the pendulum. He points out the difficulties posed by this particular challenging object:

Until now I have demonstrated without transgressing the terms of mechanics; but I cannot manage to demonstrate how the arcs [...] have been passed through in equal times and it is this that I am looking for.

When Galileo finally published his Discorsi he still lacked a satisfactory theory of the pendulum. Since he did not manage to include his results on the pendulum within the deductive framework of his theory of motion he evidently saw no other solution for making these results public but to include them not within the fictive treatise presented in the Discorsi but to casually mention them in the dialogue parts. He thus missed his last chance to place these results into the records of published discoveries, although the solution to this problem would have been only a small step within his theory.

35 See [Galilei, G., ca. 1602 - ca. 1637, folio 166 recto] and [Hill, D. K., 1994].
36 See the letter to Guidobaldo dal Monte, November 29, 1602, [Galilei, G., 1890-1909, X: 97-100]
37 For a treatment of the pendulum in the Discorsi see e.g., [Galilei, G., 1890-1909, VIII: 139ff.].
In fact, there is evidence that the story was not over yet. Shortly after Galileo’s publication of the *Discorsi* Baliani also published a theory of motion which, however, turned the problem upside down. Baliani used as an axiom the law of the pendulum already known to him and derived from it Galileo’s law of fall. A hitherto unknown letter of Galileo to Baliani, a draft of which we identified among the notes of one of his disciples, contains a critique of Baliani’s proof of the law of fall based on the law of the pendulum and proposes an improvement which, read in reverse direction, immediately gives a proof of the law of the pendulum in the framework of Galileo’s own theory. There can hence be no doubt that Galileo’s theory of motion would have been able to include a theory of the pendulum if he had had the chance to compose it after he read the treatise of Baliani.

**EXAMPLE 6: SHARED IMAGES OF KNOWLEDGE AND THE CHALLENGE OF ASTRONOMY**

Like mechanics, astronomy represents an ancient body of knowledge that received growing attention in the early modern period due to its increased practical significance, in particular in the context of the challenges for navigation posed by the discovery of new parts of the globe. At the same time, astronomical knowledge was tidily interwoven with the dominating worldview embraced by the Church which comprised the geocentric cosmology of Aristotle and Ptolemy.

Since the nineteenth century, the clash between this worldview and Galileo’s defense of Copernicanism has repeatedly been interpreted as Galileo’s systematic battle against an ancient dogma in order to defend his new science of motion. But as Galileo’s unpublished treatises dealing with astronomical issues amply illustrate, his encounters with Copernicanism were neither rooted in a strategically planned battle nor were they the consequence of a conversion experience that made him a firm believer in the new world system. From the viewpoint of historical epistemology, they rather appear as the results of unavoidable encounters of an engineer-scientist with the shared knowledge of his time. These encounters gave Galileo, as was the case with his

38 See [Baliani, G. B., 1638] and the revised and substantially extended edition [Baliani, G. B., 1646].
contemporaries, occasion to react to this knowledge in ways that were determined both by local contexts and by the dominating image of astronomical knowledge as an underpinning of the ruling worldview.

When Galileo wrote, for instance, his unpublished treatise on the sphere in the context of his teaching activities, he did not even bother to discuss the Copernican system. He even included a defense of the geocentric world system by physical arguments although he declared himself to Kepler in a letter of 1597 as a supporter of the new astronomical system. He did so at the same time as he gave, at the university of Padua, a course on the geocentric world system based on the medieval treatise on the sphere. Also, he could not resist proving a philosopher colleague wrong when the latter criticized the Copernican system with unsound arguments. It was this academic rivalry that occasioned Galileo’s first treatise on Copernicanism, circulated only in the form of a letter.

However, the omnipresence of Aristotelian natural philosophy did not only account for the fact that engineer-scientists of the time almost unavoidably encountered related astronomical issues. Rather its embedding within the dominating worldview created boundary conditions that no attempt at a new science of motion at this time could ignore. In particular, the fact that in Aristotelian natural philosophy terrestrial physics was an integral part of a global world view enforced a cosmological meaning on every mechanical model of astronomical phenomena. This is true, for instance, for an attempt by Galileo that we have been able to reconstruct from his manuscripts. In fact, he tried to explain the tides by using the swinging pendulum as a mental model of the motion of the sea and by scaling-up the relation between the length of the pendulum and its period to the dimensions of the diameter of the earth in order to determine the period of the tides. This is true also for his attempt to explain the periods of the planets in their orbits around the sun taken from Kepler’s publication by using projectile motion as a mental model of the divine creation of the planetary system and by scaling-up his experiments with free falling bodies deflected into the horizontal to cosmic dimensions. Although both attempts failed to match the observed data, he published these ideas in the Dialogue and in the Discorsi, not, however,

40 See [Galilei, G., 1890-1909, II: 203-255].
41 See the letter to Johannes Kepler, August 4, 1597, [Galilei, G., 1890-1909, X: 67f.].
42 See [Galilei, G., 1890-1909, XIX: 120].
44 See [Galilei, G., ca. 1602 - ca. 1637, folio 154 recto].
as the systematic treatises he probably had hoped to write originally but as grand visions supporting the Copernican system, leaving out the details we know from his unpublished manuscripts.46

EXAMPLE 7: PATRONAGE AND THE SHARED SOCIAL STRUCTURES OF KNOWLEDGE

The last example of unpublished treatises discussed here is again of a quite different nature and points to a framing condition of the development and reception of scientific knowledge in this period that has been studied under the heading “Galileo Courtier.”47 At the end of 1630, Galileo completed an expertise that he had written by request of the Tuscan government. It represents an unpublished treatise on the regulation of the Bisenzio river in which Galileo mustered almost the entire arsenal of his science of motion to cope with a technological challenge of his time.48

The writings he composed as a courtier under the patronage of the Medici family make it clear that the creation and dissemination of scientific knowledge in the early modern period is profoundly shaped by a social organization quite different from later periods. Gestures such as the appeal to the authority of princes, the exchange of gifts, here take the role of scientific credentials obtained by the recognition of peers.

For instance, when Galileo negotiated for his position at the Florentine court, he felt pressed to pretend that he could offer a cornucopia of unpublished treatises which in fact can only partly be identified among his manuscripts, probably because much of what he announced was actually never written:

I also have some minor works on natural topics, like De sono et voce, De visu et coloribus, De maris estu, De compositione continui, De animalium motibus, and still others. I have also the intention to write some books concerning the soldier, educating him not only in abstract, but by teaching [him]

45 See [Galilei, G., ca. 1602 - ca. 1637, folios 134,135 and 146]. For a detailed account of Galileo’s cosmogony see the contribution by Jochen Büttner in this preprint.
47 See [Biagioli, M., 1993].
48 See [Galilei, G., 1890-1909, VI: 627-647].
by means of very good rules, all what is convenient to know and depends on Mathematics, like knowledge of castrametations, regulations, fortifications, assaults and captures, taking plants away, measuring with the sight, knowledge concerning artilleries, uses of various instruments, etc.\textsuperscript{49}

However, it would mean to greatly underestimate the role of this social organization for early modern science if its impact is only considered with regard to the images of science on which the appreciation of scientific achievements by the ruling classes was based. Galileo’s writings on hydraulics, such as his unpublished treatise on the regulation of the Bisenzio river show, that the social context of the early modern engineer-scientists provided in fact the precondition for the merging of bodies of knowledge such as practitioners knowledge about hydraulic engineering and scholastic theories on the dynamics of moving bodies whose traditions had been separated over centuries by a gulf of social status.

\textbf{THE IRRESISTIBLE SPREAD OF SHARED KNOWLEDGE}

Let us end as we began with a thought experiment. Let us assume that an early modern scientist had made an important discovery such as, for instance, the parabolic shape of the projectile trajectory, but that he hesitated to publish it because he consciously attempted to hide his discovery hoping to eventually make a fortune from it. Would it simply be possible to hide it or would it have been rediscovered by someone else or spread in spite of attempts to conceal it. As long as we conceive of the dissemination of scientific knowledge in terms of a chain of discoveries, their subsequent publication, and finally their reception, both events seem to be extremely unlikely. Once again, our thought experiment turns out to correspond to a real story that makes it possible to actually verify the outcome. It is the story of Galileo’s failed attempt to hide the discovery of the parabolic trajectory for more than forty years.

When Bonaventura Cavalieri in 1632, shortly after the publication of Galileo’s \textit{Dialogo}, published his book \textit{Lo Specchio Ustorio overo Trattato delle Settioni Coniche} on parabolic mirrors, he sent Galileo a letter which contains the following information:\textsuperscript{50}

\textsuperscript{49} See the letter to BelisarioVinta, May 7, 1610, [Galilei, G., 1890-1909, X: 348-353].
\textsuperscript{50} Bonaventura Cavalieri to Galileo, August 31, 1632, [Galilei, G., 1890-1909, XIV: 378].
I have briefly touched the motion of projected bodies by showing that if the resistance of the air is
excluded it must take place along a parabola, provided that your principle of the motion of heavy
bodies is assumed that their acceleration corresponds to the increase of the odd numbers as they fol-
low each other from one onwards. I declare, however, that I have learned in great parts from you
what I touch upon in this matter, at the same time advancing myself a derivation of that principle.

This announcement shocked Galileo. In a letter written immediately afterwards to Cesare Mar-
sili, a common friend who lived like Cavalieri in Bologna, he complained:51

I have letters from Father Fra Buonaventura with the news that he had recently given to print a trea-
tise on the burning mirror in which, as he says, he has introduced on an appropriate occasion the the-
orem and the proof concerning the trajectory of projected bodies in which he explains that it is a
parabolic curve. I cannot hide from you, my dear Sir, that this news was anything but pleasant to me
because I see how the first fruits of a study of mine of more than forty years, imparted largely in con-
fidence to the said Father, should now be wrenched from me, and how the flower shall now be bro-
ken from the glory which I hoped to gain from such long-lasting efforts, since truly what first moved
me to speculate about motion was my intention of finding this path which, although once found is
not very hard to demonstrate, still I, who discovered it, know how much labor I spent in finding that
conclusion.

Galileo received an immediate answer from Cavalieri. 52

I add that I truly thought that you had already somewhere written about it, as I have not been in the
lucky situation to have seen all your works, and it has encouraged my belief that I realized how much
and how long this doctrine has been circulated already, because Oddi has told me already ten years
ago that you have performed experiments about that matter together with Sig.5 Guidobaldo del Mon-
te, and that also has made me imprudent so that I have not written you earlier about it, since I be-
lieved, in fact, that you do in no way bother about it but would rather be content that one of your
disciples would show himself on such a favorable occasion as an adept of your doctrine of which he
confesses to have learned it from you.

He offered all kinds of options for a reconciliation and even proposed that he would finally

... burn all copies so that with them the reason is destroyed for which it is possible that I have given
disgust to my master Galileo so that he could say like Cesar to me “tu quoque, Brute fili”

51 Galileo to Cesare Marsili, September 11, 1632, [Galilei, G., 1890-1909, XIV: 386].
52 Bonaventura Cavalieri to Galileo, September 21, 1632, [Galilei, G., 1890-1909, XIV: 395]; see also the discus-
sion of this correspondence in [Wohlwill, E., 1899].
In short: at a time when Galileo’s collaborators were using and spreading the knowledge of Galileo’s discovery, convincing themselves that it must have been long published, Galileo himself still fought like a Don Quichote to keep it secret.

To sum up: Galileo was not the lonely hero he was considered to be in the nineteenth century. From the viewpoint of historical epistemology Galileo was working on the basis of structured bodies of physical knowledge which he shared with his contemporaries. This knowledge opened up a field of standard applications to specific objects as well as to the construction of global world models, and raised a number of open problems and alternative options for solving them. His work was furthermore constrained by certain images of knowledge, in the sense of controversial interpretations of the social status and the cultural meaning of these knowledge systems. Within this epistemic cosmos, defining the space for the trajectories of individual scientists, both common aims and tools of Galileo and his contemporaries, as well as the space of alternative interpretations, were determined.

A reconstruction of the emergence and dissemination of a new theory of motion in early modern Europe cannot really be successful as long as it is merely understood as the consequence of the reception of the isolated discoveries of Galileo. If those who built on his achievements were standing on the shoulders of a giant, this giant was represented not so much by these discoveries but rather by the shared heritage of early modern Europe that made the pioneering achievements of Galileo and his contemporaries meaningful in the first place. Such an understanding of scientific progress requires analyzing not only sources in order to identify exceptional events which can then be designated as “discoveries.” It rather makes it necessary to reconstruct the bodies and images of knowledge representing what one might call “normal planning, reflecting, and teaching activities.” From this perspective, the study of Galileo’s unpublished treatises documenting such activities may turn out to be even more revealing than any attempt to identify singular discoveries in his published works.
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Galileo’s insight into the parabolic shape of projectile trajectories is commonly considered not only as having provided a major turning point in the history of ballistics, but also as having constituted a fundamental step towards the establishment of classical mechanics. In fact, from a modern perspective, the parabolic trajectory is closely associated with three of the most fundamental principles of classical mechanics, the law of inertia, the law of fall, and the superposition of motions without interference.

According to classical mechanics, the projectile trajectory results from a composition of two motions, an inertial motion along the line of the shot, and the accelerated motion of free fall vertically downwards. Given the knowledge of the laws governing these two fundamental kinds of motion, points on the trajectory can be geometrically constructed by considering the distances traversed by the two motions in equal intervals of time. Figure 1 illustrates such a construction. To represent the uniform inertial motion along the line of the shot, equidistant points are plotted on the oblique line, marking the distances traversed in equal intervals of time. From these points, the distances the mobile traverses in free fall in the time that has passed since the beginning of the shot are measured vertically downwards. Since the space traversed in free fall grows quadratically in time, these distances in-
crease according to the sequence of square numbers: 1, 4, 9, etc. The resulting points mark the actual positions of the projectile after equal intervals of time. The trajectory, represented as a dotted line in Figure 1, is drawn by joining these construction points smoothly.

This construction appears to be so immediately plausible that one is tempted to assume that whoever conceives of the projectile trajectory as a parabola resulting from a composition of two motions, must understand it in this way and hence also attain the insight into the law of inertia and the law of fall underlying the construction. Accordingly, the fact that Galileo, in his late work on mechanics, the *Discorsi*, formulated a classical result, namely the parabolic trajectory, is usually understood to imply that he was already working within the framework of classical mechanics. Even evident deviations in his works from the reasoning expected according to classical physics are usually not understood as indicating that Galileo’s arguments were actually not rooted in the framework of that science.

An example of such a deviation is provided by a striking gap in the deductive structure of the *Discorsi*, the major work on mechanics of the mature Galileo, after all. Galileo derives the form of the projectile trajectory only in the case of horizontal projection, by composing the horizontal component with the vertical motion of fall—precisely according to the construction presented above. For the case of oblique projection, however, he just states that the resulting trajectory would likewise be a parabola, without offering any proof for this statement.¹ In the light of the fact that, in classical mechanics, the trajectory of oblique projection follows from the same construction as that for horizontal projection, this omission is hard to understand. In classical mechanics, the case of horizontal projection is, after all, merely a special case of the more general class of projections in any direction. However, if one takes into account not only Galileo’s published works but also the numerous unpublished manuscripts documenting his ongoing research, one can indeed, as I shall illustrate in the following, identify clues illuminating why oblique projection represented a problem for him.

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¹ Galileo [1968, Vol. VIII, p. 296] claims, but does not prove, that a bullet shot at a given angle would traverse the reverse path of a bullet shot horizontally and hitting the ground at the same angle. The deficiency in Galileo’s argument was already pointed out by Descartes [1964 ff., Vol II, p. 387 (letter no. 146)] in his famous critique of the *Discorsi*, and later discussed by Wohlwill [1884, pp. 111 f.].
Figure 2 illustrates the construction of a projectile trajectory for oblique motion as can be reconstructed from a drawing found in Galileo’s manuscripts. In this representation the bullet is projected in the lower left corner. After being projected it participates in two motions. One is the motion along the line of the shot, the other is the motion vertically downwards. The construction thus looks very similar to that of classical mechanics (see Figure 1). There is, however, one major difference between Galileo’s construction and that of classical mechanics: the motion along the line of the shot in Galileo’s construction is not the uniform inertial motion of classical mechanics. Rather, this motion is decelerated. The spaces traversed along the oblique line in equal intervals of time decrease in such a way that the resulting motion behaves like a reversed motion of fall. The motion along the line of the shot therefore appears to be modeled in analogy to the motion along an inclined plane. From the perspective of classical mechanics this conception is fallacious. Only in the case of horizontal projection does the result coincide with that of classical mechanics, since in that case the inclination of the plane is zero and no deceleration due to gravity occurs.

Why did Galileo adhere to such a strange idea? Was he just infatuated with inclined planes as others claimed he was with circles? And if so, was it then just a mere coincidence that Galileo treated the case of horizontal projection correctly and thus hit upon what was to become a key insight of classical mechanics?

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2 Folio 175v of MS 72, Biblioteca Nazionale, Florence. Naylor [1980, pp. 557–561] was the first to interprete this drawing as a theoretical analysis of oblique projection. Damerow, Freudenthal, McLaughlin, and Renn [1992, pp. 206–209] follow Naylor but offer a different reconstruction of the conceptions underlying the construction. Here, I follow the interpretation given by the latter.

3 Another example for such a deviation of the reasoning on projectile motion in Galileo’s work from that expected according to classical mechanics points in the same direction. In the dialogue part of the Discorsi it is argued that, due to the curvature of the earth’s surface, the line of the shot would actually have to be considered as inclining even in the case of horizontal projection, so that the motion along this line cannot be uniform. Only the smallness of this effect is mentioned in order to refute the objection [Galileo, G., 1968, Vol. VIII, pp. 274 f.]. See also [Wohlwill, E., 1884, pp. 112 f.].
If one conceives of scientific ideas as being merely a product of individual thinking—great ideas as the product of genius, and lousy ideas as the product of infatuations that may affect even a genius—virtually no other explanation remains. However, if one takes into account the shared knowledge on the basis of individual thinking, what at first sight appears to be merely the individual blunder of a hero of science, may actually become plausible as an expression of a differently structured body of knowledge. From this perspective, the emergence of classical mechanics would hence not have to be explained as being due to an accidental discovery, but could be accounted for as the result of a transformation of the shared knowledge underlying also Galileo’s thinking.

Are there any indications that Galileo’s apparently eccentric idea of constructing the trajectory of oblique projection by means of an inclined plane is actually not just an individual idiosyncracy but strongly suggested by the shared knowledge of his time, a knowledge possibly structured by other principles than those of classical physics? Evidently, this question cannot be answered by looking at Galileo’s work alone, and has, accordingly, been neglected by Galileo scholars. What did, in particular, early modern practitioners and theoreticians of artillery—Leonardo da Vinci, Giusto Aquilone, Paulus Puchner, Sebastian Münster, Daniel Santbech, William Bourne, Niccolò Tartaglia, Alessandro Capobianco, Luys Collado, or Diego Uffano, to name a few—think about projectile motion? Did they each have their distinct individual views, or is it possible to recognize structural similarities in their conceptions, revealing a body of non-classical shared knowledge? A systematic study of such similarities has hardly begun. Here, I would like to offer a glimpse at the work on projectile motion of one such contemporary of Galileo, the English natural philosopher Thomas Harriot, using a reconstruction of his work from the extant manuscripts.4

Harriot lived from 1560 to 1621. He filled more than 8000 manuscript pages with notes on various topics of contemporary mathematics, natural philosophy, and engineering, but did not publish any of his scientific achievements. Though Harriot eventually became familiar with Galileo’s astronomical work through the latter’s publications, his work on ballistics has to be regarded as independent of Galileo’s, no personal contact between the two being known, and Harriot’s work having been completed long before Galileo published on ballistics.

4 A comprehensive reconstruction of Harriot’s work on ballistics is presently carried out by the author. This contribution refers to a few preliminary results.
One of the folios in Harriot’s manuscripts bearing notes on ballistics is preserved as f. 67r, Add MS 6789, in the British Library. On its upper part, there is a drawing of the curve reproduced in Figure 3. Below this drawing Harriot noted:

The species of the line that is made upon the shot of poynt blanke is as is here described & is a parabola as of the upper randons.

The “shot of point blank” thereby denotes the horizontal shot, while with the “upper randons” Harriot refers to the shots at an elevation above the horizontal. The trajectory is evidently constructed in the manner previously described, i.e. by composing motions traversed in equal intervals of time. Along the horizontal, equal distances are marked, thus representing a uniform motion. The lengths of the verticals obviously represent the motion of fall and grow quadratically as Harriot noted by writing down the numbers 1, 4, 9, and 16.

In short, Harriot’s construction and the accompanying text, which must have been composed before 1621, the year of his death, document his knowledge of the law of fall and of the parabolic shape of the projectile trajectory to the same extent as is known from Galileo’s Discorsi of 1638. In the light of this document alone, it would thus seem to be justified to consider Harriot the Galileo of England. Indeed, if only this single document were known, Harriot could be credited as much as Galileo with the foundation of the classical theory of ballistics.

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5 British Library Add MS 6789, f. 67r.
6 On the basis of Harriot’s handwriting, Shirley [1983, p. 261] dates these notes to 1607.
On closer inspection, however, it turns out that Harriot’s construction not only produces the same insights but also displays the same weaknesses as Galileo’s exposition in the *Discorsi*. In fact, it is only in the case of horizontal projection that the depicted parabola results from Harriot’s construction. On this folio, at least, the parabolic shape of the trajectory for the case of oblique projection is only claimed but not proven.

While Harriot did not publish on ballistics, he did leave us with many more manuscripts than Galileo, allowing us to reconstruct how he thought about oblique projection. In particular, the folio previously mentioned turns out not to be a disparate fragment, but rather part of a larger group of folios dealing with projection at arbitrary angles. Let us take a look at another one.

The drawing on f. 64r, Add MS 6789, reproduced in Figure 4, illustrates a shot at an elevation above the horizontal (here at an angle of about 53˚). The dotted curved line represents the trajectory, the oblique line tangent to it at its origin represents the line of the shot. From points on this line in decreasing distances, lines are drawn vertically downwards. The distances marked on these vertical lines from the line of the shot to the trajectory are of increasing length. Both, the deceleration of the motion along the line of the shot, as well as the acceleration of the motion along the vertical, obey a quadratic law. This suggests that also in Harriot’s case, the motion along the line of the shot was conceived in analogy to the motion along an inclined plane, just as it has been the case in Galileo’s construction.7

It thus seems that the use of the inclined plane in order to construct the trajectory of oblique projection was not an eccentric idea of Galileo, but rather a plausible option for anybody who, at that time, attempted to obtain the projectile trajectory from the composition of the motion along the line of the shot and the vertical motion of fall.

7 Lohne [1979, pp. 236 f.] interprets the deceleration along the oblique occurring in Harriot’s trajectories as being due to air resistance. Although Harriot did indeed consider motion through a medium to be decelerated according to a quadratic law, this interpretation is untenable. As I will explain below, the deceleration of the oblique motion depends on the angle of elevation in a specific way, supporting the interpretation in terms of the inclined plane. In accordance with his understanding, Lohne interprets the drawing on f. 67r previously discussed, as representing the trajectory for the case that air resistance is neglected [Lohne, J. A., 1964, p. 19], obviously ignoring the fact that the folios are related and the drawings on them illustrate the trajectory for different angles of elevation.
There is even stronger evidence to support the interpretation that Harriot’s construction makes use of the inclined plane. As one can easily see in Figure 4, there are further construction lines drawn perpendicularly to the line of the shot, giving rise to triangular structures setting the distances traversed in the motion along the oblique in relation to those traversed along the vertical. Without going into details of the construction, I would like to mention that these structures assure that the motion along the oblique does indeed obey the law of the inclined plane.8

![Figure 4: Folio 64r, BL Add MS 6789.](image-url)
Harriot did not only construct these curves, he also analysed their mathematical character, finding that they are indeed parabolas. However, they are tilted at an angle depending on the elevation of the shot. Furthermore, Harriot used the composition of motions involving the inclined plane to solve problems that Galileo was also concerned with. In particular, like Galileo,

8 The law of the inclined plane states that the acceleration of a motion along the plane equals \( \sin \alpha \) times the acceleration of free fall, where \( \alpha \) denotes the angle of inclination. That this law holds in Harriot’s construction can be seen as follows. The distances marked on the line of the shot by the vertical lines decrease according to the sequence of odd numbers, i.e. they are 11, 9, 7, 5, 3, and 1 units wide, so that a square law results when adding them up from above (1, 4, 9, etc.). In addition to this first motion along the line of the shot, Harriot considers a second one represented by the distances marked on the line of the shot by the lines perpendicular to the latter. In comparison to the first motion, this second motion is doubly decelerated, i.e. the spaces traversed in succeeding equal intervals of time are 10, 6, and 2 units, the double of the last three distances of the first motion. Then the doubly decelerated motion proceeds downwards again, traversing the same distances in reverse order. The difference of the spaces traversed by these two motions grows according to a square law as 1, 4, 9, etc., i.e. exactly as the motion along an inclined plane. Now consider the right triangles having the line segment representing the differences of the spaces traversed by the two motions as one leg, a line segment perpendicular to the line of the shot as the other, and a vertical line segment as the hypothenuse. The lower corners of these triangles are taken to be points on the trajectory, i.e. the hypothenuses represent the spaces fallen in free fall. But as they are as \( 1/\sin \alpha \) to the opposite legs representing the distances traversed along the inclined plane, where \( \alpha \) denotes the elevation angle, the law of the inclined plane is satisfied.

9 Besides many folios documenting Harriot’s attempts at such a proof, f. 69r, BL Add MS 6789 bears the ultimate proof. For a transcription of this folio, see [Lohne, J. A., 1979, pp. 258 f.].

10 The tilting angle \( \beta \) is given by \( \tan \beta = \sin \alpha \sin \beta / (1 + \sin^2 \alpha) \), where \( \alpha \) denotes the elevation angle, see [Lohne, J. A., 1979, p. 238].
leo, Harriot was interested in solving the “gunner’s question” of how the range of a shot depends on the gun’s angle of elevation. Now, consistently applying the construction just explained to shots at different angles, a plausible answer to the gunner’s problem may indeed be found.

Consider the drawing reproduced in Figure 5. There are five trajectories drawn at the angles of 15, 30, 45, 60 and 75 degrees. They differ in their appearance, particularly as regards the range of a shot. At first sight it is not clear how these trajectories were obtained since no construction lines are visible. On illuminating the folio with raking light, however, an abundance of construction lines carved into the paper but not drawn in ink become visible, some of which are redrawn in Figure 6. On closer analysis these lines reveal that the basic construction principles for all five trajectories are exactly those previously outlined.\(^{11}\)

Obviously, the construction also implies the existence of an elevation angle of maximum range. Harriot was even able to calculate this angle, determining the maximum range to be at an elevation of about 27˚55’.\(^{12}\)

Not only has Harriot pursued this line of thought further than Galileo, enabling its various consequences to be studied, Harriot’s manuscripts also provide an insight into the origins of this conception of projectile motion.

\(^{11}\) Although the resulting curves obey the same principles as those on f. 64r, BL Add MS 6789, their actual construction is different. In Figure 6 there are four obliques representing the line of the shot for the respective elevations and one vertical. The concentric circles divide these lines into equal sections. The intersection points would thus represent a uniform motion. On the vertical line, the distances a body falls in a given time are measured from these intersection points downwards and marked with horizontal bars. From the first intersection point beginning from below, the distance a body falls in one interval of time is measured, from the second point that fallen in two intervals of time, and so on. Accordingly, these distances increase quadratically. The same distances are also measured vertically downwards from the respective intersection points of the circles with the obliques. From the endpoints of the vertical line segments thus obtained, a line is drawn perpendicular to the respective line of the shot. The intersection point of this perpendicular with the line of the shot marks the position the motion along the line of the shot reaches in the given time. From this point vertically downwards the respective distance of fall is laid down. In this way the points on the trajectory are generated.

While in the construction on f. 64r, BL Add MS 6789, the vertical distances were adapted to the oblique ones in order to satisfy the law of the inclined plane, here the oblique distances are adapted to the fixed vertical distances of fall, thus allowing for a comparison of the trajectories for shots at different angles.

\(^{12}\) British Library Add MS 6788, f. 165v.
Figure 6: Folio 216v, BL Add MS 6788 (construction lines not drawn in ink are represented as thin lines).
Figure 7 shows a drawing from what probably represents an earlier period of Harriot’s occupation with projectile motion. While the basic idea of the composition of two motions is the same as in the later constructions, neither the correct law of fall, nor the law of the inclined plane are actually applied in the construction. The fact that even the motion along the horizontal is decelerated suggests that it is not the specific idea of an analogy to the inclined plane that constitutes the basis for this construction. Rather, it appears to be the natural exhaustion of the violent mo-
tion that leads to a deceleration along the line of the shot. When this violent motion has ceased, only the natural motion remains and the projectile falls vertically downwards, as one can see in Figure 7 in the case of the steepest shot.

This example may serve to illustrate what a more thorough analysis of numerous extant manuscripts amply confirms: that Harriot’s constructions of trajectories, and in particular also his construction with the help of an inclined plane, can be understood as specific implementations of the Aristotelian dynamics of violent and natural motion. In fact, from this perspective, the inclined plane allows to specify in a physically plausible and mathematically tractable way the decrease of the violent motion and the way it depends on the angle of elevation.

As is well known, Galileo—even in his *Discorsi*—continues to make use of the concepts of violent and natural motion. While this is normally treated as nothing but a traditional way of speaking, on the background of the analysis just given it becomes evident that this has deeper implications: In fact, the deviations of Galileo’s arguments from classical mechanics become understandable as the expression of a non-classical conceptual organization of knowledge that surfaces in the use of this traditional terminology. The knowledge, which Galileo shared with his contemporaries, is still rooted in the dynamical conceptions of Aristotle, but also comprises the experiences accumulated by the practitioners of ballistics, for instance the insight that there is an angle at which the shots obtain a maximum range.

The analysis of this shared knowledge cannot be covered by this limited contribution, whose aim is to point to the existence of this knowledge, which becomes strikingly visible in the remarkable similarities between the otherwise unrelated work of Harriot and Galileo. These similarities can hardly be explained by the traditional paradigm of influence and reception but have to be understood as the outcome of common challenges and shared means to address them. In short, studying the science of Thomas Harriot also entails learning about the roots of Galileo’s contributions to classical mechanics.
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INTRODUCTION

When Galileo was in Padua, he gained significant theoretical results on the isochronism of the pendulum, the theory of motion and the theory of solid bodies. And so, it is generally acknowledged that this is the period in which he achieved many of his most important theoretical discoveries. In this paper are presented the first results of research done in the context of a Dissertation that focuses on this period of Galileo's life but on different aspects.

We have many documents that allow us to throw light on Galileo's work in Padua. One of the most important documents is certainly the famous letter to Paolo Sarpi of 14th of October 1604. This letter and the one to Guidobaldo del Monte from 1602, as well as the great theoretical results achieved in this period, evoke the image of Galileo sitting down, alone, with the hand put on his forehead, in a quiet room, pondering on his ambitious new theory of motion.

On the other hand, therefore, details of his life that do not fit in with this image. Consider, for instance, his request for a patent for a lifting water machine, for the building of whose model he borrowed money from Niccolò Contarini in 1601. Also, thanks to the statements of his contemporaries, we know about his special skill in polishing lenses for telescopes, in the latest period of his stay in Padua. Furthermore, we know from the first sentence of the Discorsi that he often visited the Arsenal in Venice. There, he became aware of the practical problems related

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1 This paper has been written at the Max Planck Institute for the History of Science, Berlin. I would like to thank the Biblioteca Nazionale Centrale of Florence for the permission to publish an extract from the Manuscript Gal. 26. Thanks also to David McGee for his encouragement during the preparation of the talk.
to shipbuilding, which constituted the normal agenda of the work at the Arsenal. In fact, the
_Proti_, foremen of the Arsenal, necessarily faced the problem, later at the core of one of Galileo’s
two new sciences, namely that of the strength of materials, and this because of the need to
change some aspects of the design of the “large galleys”, exactly in those years when Galileo
was in Padua. The two letters between Galileo and the then _Provveditore_ at the Arsenal,
Giacomo Contarini, are, in fact, related to some of these problems.²

These details suggest that it makes sense to investigate the context in which Galileo was
working during his time in Padua. Some points of this context are well known, but they are often
considered to be marginal with respect to his major achievements. In order to reconstruct this
context more extensively the first step is to analyze the environment in which he was working
and, therefore, also the work in his house. Although some details of this environment are well
known, a general and quantitative view of his work at home has not been given.

**MANUSCRIPT**

The text called _Ricordi Autografi_ represents what we would call a multi-column register for
Galileo’s household. The _Ricordi Autografi_ are the result of a reconstruction made by Antonio
Favaro, whereas the real manuscripts, Manuscripts number 26 and 49, from which the _Ricordi_
were taken out, include also many calculations for Medici satellites and some notes on Motion.
Moreover, the entries are ordered in groups and normally in chronological order, whereas these
groups are scattered on the original pages often without any order. Most of the entries refer to
the Paduan time and especially to the first years of the XVIIth century. However, since there are
similar entries for the earlier and later periods, until 1620, one can suppose that a large section
of these entries went lost.

² For the relations between the first of Galileo’s two new sciences, that of the strength of materials, and the
practical knowledge of the foremen of the Venetian Arsenal, see Renn, Valleriani [2001].
The analysis of the Ricordi Autografi allows us to describe the two main activities at Galileo's house - the workshop and the private lessons. I concentrate here on the period between 1602 and 1604, as delimited by the letter to Del Monte (November 29th, 1602) about the isochronism of the pendulum, and that to Paolo Sarpi (October 16th, 1604), where signs of the construction of the theory of motion are present. We know that this is also the period in which Galileo extensively worked on the folios constituting the now famous Manuscript 72 on Mechanics.
Taking a glance at the Manuscript 26 in a general way, the first remarkable information concerns the number of persons who were living there, the quantity of work which was being done and the great amount of money circulating at Galileo's house. All of this is documented by a large amount of entries regarding the general administration of the house.

**WORKSHOP**

Galileo's house also comprised a workshop. There Messer Marcantonio Mazzoleni, who lived in the house together with his family, worked as a smith. With Galileo's words of July 5th, 1599:

> Memory that, on the mentioned day, Mess. Marcantonio Mazzoleni, came to live in my house, in order to work mathematical instruments for me and on my charge; and having obliged myself to make all the expenditures for him, his wife, and his daughter, and, moreover to give him 6 ducati a year, the money that he will receive from me will be annotated here.4

Studying how this and other workshops functioned, it is possible to learn not only about the practical method used to manufacture instruments, but also about the *shared knowledge* that also made Galileo's realization of experiments possible.5

In the workshop, Galileo achieved a quite systematic production of military and surveying compasses of different kinds. The military compasses differed from each other with respect to their materials, their size and their functionality. The workshop fabricated three kinds of military compasses: a simple one with two points, one with four points and the military compass with four bent points. Ordered from the lowest to the highest quality, the materials used were

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4 Galilei G, «Ricordi Autografi», in Favaro [1968, Vol. XIX, p. 131]: "Memoria come a dì detto è venuto a stare in casa mia Mess. Marcantonio Mazzoleni, per lavorare per me et a mie spese strumenti matematici; et essendomi io obligato di far le spese a lui, sua donna et alla sua puttina, et di più darli 6 ducati l’anno, qui a presso saranno notati i danari che da me haverà ricevuti."

5 For an extensive discussion about the concept of *shared knowledge*, see Büttner, Damerow, and Renn in this preprint. Concerning Galileo's skill in making instruments, see also Settle [1996].
basin brass, Italian brass, German brass and silver. Points were of steel. In reference to the size the military compasses are registered either as normal or as big. Normally Messer Marcantonio and Galileo purchased the materials themselves. But sometimes also friends or relatives did so. The material was ordinarily bought in form of plates and then sent to a foundry. Once the pieces came back in the proper shapes and sizes, Mazzoleni completed them by refining and assembling them. Unfortunately it is not clear who normally marked them because only one entry of the Ricordi is concerned with it. Often Galileo registered payments for finished compasses. From his letter of the November 11\textsuperscript{th}, 1605 to Cristina di Lorena, to whom two military compasses of silver had been promised, we know that Galileo was waiting for silver blocks in order to mark them personally, whereas Messer Mazzoleni probably marked them normally:

I am waiting that the two instruments of silver are sent to me, in order to be able to marke them and to send them perfect back.\textsuperscript{6}

In the period taken into account here, 23 compasses were produced, of which one of silver and two big. Most of them were sold to his private students. Besides military compasses the workshop was producing also surveying compasses. But in this case it is documented that not only Mazzoleni in the workshop was manufacturing them but also that Galileo purchased them in Venice and in Florence. Surveying compasses were mostly sold to his private students as well. Finally, the workshop also produced iron tools and parts of tools like screws, perpetual screws and clamps.

**PRIVATE LESSONS: STUDENTS**

The workshop with its variety of products was not an isolated commercial activity, independent from Galileo’s intellectual pursuits. In fact, the instruments produced and sold in Galileo’s household were only useful together with the knowledge of how to operate them. The

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transmission of this knowledge was, therefore, another essential activity, going on in Galileo’s household and intimately related to the workshop. Private lessons were Galileo’s way of transmitting this knowledge.

Galileo gave private lessons on a variety of themes. Their overarching topic was, as it turns out, fortification. His students were often young persons from distinguished or, anyhow, rich families who wanted to complete the curriculum at the University of Padua before starting their military career as officers. At the time, taking private lessons was quite normal. They offered the student the possibility of improving his knowledge of special topics in comparison with what could be learned from attending the University lectures alone. In this sense, it was perfectly normal that a student destined for a military career took private lessons on Fortifications. Accordingly, fortifications and military Architecture formed a part of the shared knowledge of many Engineers and Architects.

During this short period of two years, as many as 28 private students are registered in the Ricordi, sixteen of whom also lived in his house as roomers either for the whole period or part of it. At least one servant or friend also accompanied most of his roomers, so that the total number of roomers documented during that period even increases to 33.

<table>
<thead>
<tr>
<th>Private students who were also roomers</th>
<th>Other private students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schweinitz G. (+2)</td>
<td>Alffeldt (von) C.</td>
</tr>
<tr>
<td>Lazoczki (+1)</td>
<td>Filippo d’Assia</td>
</tr>
<tr>
<td>Lentowicz M.</td>
<td>Vinciguerra Coll’Alto</td>
</tr>
<tr>
<td>Bucau B. (+1)</td>
<td>Reisener B.</td>
</tr>
<tr>
<td>Buc</td>
<td>Luzimburg</td>
</tr>
<tr>
<td>Plesch M.</td>
<td>Noailles (de) F.</td>
</tr>
<tr>
<td>? Giovanni - from Lithuania</td>
<td>Batavilla</td>
</tr>
<tr>
<td>Ferrante (+1)</td>
<td>Reigesberg G</td>
</tr>
<tr>
<td>Ricques D.</td>
<td>Dietrichstein (de) P.</td>
</tr>
<tr>
<td>Zator G. (+8)</td>
<td>+3 students whose names are unknown</td>
</tr>
<tr>
<td>Lesniowski R.</td>
<td></td>
</tr>
<tr>
<td>Soell G. C.</td>
<td></td>
</tr>
<tr>
<td>Het B.</td>
<td></td>
</tr>
<tr>
<td>Montalban A. (+2)</td>
<td></td>
</tr>
<tr>
<td>Morelli Andrea (+1)</td>
<td></td>
</tr>
<tr>
<td>Caietano Giulio Cesare (+1)</td>
<td></td>
</tr>
</tbody>
</table>

Total: 33 persons

Total students: 28

*Figure 2: Private students (November 29,1602-October 16, 1604). Friends and servants in brackets.*
By going through the Manuscript, it is easy to reconstruct that at least ten roomers were simultaneously living in Galileo's house all the time. Taking servants and workers into account, there were at least fifteen persons in Galileo's house permanently. Now, by considering also the presence of the workshop and of all these students and roomers, it becomes immediately clear that the image of Galileo, the lonely thinker, sitting down, alone, in a quiet room is probably wrong.

**PRIVATE LESSONS: TOPICS**

Entries regarding private lessons are, as a rule, labelled according to their topic. These topics are: Geodesy, Mechanics, the Sphere, Perspective, Euclid, Arithmetics, Fortifications, and Use of the Military Compass. Although lessons specifically labelled as Fortifications are only one type among many others, a comparison with a traditional and most used treatises on Fortifications of that time, for example, with that of Boniauto Lorini of 1609,\(^7\) suggests that all registered topics, taken together, constitute a rather typical treatise on fortification for that time.

Four of these different kinds of lessons were based on the treatises known as *Le Mecaniche,\(^8\) La Sfera ovvero Cosmografia,\(^9\) Le Fortificazioni\(^{10}\) and *L'Uso del Compasso Militare*\(^{11}\) and the entries for these lessons are registered exactly under these names. Furthermore, in 1603 a copyist, Messer Silvestro, was also working at Galileo's house providing the private students with handwritten copies of the treatises:

Nota delle scritture haute da Mess. Silvestro:

Fortificazioni, copie 2, per il S. Giovanui Svainitz et S. Lerbac

Item, copie 1 al S. Bucau

Item, copie 1 al S. Alfelt

Item, copie 1 al S. Staislao

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\(^7\) Lorini [1609].


Item, copie 1 al S. Niccolò Beatavil
Per una copia dell’Uso del Compasso, data al S. Staislao
Per una copia dell’Uso del Compasso, data al S. Beatavilla
Per una copia del detto Uso, data all’Ill.mo et Ecc.mo S. Langravio
Per una delle dette copie, data ad un gentil’homo todesco
Per una data al S. di Noaglies.12

Although we have entries regarding handwritten copies of treatises only concerned with Mess. Silvestro in 1603, there is nevertheless no reason to exclude that this was a normal procedure in Galileo's house. Moreover, as we know from Favaro's analyses of four handwritten copies of Galileo's *Trattato della Sfera*, Galileo's students were able to copy for themselves Galileo's treatises. This is the case, for instance, for Abbot Giugni's copy of this treatise. In 1604 Galileo writes:

On October 28, Lord Abbot Giugni came with a priest and a servant of him.13

If the hypothesis that Galileo’s teaching was characterized by an encompassing curriculum of fortification is true, the most significant difference that distinguishes Galileo's curriculum concerns the long and detailed explanation of the uses of mathematical instruments like the compass for military purposes.

Reconstructing the curriculum of the course offered by Galileo, by means of the *Ricordi Autografi*, it becomes clear, in particular, that almost all of the students were taking lessons on the *Uso del Compasso Militare*. The possibility of obtaining such an instrument together with its instructions was not easy at the time and, since in Galileo's house it was also possible to obtain both the instrument and private instructions on its use, this must have been a great opportunity for a student of Fortifications, who could also obtain a horoscope from Galileo into the bargain.

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Most of the students alternated these lessons on the military compass with those specifically based on the treatise called *Le Fortificazioni*, like, for instance, Lord Alfelt, who in 1602

On December 10 has began again Fortifications and Instrument. 14

Before these lessons many students took others on different topics. As mentioned above, Galileo called one of these kinds of lessons *Le Mecaniche*. 15 In fact many contemporary treatises on fortification, such as that of Lorini, have a section dedicated to the same topic. It thus becomes clear that Galileo’s treatise *Le Mecaniche*, usually only considered in the context of Galileo’s emerging science of motion, was actually part of a curriculum on fortification. 16

The topics of the other lessons are compatible with this interpretation. Galileo’s reading of Euclid and Perspective, for instance, was oriented towards the practical needs of a military man faced with challenges such as designing fortresses, aiming a cannon, and surveying a terrain. Like every other treatise on fortification, Galileo’s taught how to divide lines and angles, how to draw geometric figures, and how to apply some useful geometric theorems, and this also referring to Albrecht Dürer. 17 Another topic taught by Galileo was the Sphere. For these lessons there are many entries in the *Ricordi*. Most of the students of the Sphere were taking these lessons after Mechanics and before starting Fortifications and Use of the Compass and some of them are registered only for this topic. Finally we have one entry regarding lessons in Geodesy and one for Arithmetics.

For the way the topics of the lessons are labelled and, more concretely, for the way in which students learned, it is possible to conclude that all topics, apart from the main lessons in Fortifications and Use of the Military Compass, were propaedeutic to the latter.


15 A paradigmatic entry in the *Ricordi Autografi* for the lessons called “mecaniche” is the following: in 1602, ”A dì 5 di Marzo cominciò le mecaniche il S. Marco pollacco, et il maiordomo del’Ill.mo S. Lencischi et S. Donec”; in Galilei G, «Ricordi Autografi», in Favaro [1968, Vol. XIX, p. 151].

16 Problems related to the dating of the Manuscript *Le Mecaniche* and that reveal the link between this Manuscript and Galileo's course on Fortifications are described in Gatto R., «Sull'edizione critica de *Le Mecaniche* di Galileo», in Montesinos, Solís, [2001, pp. 203-216].

Figure 3: Galileo's private lessons: the core was represented by courses in Fortifications and Use of the Military Compass. The students could take classes in one or more propaedeutic topics (the six external circles).

To sum up, Galileo's course was organized around Fortifications and the Use of the Military Compass. These lessons were based on the treatises *L'Uso del Compasso Militare* and *Le Fortificazioni*. Other topics were offered to the student probably in form of propaedeutic lessons such as those on Mechanics, Euclid, Perspective, Arithmetics, the Sphere and Geodesy. On some of these subjects, students could buy treatises in form of handwritten copies or copy Galileo's original treatises for themselves.

Obviously the teaching of these subjects must have been combined with and supported by an introduction into the usage of the mathematical instruments produced and sold by Galileo’s workshop. The fact that the students could learn all of this while staying in Galileo’s household was certainly an advantage and effectively made this household resemble more a boarding school for future officers than a scholarly studio.
CONCLUSION

In conclusion, it is clear that the two main activities at Galileo’s home - the workshop and the private lessons - were absorbing a great amount of his time and this already without counting the general administration of the household. Furthermore, the presence of the workshop finds a full explanation only if related to the large number of private students on Fortifications. What is more important is that the four treatises - *Le Mecaniche, Trattato della sfera, Le Fortificazioni* and *L'Uso del Compasso Militare* - are related to each other as parts of a complete course, where only the last of those, the one on the Military Compass, represents something new in comparison with common treatises on Fortification of the time. In other words, the subjects of *Le Mecaniche, Trattato della Sfera, and Le Fortificazioni* perfectly fit in the shared knowledge of military Architects of the time.

Finally, all this suggests that the image of Galileo, the lonely theoretician in Padua, is probably not adequate and must be complemented by that of Galileo *Ingegnere*.

REFERENCES


The early modern period saw the rapid development of two fundamental bodies of knowledge, astronomy and mechanics. The integration of these two bodies of knowledge later resulted in one of the most successful scientific theories, Newtonian classical mechanics. One of the first major steps in this development was Galileo’s attempt to present a coherent world picture linking Copernican astronomy with his new theory of motion, presented in the Dialogue of 1632. Although this attempt fell, in hindsight, short of its ambitions, it does testify to the fertility of integrating these two bodies of knowledge and has hence given rise to questions about which of them was the driving force. Did Galileo develop a new mechanics as a strategic plot in order to justify Copernican astronomy? Or did, on the contrary, his mechanical thinking necessitate his adherence to Copernicanism? Consequently Galileo’s ambitious attempt, first published in the Dialogue, to join a key insight of his mechanics, the law of fall, with the most advanced astronomical data in an attempt to explain the cosmogony of the planetary system appears to be uniquely suited to discuss these questions.

In a short passage in the Dialogue Galileo sketches this cosmogonical hypothesis and claims to have based calculations on it that agree “truly wonderfully” with observations.¹ Six years later in the Discorsi the hypothesis is reiterated.² The existing manuscript evidence of Galileo’s work on cosmogony has, however, received considerably less attention than the passages in the published work.

This paper presents some of the results of a new interpretation of six folio pages from a manuscript comprising Galileo’s notes on motion which is preserved as Ms. Gal 72 in the Biblioteca Nazionale Centrale in Florence. The diagrams and calculations scattered over the pages pertain to Galileo’s cosmogonical hypothesis. This paper will focus on showing that Galileo attempted to prove the empirical adequacy of this hypothesis when his new science of motion was still in its infancy and not yet sufficiently developed to tackle such a complex problem. Such a positioning of the work on the cosmogonical hypothesis to a time when Galileo probably was neither firmly committed to Copernicanism nor had a fully developed science of motion promises to shed new light on the interrelations between his mechanical and astronomical thinking.

In the *Dialogue* Galileo introduces an idea concerning the genesis of the planetary system, crediting Plato with its authorship. According to this idea, the “divine Architect” created the sun and, at a certain distance from it, the planets. The planets, according to their “assigned tendencies”, then began to fall towards the sun in naturally accelerated motion. Upon reaching their predestined orbits, their linear motions were diverted into circular motions by the “divine Mind”, thereby retaining their acquired velocities. Galileo’s spokesman Salviati raises the question of whether all planets could have been created in the same place—referred to in the following as the *creation point*—in order to account for the observed orbital velocities of the planets. After a fairly detailed description of how this hypothesis would have to be tested, Galileo informs us that he has carried out the required computations and that they agree “truly wonderfully” with his observations. When the topic is brought up again six years later in the *Discorsi*, Salviati once more emphasizes that Galileo had done the computation and “found it to answer very closely to the observations”.

These two passages in Galileo’s major works provoked great interest among his contemporaries and have also been discussed by historians of science. Intrigued by the obvious falsity of Galileo’s claim, first noticed by Mersenne in 1637 [Mersenne, M., 1637, pp. 103-107], historians have tried to come to terms with the motives for Galileo’s insistence on his cosmogonical hypothesis. Interpretive attempts focused initially on reconstructing computations possibly made


4 This reiteration of the cosmogonical hypothesis with its clear copernican undertone in the *Discorsi* is especially remarkable in the light of the fact that the *Discorsi* were published after Galileo’s trial—a trial that was itself triggered by his engagement in Copernicanism.
by Galileo that would justify his claim of empirical adequacy. This situation changed when Stillman Drake in 1976 discovered that Galileo, in various calculations scattered over three folios of Ms. Gal 72, had used numbers taken from Kepler’s *Mysterium cosmographicum*. He successfully linked these calculations to Galileo’s work on cosmogony and offered a preliminary interpretation.

However, before turning to an interpretation of Galileo’s elaborations of his cosmogonical hypothesis, two theorems that are essential for their understanding—the law of fall, and the so-called double distance rule—will be discussed. What is needed, in particular, is the law of fall in its geometrical or mean proportional form which can be found in the *Discorsi* as the second corollary to proposition II on accelerated motion:

> It is deduced, second, that if at the beginning of motion there are taken any two spaces whatever, run through in any [two] times, the times will be to each other as either of these two spaces is to the mean proportional space between the two given spaces.

The second theorem important for understanding Galileo’s work on cosmogony is the double distance rule. Even though never explicitly formulated as a theorem in the *Discorsi*, this rule was one of Galileo’s earliest and most important conceptual tools. The double distance rule states that a body whose motion is diverted into a uniform motion after fall through a certain distance will, in the time it took it to fall, traverse in uniform motion twice the distance fallen.

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5 A series of articles on Galileo’s cosmogony was published in the 1960s. The lively but short lived discussion revolved essentially around the two questions “why did Galileo attribute his hypothesis to Plato?” and “why was he so deeply committed to his hypothesis?”. The discussion can be reconstructed starting from [Cohen, I. B., 1967].

6 See [Kepler, J., 1981], henceforth referred to as *Mysterium*. From a note of thanks sent back to Kepler we know that Galileo had received a copy of Kepler’s *Mysterium* in 1597. See [Rosen, E., 1966] for an account of this first contact between Galileo and Kepler.

7 For Drake’s initial account see [Drake, S., 1973]. Drake later admitted a misreading of an abbreviation that, even though seemingly crucial to his interpretation, did not lead to its revision [Drake, S., 1978].

8 See [Galilei, G., 1974, pp. 170-171]. With A and B denoting the distances run through, and $T_A$ and $T_B$ the respective times of fall, the law of fall can (in modern notation) be expressed as $T_A : T_B = A : mp(A|B)$. Here $mp(A|B)$ specifies the mean proportional between the distances A and B, synonymous with the geometric mean of the two distances.

9 For a comprehensive account of the double distance rule and its role within the developing conceptual framework of Galileo’s science of motion see [Damerow, P., et al., 1992].
If Galileo’s cosmogonical hypothesis were correct it should be possible to derive essential features of the planetary system with the help of these two theorems. Indeed, given the point from which the divine creator drops the planets, their accelerated motion is determined by the law of fall, while properties of their uniform orbital motion can be inferred with the help of the double distance rule. But how exactly did Galileo test his cosmogonical hypothesis? Three folios containing material relevant to Galileo’s work on the cosmogonical hypothesis are readily identified by the appearance of the number 10759, the revolution time of Saturn in days as given by Kepler in his Mysterium.\textsuperscript{10} As will be shown, two of these, folios 134 and 135, document Galileo’s first approach to testing his hypothesis.\textsuperscript{11} They contain calculations and diagrams referring to the same geometrical set-up and are hence interpreted here as belonging to the same approach.

On folio page 135 verso Galileo starts this first attempt by identifying a circle with a radius of 35 units and consequently a circumference of 220 units with Saturn’s orbit.\textsuperscript{12} According to Galileo’s cosmogonical model there must be a creation point from which Saturn was originally dropped and whose height has to be determined from the astronomical data found in Kepler’s book. The size of the orbit, fixed by Galileo’s assumption, together with the period of Saturn’s revolution fully determine the planet’s uniform motion along its orbit. As mentioned above, the double distance rule makes it possible to determine from an accelerated motion over a given

\textsuperscript{10} The orbital data used by Galileo in the cosmogonical calculations can be found in the last two chapters of Kepler’s Mysterium. A detailed discussion of the approach Kepler took on the question of planetary motion in the Mysterium can be omitted here, since it is essential only to Galileo’s second approach to the cosmogonical problem, in folio 146, which will not be discussed in this article. In the first approach to the problem, Galileo simply used the revolution periods of the planets he found in a table in chapter 20 of the Mysterium. The exact amount given for the period of Saturn in this table is 10,759 and twelve sixtieth days.

\textsuperscript{11} Folios 134 and 135 are physically connected and together form a larger sheet of paper. Both bear a double watermark of the type crown/crossbow and can hence with high probability be attributed to Galileo’s Paduan period. These two folios have been discussed in the literature. Stillman Drake dedicated a short paragraph to their discussion without linking their content directly to Galileo’s cosmogony. Drake does not deal with the content of folio 135 at all, while the content of the folio 134 is only touched upon briefly in a rather speculative paragraph. Following up on Drake’s work Eric Meyer realized that the calculations on these two pages represent an early approach by Galileo to testing his cosmogonical hypothesis but failed to give a satisfactory interpretation because of a puzzle he had encountered but did not solve, see [Meyer, E., 1989].

\textsuperscript{12} This choice of a radius for Saturn’s orbit merely fixes the unit in which the cosmic distances are to be measured. Since Galileo uses the numerical value 3 1/7 for Pi, a diameter of 7 naturally offers itself if one aims to arrive at an even number for the circumference. Scaled by 10 this value represents a convenient choice for subsequent calculations.
space and time the distance traversed in the same time by a uniform motion resulting from a
deflection of the accelerated motion. What is actually needed in this case, however, is a reversal
of this procedure allowing to determine a distance of fall from a given uniform motion. It was
therefore plausible for Galileo to assume that Saturn, since it covers in its uniform orbital mo-
tion a distance of 220 units during its revolution time of 10759 12/60 days, has in the preceding
free fall covered in the same time half the distance, that is 110 units (Fig. 1).

Figure 1: Determination of the creation point from the orbital motion of Saturn.

As a next step Galileo tried to test the empirical adequacy of his hypothesis by determining the
motions of the inner planets resulting from a fall from the creation point whose position he had
determined from Saturn’s motion. How can the motion of one of the inner planets be deter-
mined? If the size of the orbit is assumed to be known, the time it takes an inner planet to fall
from the given creation point to its orbit can be found with the help of the law of fall. After being
diverted into its orbit, the inner planet will, according to the double distance rule, traverse twice
the distance fallen in the time of the motion of fall. This distance will no longer, in contrast to
the case of Saturn, coincide with half the length of the orbit’s circumference. Accordingly also
the revolution time of the planet will no longer coincide with the time of fall but it can be cal-
culated from the ratio between double the distance of fall and the length of the orbit. In short,
the determination of the period for an inner planet from its orbital geometry involves three steps,
first the application of the law of fall to the distance between the creation point and the orbit,
then an application of the *double distance rule* to the relation between the accelerated motion of fall and the uniform orbital motion, and finally an application of a basic theorem on uniform motion yielding the period of revolution (Fig. 2).

**Figure 2: Determination of the motion of an inner planet.**

If the proportions yielded by these three steps are combined one arrives, by employing basic proportional theory, at the proportion shown in Figure 3, henceforth referred to as the *first cosmogonical proportion*.13

**Figure 3: First cosmogonical proportion used on folio page 135 verso.**

As a matter of fact, the *first cosmogonical proportion* provides Galileo with two alternatives for testing whether the actual motion of the planets is in accordance with the cosmogonical hypothesis. It can either be exploited to determine the planetary geometries from given periods or it can be used to determine the periods from a given planetary geometry. If the hypothesis were
adequate, the results of both approaches would coincide and yield the empirical data. Given the fact that Galileo made these calculations at a time when the periods of the planets had been observed with pretty high accuracy but little was known about the actual planetary geometry, he reasonably chose the first approach, namely to determine the planetary geometry from the periods. However, this approach was hampered by a difficulty resulting from the fact that he could not resolve the *first cosmogonical proportion* for one characteristic magnitude of the orbital geometry but rather had to rely on a laborious iteration procedure in order to determine such a magnitude from the given periods and the given position of the *creation point*.14

On folio page 135 verso the *first cosmogonical proportion* is exploited to determine Jupiter’s planetary geometry from the periods of the planets given by Kepler. From a first guess of Jupiter’s planetary geometry, a period for Jupiter is calculated. According to the result the geometry is then varied until the period resulting for Jupiter is within an acceptable limit identical to the one given by Kepler.15

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13 In this and in the following footnotes capital letters represent a quantity that is a time or a distance. A subscripted letter further specifies the quantity as pertaining to one of the planets, where a subscripted S denotes Saturn, a subscripted J Jupiter. Accordingly $T_S$ and $T_J$ represent the times of fall of Saturn and Jupiter over the respective distances of their orbits from the *creation point*, $F_S$ and $F_J$. As a consequence of Galileo’s earlier choice $T_S$ is identical to the period of Saturn $P_S$. Then according to the law of fall $T_J : T_S = mp(F_J | F_S) : F_S$. According to the *double distance rule* Jupiter after being diverted in its orbit will cover $2F_J$ in the time $T_J$. Then since in uniform motion the times are in the same proportion as the distances covered $P_J : T_J = O_J : 2F_J$, where $O_J$ denotes the size of Jupiter’s orbit. Hence by compounding proportions $P_J : P_S = (mp(F_J | F_S) : F_S) \times (O_J : 2F_J)$ which according to proportional theory is equivalent to $P_J : P_S = O_J : 2mp(F_J | F_S)$, the *first cosmogonical proportion*.

14 Doubts concerning the precision of the sizes of the Copernican orbits could have been furthered by Galileo’s reading of chapter 18 of the *Mysterium*, in which Kepler elaborates on the disagreement between his theoretical values and the sizes of the Copernican orbits as well as on the precision of astronomy in general. Yet Galileo with the mathematical tools available to him could not exploit the *first cosmogonical proportion* to determine $O_J$ from the given ratio of the revolution times directly. A complicated relation relates the size of Jupiter’s orbit to the mean proportional of the distances of Saturn’s and Jupiter’s orbits from the *creation point*. Consequently Galileo can only determine $O_J$ from a given mean proportional $mp(F_J | F_S)$ in three steps. First $F_J$ is determined from $F_S$ and the mean proportional $mp(F_J | F_S)$ according to $F_J = mp(F_J | F_S)^2 : F_S$. In the next step the radius of Jupiter’s orbit $R_J$ is calculated as the difference between the distance of the *creation point* from the center and the distance fallen by Jupiter, $F_J$. The size of Jupiter’s orbit is calculated from knowledge of the size of its radius according to $O_J = 2 \times 3 \frac{1}{7} \times R_J$. Finally in a last step the resulting period of Jupiter is calculated according to $P_J = O_J : 2mp(F_J | F_S) \times P_S$, in conformity with the *first cosmogonical proportion*. 
One would expect that Galileo went through this procedure four times to determine the orbital geometries of the other inner planets Mars, Earth, Venus and Mercury. Indeed on folio page 134 verso we find the calculations for Mars. But only at first glance do the calculations follow the same scheme. A closer look reveals that in place of the *first cosmogonical proportion* Galileo had used on folio page 135 verso, he here uses a modified proportion, in the following called the *second cosmogonical proportion*, to determine the orbital geometry of Mars from its period (Fig. 4).

![Diagram](image)

**Figure 4: Conflicting proportions used in the calculations for Jupiter and Mars.**

As it turns out, the *second cosmogonical proportion* is, from Galileo’s perspective, just as justified as is the first. Instead of just using the law of fall and the *double distance rule*, the method of calculation underlying this second proportion is based on a principle of his theory of motion that is incorrect according to classical mechanics: the proportionality between the distance of fall and the “degree of speed” it reaches at the end. From a famous letter written to Sarpi in October 1604, we know that Galileo at that time adhered to this principle of fall on which he hoped to found his new science. The use in the cosmogonical calculations of the principle mentioned in this letter suggests to date the calculations previously discussed to a time period whose boundaries are determined by the fact that Galileo is already using the law of fall while still employing his early erroneous principle of fall.

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15 On folio page 135 verso Galileo applies the procedure to determine Jupiter’s period $P_J$ from a given mean proportional $mp(F_S|F_J)$ described in the preceding footnote a full five times. The subsequent choices for the numerical value of the mean proportional, 120, 116, 118, 119 and 119 1/4 result in the following periods for Jupiter, 39[67], [6608], [5282], 45[xx] 44[59]. Digits not written down by Galileo are given in square brackets. The final period calculated for Jupiter has to be compared to the actual period of 4333 days.

16 See the letter to Paolo Sarpi, October 16, 1604, [Galilei, G., 1890-1909, X: 115f].
If this principle is applied to the case under consideration, the necessary calculations may be simplified a bit, which possibly constitutes the reason why Galileo adopted this second method – which from his point of view must have been equivalent to the first – to the determination of the orbit of Mars. According to Galileo’s erroneous principle, the degrees of speed of Saturn and an inner planet upon reaching their respective orbits are to each other in the same ratio as the distances fallen. A proposition on uniform motion then allows one to determine the ratio of times of the two uniform planetary motions covering orbits of different sizes with the different speeds resulting from their fall. Since this reasoning yields exactly the second cosmogonical proportion used on folio page 134 verso, it becomes probable that Galileo’s principle mentioned in the letter to Sarpi constitutes indeed the basis for this calculation.\(^{18}\)

Ergo both cosmogonical proportions, even though contradictory from the perspective of classical physics, turn out to be justified within the framework of Galileo’s science of motion in a particular stage of its development. But did Galileo realize this internal contradiction in his approach to the problem? At least not immediately, because the diagram he drew for the three orbits of Saturn, Jupiter, and Mars on the reverse of folio 134 contains a further orbit showing that Galileo had carried out computations also for the earth without in any way revising his earlier results.\(^{19}\)

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17 Such a dating is in accordance with other proposed evidence for dating, especially with the conjecture that a letter by Edmund Bruce written to Kepler in 1602 refers to Galileo’s work on cosmogony. It is however less speculative and at the same time has the prospect of being refined as we learn more about the development of Galileo’s science of motion.

18 With GV\(_S\) and GV\(_M\) representing the degrees of speed of Saturn and Mars upon reaching their orbits, the principle that the degrees of speed grow in proportion to the distances fallen can be expressed as GV\(_S\) : GV\(_M\) = FS : FM. Together with Galileo’s common assumption that in the deflection of an accelerated motion into a uniform motion the ratio of the degrees of speed is preserved in the ratio of the speeds of the resulting uniform motion, the proportion also holds for the orbital speeds VS : VM = FS : FM. Since in uniform motion with unequal speeds, the ratio of the times is compounded from the ratio of spaces and from the inverse ratio of speeds and the size of Saturn’s orbit amounts to twice the distance fallen by Saturn the following proportion holds PM : PS = OM : 2 FM, the second cosmogonical proportion. Just as before this proportion is exploited to determine the size respectively the radius of Mars’s orbit.
There is no indication whatsoever, whether, for the time being, Galileo judged his cosmogonical calculations to be successful or not. What had been accomplished was, in any case, the calculation of orbits so that the planets would move in these orbits with the observed orbital periods. However, a comparison of these calculated orbits with the sizes of the Copernican orbits used in Kepler’s book shows crude deviations.

While the implications of Galileo’s calculations for the understanding of cosmogony may have been doubtful, these calculations probably had a profound impact on Galileo’s science of motion. Folio page 134 recto contains a diagram representing a motion of fall, interrupted at two points to generate a uniform horizontal motion. This diagram represents, in a nutshell, the essential mechanism of Galileo’s calculations based on three fundamental ingredients, the law of fall, the double distance rule, and the erroneous proportionality between the degrees of speed and the distances of fall. As is known from the analysis of other folio pages of Ms. Gal. 72, Galileo eventually realized and elaborated the internal contradiction between his erroneous principle and the law of fall on the basis of considering exactly the same diagram.\(^{20}\) Indeed, Galileo’s refutation of his erroneous principle of fall has precisely the same structure as his cosmogonical hypothesis, which also involves a comparison between different uniform motions generated by a motion of fall, and may well have been triggered by the work on this hypothesis. In other words, Galileo’s elaboration of his theory of motion to include cosmogony may have had far-reaching repercussions on its very foundations. The insight into the contradiction between the law of fall and the early erroneous principle led to a conceptual revision of the foundations of his theory of motion, requiring in particular a shift to a new principle of fall, according to which the degrees of speed increase with the times fallen.

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19 Meyer has pointed out that the radii of the three outermost of the four concentric circles on folio page 134 recto are in the same ratio as the radii of Saturn, Jupiter, and Mars as resulting from Galileo’s considerations. Meyer further claims that the scattered calculations on this page represent attempts to reduce the calculated ratios to a series of simple ratios. The inner circle represents Earth’s orbit drawn to scale with a ratio resulting from a calculation similar to the ones discussed. Whether the first cosmogonical proportion or the second cosmogonical proportion have been used in the determination of Earth’s radius cannot be established with certainty since both approaches involve a break off of the calculation procedure when a satisfactory agreement with Earth’s actual period of 365 days is reached.

20 These considerations are documented on folio page 152 recto. For a comprehensive interpretation see [Damerow, P., et al., 1992, pp. 185-194].
When Galileo returned to the problem of cosmogony for a second time after the revision of his theory, he consequently had to modify his approach. His later approach, which will not be discussed here, avoided also other shortcomings of his first attempt. In fact, he now not only used the correct proportionality between the degrees of speed and time, but also managed to reproduce with his model the planetary data, orbits as well as periods, for two planets, and he developed a more adequate method of calculation. Even though Galileo again did not achieve full correspondence between Kepler’s astronomical data and the results of the calculations based on his own hypothesis, this second approach represented a far more adequate appropriation of the problem to his science of motion. The relative success of the second approach may well have constituted the background of Galileo’s self-assured public statements on the cosmogonical hypothesis.

Historians of science have extensively discussed the interrelations between Galileo’s mechanical thinking and his astronomical thinking. They have in particular tried to answer the question of whether it was a fully developed Copernican program that shaped Galileo’s science of motion or whether it was only his theory of motion that led to the development of such a program. The example presented in this paper implies, however, that the question is posed too narrowly. The interpretation given shows that Galileo tried to integrate his new science of motion with Copernican concepts already at an early stage, that is at a time when neither his science of motion nor his Copernican position were fully developed. As a matter of fact, in Galileo’s time, the omnipresence of Aristotelian natural philosophy and the astronomical issues related to it created boundary conditions that no attempt at a new science of motion could ignore. Galileo’s early

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21 As discussed above, Galileo, in his determination of the position of the creation point, had assumed that Saturn had fallen from the creation point to its orbit in exactly its revolution period. While this condition may have appeared natural in view of the double distance rule, it actually turned out to be too restrictive. In fact, it fixes the ratio between distances traversed and times consumed in free fall, in modern terms it fixes a constant of acceleration. It is, on the other hand, exactly the freedom in this choice of the relation between distance fallen and time consumed that allows Galileo in his second attempt at the problem to determine the creation point from the orbital data of two planets such that vice versa fall from this creation point yields the correct orbital data for these two planets. I plan to include a more detailed study of Galileo’s work on cosmogony including his second attempt documented on folio 146 in my dissertation project on the development of Galileo’s science of motion.

22 See for example [McMullin, E., 1967].

23 For a more detailed account of this view of the interrelations of Galileo’s mechanical and astronomical thinking see the contribution by Büttner, Damerow and Renn in this preprint.
attempt of an integration of his new science of motion with cosmological issues hence does not bear the characteristics of a strategically planned step, but has rather to be interpreted as an unavoidable encounter that affected both, Galileo’s understanding of cosmology and his theory of motion.

REFERENCES


