

Geometric stabilization of the electrostatic ion-temperature-gradient driven instability: nearly axisymmetric systems

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Abstract

The effects of a non-axisymmetric ($3D$) equilibrium magnetic field on the linear ion-temperature-gradient (ITG) driven mode are investigated. We consider the strongly driven, toroidal branch of the instability in a global (on the magnetic surface) setting. Previous studies have focused on particular features of non-axisymmetric systems, such as strong local shear or magnetic ripple, that introduce inhomogeneity in the coordinate *along the magnetic field*. In contrast, here we include non-axisymmetry explicitly via the dependence of the magnetic drift on the field line label α , *i.e. across the magnetic field*, but within the magnetic flux surface. We consider the limit where this variation occurs on a scale much larger than that of the ITG mode, and also the case where these scales are similar. Close to axisymmetry, we find that an averaging effect of the magnetic drift on the flux surface causes global (on the surface) stabilization, as compared to the most unstable local mode. In the absence of scale separation, we find destabilization is also possible, but only if a particular resonance occurs between the magnetic drift and the mode, and finite Larmor radius effects are neglected. We discuss the relative importance of surface global effects and known radially global effects.

I. INTRODUCTION

With the start of the Wendelstein 7-X experiment [1] and the coming of ITER [2], we are urged to explore the role of non-axisymmetric magnetic fields in fusion devices, for they are crucial in the control of edge localized modes (ELMs) in Tokamaks and they will necessarily influence core transport in Stellarators [3]. There is general agreement on the use of gyrokinetic theory [4] for the prediction of kinetic instabilities and transport levels in the core of fusion plasmas, however, there is not a simple prescription on *how* to solve this equation.

Gyrokinetic theory describes strongly anisotropic, slowly evolving plasmas. It is an asymptotic theory based on the fundamental assumption that perturbations can vary across the equilibrium magnetic field on a short kinetic length, l_{\perp} (of the order of the Larmor radius ρ),

$$\nabla_{\perp} \delta f_1 \sim l_{\perp}^{-1} \delta f_1 \sim \rho^{-1} \delta f_1, \quad (1)$$

but equilibrium quantities vary slowly on a macroscopic length L ,

$$\nabla_{\perp} f_0 \sim L^{-1} f_0. \quad (2)$$

Perturbations are assumed to vary slowly along the equilibrium magnetic field

$$\frac{\nabla_{\parallel} \delta f_1}{\nabla_{\perp} \delta f_1} \sim \frac{l_{\perp}}{L} \sim \frac{\rho}{L} \ll 1. \quad (3)$$

A further separation of *time* scales,

$$\frac{\omega}{\Omega_c} \sim \frac{\rho}{L} \ll 1, \quad (4)$$

where Ω_c is the cyclotron frequency, allows an average over the gyro-motion, to obtain a closed nonlinear kinetic theory that retains Larmor radius effects. The distribution function of particles is found perturbatively in $\rho_* = \rho/L$, that is $f = f_0 + \delta f_1 + \dots$, with $\delta f_n \sim \rho_* \delta f_{n-1}$, where the expansion parameter, $\rho_* \ll 1$, is chosen to be l_{\perp}/L : the ratio of the characteristic scales of variation of perturbations and equilibrium quantities, respectively. Thus, one of the fundamental assumptions of gyrokinetics is the separation of length and time scales

$$\rho \ll L, \quad (5)$$

so that conditions (3) and (4) are fulfilled. Most numerical codes solve for δf_1 , giving answers which are correct only to zeroth order in the expansion parameter.

The natural representation for modes supported by the gyrokinetic equation in toroidal geometry is the twisted slice of Roberts and Taylor [5]. The same representation, suitably extended to allow for toroidal topology, was adopted to solve the problem of ballooning modes in ideal MHD [6] and is the basis of a fruitful approach to numerical simulations of microturbulence: the local flux-tube [7]. In a local flux-tube simulation, the information about the variation of equilibrium quantities is carried by characteristic constant length scales. For instance, the equilibrium density gradient that drives drift-wave turbulence is treated as

$$\frac{\nabla n}{n} \approx \frac{1}{n_0(r_0)} \nabla n_0(r_0) \left[1 - \frac{r}{L_n} \right] \approx -\frac{1}{L_n}, \quad (6)$$

where $n_0(r_0)$ is a constant value at a given location r_0 . Local flux-tube simulations therefore can preserve separation of scales in a simple and effective way.

In some circumstances, however, we might be interested in equilibrium quantities with less trivial variations or in large turbulent structures that do not fulfill conventional scale separation. This motivated the development of “global” gyrokinetic codes. There are mainly two families of such codes: those which retain radial variations of equilibrium quantities (radially global) [8–15], and those which retain variations of these quantities across the equilibrium magnetic field (within but not across magnetic surfaces, ergo global on the surface) [3, 16]. In both cases, allowing for spatial variation of equilibrium quantities to interact with that of the fluctuations seems at odds with the underlining assumption of separation of scales, Eq. (5), especially if the gyrokinetic equation solved is asymptotically correct only to zeroth order in $\rho_* \ll 1$. However, we can identify two scenarios in which it is both consistent and desirable to solve the gyrokinetic equation in the full-surface setting.

In the first scenario, conventional gyrokinetics is assumed to apply, with good scale separation between the background and fluctuations. Fortunately, recent turbulence simulations [17] indicate that the properties of fluctuations are then determined “locally”, by conditions in the neighborhood of a given field line, validating the underlying assumption of scale separation. An intriguing question is whether this is also true of linear modes: If one (naively) solves the linear gyrokinetic equation in a flux-surface domain, are then the linear modes affected by the magnetic geometry of the entire surface, or are they determined only by the properties at a specific location, *i.e.* where the modes peak?

The second scenario in which full-surface gyrokinetic simulations are needed is when there is a certain loss of scale separation, namely that between the magnetic field and the

fluctuations. Let us introduce a new scale $a \gg \rho$, taken to be the smallest scale on which the background magnetic field \mathbf{B} can vary. The field may of course still have variation on the larger equilibrium scale L , but now also includes a range of scales between a and L . Fluctuations exhibit a range of scales, and we will denote the largest scale as l_{\perp}° , assuming

$$l_{\perp}^{\circ} \sim a \ll L. \quad (7)$$

As the magnetic drifts must remain small, we require that the gradient of the magnetic field can still be ordered as

$$\nabla B \sim B/L, \quad (8)$$

implying that any small scale component of \mathbf{B} is itself small in magnitude. For example, if \mathbf{B} can be decomposed as $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_a$, then $B_a/B_0 \sim a/L$ is required to ensure $\nabla B_a \sim \nabla B_0$; basically, a small scale variation of the equilibrium magnetic field can be tolerated, if such variation pertains to a small corrugation of the field. We note that the above ordering constitutes a slight variation on standard gyrokinetic theory; further details of such a theory are left for a future publication.

A concrete example where such an intermediate scale is necessary is a stellarator where the character of the magnetic curvature changes (from good to bad and back again) over a poloidal length-scale that is shorter than the major radius but still much larger than the ion gyroradius.

In the case of an equilibrium magnetic field imposed with external coils (as for Stellarators), the long-time evolution of the intermediate equilibrium scale, a , does not depend on transport processes, as opposed to radially global structures such as the temperature profile. Thus, we can expect Eq. (7) to be a structural property of a device that can be always be evaluated a priori ($a \ll L$) and verified a posteriori ($l_{\perp} \lesssim a$).

In this work, we present a study of global (on the surface) effects on the electrostatic ion-temperature-gradient (ITG) driven mode. We focus of the strongly driven, toroidal branch of the instability.

The equations we will use to study this problem are presented in Section (II), with a discussion of the assumptions and simplifications, the most significant being the neglect of parallel ion motion. This is an important simplification because it focusses attention on the

effects of non-axisymmetry in particular, as opposed to the effects of the variation of geometric quantities along the magnetic field line, which have been studied in previous works [18, 19]. In Section (III) we study perturbatively the effect of non-axisymmetric corrections to the magnetic drift, in the limit that the scale of the mode is much smaller than the scale of the magnetic drift. An important conclusion is that, in non-axisymmetric geometry, modes are generally dependent on the magnetic geometry of the entire surface, *i.e.* they are not driven by the resonance of the local drift, but to first order act according to the surface-average drift. This means that, in non-axisymmetric systems, one must distinguish between local (flux tube) and global (full-surface) linear modes, even in the conventional limit $\rho_* \rightarrow 0$. This is different from previously studied (radially) global effects that require a finite ρ_* . In Section (IV) we relax the assumption of separation of scales, between the mode and equilibrium. What we find is consistent with the case of separation, but there now appears a possibility of a destabilizing resonance with magnetic drift. However, this resonance occurs under special conditions, and is counteracted by the aforementioned averaging effect, and we conclude that the generic effect of non-axisymmetry is one of stabilization. In Section (V), we discuss the relative importance of radially and surface-global effects, concluding that the surface global effects predominate as long as the relative strength of non-axisymmetric component of the magnetic drift exceeds the gyrokinetic expansion parameter ρ/L . Conclusions are presented in Section (VI).

II. MODEL EQUATION

Our starting point is the linear gyrokinetic equation for the ions [20, 21]

$$iv_{\parallel} \frac{\partial h_{\mathbf{k}}}{\partial z} + (\omega - \hat{\omega}_d) h_{\mathbf{k}} = (\omega - \hat{\omega}_*^T) J_0 \left(\frac{k_{\perp} v_{\perp}}{\Omega_i(B)} \right) \frac{e\varphi_{\mathbf{k}}}{T_{0i}} F_{0i}, \quad (9)$$

where $h_{\mathbf{k}} = h_{\mathbf{k}}(\mathbf{R}, \mu, \mathcal{E}, t)$ denotes the Fourier component of the non-adiabatic part of the perturbed distribution function $\delta f = -F_{0i} Z e \varphi(\mathbf{r}, t) / T_{0i} + h(\mathbf{R}, \mu, \mathcal{E}, t)$, $\mu = mv_{\perp}^2 / (2B)$ the magnetic moment, and $\mathcal{E} = B\mu + mv_{\parallel}^2 / 2$ the total particle energy, and v_{\perp} and v_{\parallel} are the perpendicular and parallel (to the equilibrium magnetic field \mathbf{B}) particle velocities. Here

$$F_{0i} = \frac{n_{0i}}{(\pi v_{thi}^3)^{3/2}} e^{-\frac{\mathcal{E}}{T_{0i}}} \quad (10)$$

is the equilibrium Maxwellian with $v_{thi} = \sqrt{2T_{0i}/m_i}$, where T_{0i} and n_{0i} are the equilibrium temperature and density, respectively. The term on the RHS of Eq. (9) represents the Fourier

component, $J_0(k_\perp v_\perp / \Omega_i(B)) \varphi_{\mathbf{k}} = \langle \varphi \rangle_{\mathbf{R}, \mathbf{k}}$, of the gyroaveraged electrostatic potential φ . Here [22] $\mathbf{R} = \mathbf{r} + \mathbf{v}_\perp \times \hat{\mathbf{b}} / \Omega_i(B)$ is the gyrocentres' position, \mathbf{r} is the physical position of particles in real space, $\hat{\mathbf{b}} = \mathbf{B}/B$ is the unit vector in the direction of the equilibrium magnetic field, and $\Omega_i(B) = eB/m_i$ is the cyclotron frequency. Furthermore, we have $\hat{\omega}_*^T = \omega_* + \eta_i \omega_* (\mathcal{E} - 3/2)$, with $\omega_{*i} = k_\alpha \rho_i v_{thi} / (2L_n)$, and $\rho_i = v_{thi} / [a \Omega_i(B_0)]$. Equilibrium density and temperature are treated locally as shown in Eq. (6), with $L_n^{-1} = (aB_0) d \ln n_{0i} / d\psi$, $\eta_i = d \ln T_{0i} / d \ln n_{0i}$, a is a reference scale, and B_0 is a reference magnetic field strength. The form of the perturbations used is $\sim \exp[-i\omega t + iS(\psi, \alpha)]$, where field-aligned co-ordinates (ψ, α, z) define the equilibrium magnetic field [23]

$$\mathbf{B} = \nabla \psi \times \nabla \alpha, \quad (11)$$

z being the field-line following co-ordinate and ψ a scalar that defines magnetic flux surfaces. The triad of co-ordinates is completed by the field-line label $\alpha = \theta - \iota \zeta$, constructed from the poloidal (θ) and toroidal (ζ) angles, with ι the rotational transform. Our main focus in this work will be the geometric effects associated to the variable α . Within this formalism, the fast variation of the fields is contained in the eikonal S , which defines the wavevector

$$\mathbf{k}_\perp \equiv \nabla S = k_\psi \nabla \psi + k_\alpha \nabla \alpha, \quad (12)$$

where S is not periodic in α , but $k_\alpha = \partial_\alpha S$ and $k_\psi = \partial_\psi S$ are [21].

Geometric effects enter Eq. (9) mainly in two ways: through the frequency

$$\hat{\omega}_d = \hat{\mathbf{v}}_d \cdot \mathbf{k}_\perp \equiv \left(\frac{v_\parallel^2}{v_{thi}^2} + \frac{v_\perp^2}{2v_{thi}^2} \right) \mathbf{v}_d \cdot \mathbf{k}_\perp, \quad (13)$$

via the gradient of the equilibrium magnetic field in the low- β approximation of the drift frequency,

$$\omega_d = \mathbf{v}_d \cdot \mathbf{k}_\perp \equiv \frac{v_{thi}^2}{\Omega_i(B)} \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \mathbf{k}_\perp, \quad (14)$$

via the wave vector \mathbf{k}_\perp , and through the gyroaverage operation, which varies with \mathbf{k}_\perp for the following reason. If we know the solution of Eq. (9), we can readily evaluate the perturbed ion density and use it in the quasineutrality equation (assuming an adiabatic electron response)

$$\frac{1}{n_0} \int d^3 \mathbf{v} J_0 \left(\frac{k_\perp v_\perp}{\Omega_i} \right) h_{\mathbf{k}} = (1 + \tau) \frac{Ze \varphi_{\mathbf{k}}}{T_{0i}}, \quad (15)$$

where $\tau = T_{0i}/T_{0e}$. The function $h_{\mathbf{k}}$ is generally proportional to $J_0 F_{0i}$, and therefore the velocity-space integral of Eq. (15) results in a functional of argument $b \propto k_{\perp}^2$ [20, 21]. However, in general geometry

$$k_{\perp}^2 = k_i k^i = k_i g^{ij} k_j, \quad (16)$$

where g^{ij} is the metric tensor associated with the system under consideration. For instance, if we consider a large-aspect-ratio axisymmetric toroidal plasma with circular cross section, in suitably normalized units, we have [24, 25]

$$g^{\psi\psi} = 1, \quad g^{\psi\alpha} = g^{\alpha\psi} = \hat{s}z, \quad g^{\alpha\alpha} = 1 + \hat{s}^2 z^2, \quad (17)$$

where $\hat{s} = q(\psi)^{-1} dq/d\psi$ is the magnetic shear, q the safety factor, and we find the familiar expression [6]

$$k_{\perp}^2 = k_{\psi}^2 + 2k_{\psi}k_{\alpha}\hat{s}z + k_{\alpha}^2(1 + \hat{s}^2 z^2). \quad (18)$$

This simple case is paradigmatic of the difficulties encountered when solving Eq. (9) in toroidal geometry. In Eq. (18), the dependence of k_{\perp}^2 on the shear, \hat{s} , introduces a dependence on the field-following co-ordinate z in the driving term of the gyrokinetic equation (9). Thus, Eq. (9), in general, must be solved as a differential problem along the field line, with appropriate boundary conditions [26].

We now aim at the study of Eq. (9) when the magnetic geometry imposes variations of the equilibrium quantities in the variable α , insomuch that Eq. (9) must be considered differential in the variable α as well. This idea is at the heart of the numerical treatment of gyrokinetics on a flux surface, and will lead us in our analysis.

Let us specify the type of instability we want to investigate in this geometrical setting: the ion-temperature-gradient (ITG) driven instability. Traditionally, the theory of ITG modes has been studied by using a sound expansion of the kinetic ion response, generating simplified fluid models that retain key kinetic features such as diamagnetic and finite Larmor radius (FLR) effects [19, 27–29]. We will adopt this approach in this work.

After solving the ion gyrokinetic equation in a sound expansion (small streaming term), with a suitable ordering [19, 29], and integrating the quasineutrality condition Eq. (15), we obtain a second order differential equation for the electrostatic potential

$$\left(\tau - \frac{1}{2} \frac{\omega_T v_{thi}^2}{\omega^3} \frac{\partial^2}{\partial z^2} \right) \varphi = \left(-2 \frac{\omega_d \omega_T}{\omega^2} + \frac{\omega_T}{\omega} b \right) \varphi, \quad (19)$$

where $b = (a^2 k_{\perp}^2) \rho_i^2 / 2$. The result is standard and will not be reproduced here [see Ref. [19] and references therein]. We just note that, for sake of simplicity, we are considering strong temperature gradients, $\eta_i \gg 1$, so that $\omega_{*i} \rightarrow 0$ but $\omega_T = \eta_i \omega_{*i} = \mathcal{O}(1)$. How do we extend Eq. (19) to a magnetic surface? Let us analyze the local (in α) problem first.

The simplest way to identify classes of solutions of Eq. (19) is by expanding the RHS at the location in z where the magnetic drift has a local maximum (greatest drive). The magnetic drift frequency can be taken as quadratic in z around its maximum [19] $\omega_d = (k_{\alpha} \rho_i v_{thi} / R)(1 - \sigma^2 \theta^2)$, where $\theta = \pi z / l_{\parallel}$ is now used as the normalized field-line-following co-ordinate, σ is a constant, R a length associated with the magnetic field curvature, and l_{\parallel} the connection length. The function b , as Eq. (18) shows, can be also taken to be quadratic in z , then

$$b \approx (k_{\alpha}^2 \rho_i^2 / 2)(1 + \hat{s}^2 \theta^2), \quad (20)$$

where we anticipated that $k_{\psi} \equiv 0$. Thus, by using in Eq. (19) the ansatz

$$\varphi = e^{-\lambda \theta^2}, \quad (21)$$

one can derive, after setting to zero the coefficients of the quadratic polynomial obtained, two equations for the eigenvalue ω , and for λ [19]. The former characterizes the stability, the latter determines the localization of the mode along the field:

$$\tau + \underbrace{\frac{k_{\alpha} \rho_i}{2} \frac{v_{thi}^3 \lambda}{\omega^3 q^2 R^2 L_T}}_{streaming} - \underbrace{\frac{1}{4} \frac{v_{thi}}{\omega L_T} k_{\alpha}^3 \rho_i^3}_{FLR} + \underbrace{\frac{v_{thi}^2}{L_T R \omega^2} k_{\alpha}^2 \rho_i^2}_{curv.} = 0 \quad (22)$$

$$- \underbrace{k_{\alpha} \rho_i \frac{v_{thi}^3 \lambda^2}{\omega^3 q^2 R^2 L_T}}_{streaming} - \underbrace{\frac{1}{4} \frac{v_{thi}}{\omega L_T} k_{\alpha}^3 \rho_i^3 \hat{s}^2}_{FLR} + \underbrace{\frac{v_{thi}^2}{L_T R \omega^2} k_{\alpha}^2 \rho_i^2 \left(\hat{s} - \frac{1}{2} \right)}_{curv.} = 0, \quad (23)$$

where we are considering the $s - \alpha$ geometry [6] for illustrative purposes, since this allows us to discuss the effect of the shear. In this case R is the major radius of the device, $l_{\parallel} = q\pi R$, and $\sigma^2 = 1/2 - \hat{s}$.

We see that Eq. (19) supports two types of solutions: one when the streaming term contributes to the instability of the mode (slab branch), $\lambda \sim \mathcal{O}(1)$, and one when it does not. An ordering then follows from the requirement that the contribution of the streaming term is small in Eq. (22) but not in Eq. (23), where it is needed to determine λ :

$$k_{\alpha} \rho_i \frac{v_{thi}^3}{\omega^3 q^2 R^2 L_T} \sim \frac{1}{\lambda^2} \ll 1. \quad (24)$$

This implies a condition on R/L_T and on the connection length $l_{\parallel} = q\pi R$. The actual details of such constraint depend on the explicit form of the eigenvalue ω . We call the first case “ballooning limit”, where the shear plays a crucial role in the stability *and* localization of the mode. In the second case, when Eq. (24) holds, the eigenfunction is strongly localized along the field by the curvature term. This is what we call “strong interchange limit”. In this regime, indeed, the streaming term contribution in Eq. (22) can be neglected [because is $\mathcal{O}(\lambda^{-1})$], but the finite λ contribution in Eq. (23) sets the structure of the mode, which is assumed to be localized enough for Eq. (20) and (21) to be valid. While in the ballooning limit the shear enters in the expression for the frequency, it does not in the strong interchange limit. For instance, by neglecting the FLR term for simplicity, we find

$$\lambda \approx e^{i\frac{\pi}{4}} q k_{\alpha} \rho_i \left(\frac{R}{\tau L_T} \right)^{1/4} \sqrt{\hat{s} - 1/2} \gg 1, \quad (25)$$

for $q \gg (L_T/R)^{1/4}/(k_{\alpha}\rho_i)$, with $\hat{s} \sim \mathcal{O}(1)$. Equation (25), shows a dependence on the shear caused by the third term of Eq. (23). When we compare a positive value $\hat{s} = \hat{s}_0 > 0$ with its negative counterpart $\hat{s} = -\hat{s}_0 < 0$ (negative shear is a common situation for Stellarators and not so uncommon in Tokamaks), λ rotates in the complex plane without fundamentally affecting the stability of the mode. For $\hat{s} = 1/2$, the FLR correction must provide localization.

In the light of these considerations, we proceed by focussing on the strong interchange limit and neglect the parallel ion dynamics even if a finite shear is kept. This simplification allows us to extend Eq. (19) to the magnetic surface, and yet obtain an analytically tractable model. We defer further discussions on the effect of finite shear and proceed with our analysis.

A closer look to Eq. (14) now gives

$$\omega_d = v_d^{\alpha} \left(k_{\alpha} + k_{\psi} \frac{v_d^{\psi}}{v_d^{\alpha}} \right), \quad (26)$$

with $v_d^{\alpha} = \mathbf{v}_d \cdot \nabla \alpha$, and $v_d^{\psi} = \mathbf{v}_d \cdot \nabla \psi$. The function v_d^{ψ}/v_d^{α} depends on geometry but for $k_{\psi} \rightarrow 0$ it will not affect our results. Again, we set $k_{\psi} \equiv 0$ (but justify this later), and we are left with

$$\omega_d = \frac{v_{thi}}{\Omega_i(B)} (\partial_{\psi} B) k_{\alpha} v_{thi}. \quad (27)$$

As discussed in the Introduction, in practice this quantity varies on a length scale a that is much shorter than the major radius. The corresponding coefficient of Eq. (19) is no longer constant in a global (surface) setting and we must consider the replacement $k_\alpha \rightarrow -i\partial_\alpha$. This must be true also for ω_T and b . Thus, we have

$$\omega_d \rightarrow -i \frac{v_{thi}^2}{\Omega_i} \partial_\psi B \partial_\alpha, \quad (28)$$

$$\omega_T \rightarrow -\frac{i}{e} \frac{dT}{d\psi} \partial_\alpha, \quad (29)$$

and

$$b \rightarrow -\frac{1}{2} \rho_i^2 |a \nabla \alpha|^2 \partial_\alpha^2. \quad (30)$$

We now notice that physical periodicity in the toroidal angle ζ requires that the solution of Eq. (19) satisfies

$$\varphi(\psi, \alpha + 2\pi\iota, \theta) = \varphi(\psi, \alpha, \theta). \quad (31)$$

Since this property must hold for all coefficients on the RHS of Eq. (19), the drive of the instability must be of the form

$$\omega_d \omega_T \rightarrow -f(\alpha/\iota) \frac{\rho_i^2}{L_B L_T} v_{thi}^2 \partial_\alpha^2, \quad (32)$$

for *some* 2π -periodic function $f = f(\alpha/\iota)$. Here, ρ_i , L_T , and L_B can be considered to be independent of α , and are introduced to make the function f in Eq. (32) dimensionless. The FLR term gives

$$\omega_T b \rightarrow i \frac{v_{thi}}{4L_T} g(\alpha/\iota) \rho_i^3 \partial_\alpha^3, \quad (33)$$

where we have used $e^{-1} dT/d\psi = \rho_i v_{thi}/(2L_T)$, and $g(\alpha/\iota) = |a \nabla \alpha|^2 B_0^2/B^2(\alpha)$ is also a 2π -periodic function. In Eq. (33), we have also used $\partial_\alpha \ln \varphi \sim L/a \gg \partial_\alpha \ln B \sim 1$, from our ordering assumptions.

After introducing the new periodic variable $y = \alpha/\iota$, the normalized scale $\varrho_i \equiv \iota \rho_i/a$, and neglecting the parallel derivatives ∂_z^2 , we finally have

$$\left\{ \tau - i \frac{v_{thi}}{4L_T \omega} g(y) \varrho_i^3 \partial_y^3 \right\} \varphi(y) = \frac{v_{thi}^2 f(y)}{\omega^2 L_B L_T} \varrho_i^2 \partial_y^2 \varphi(y), \quad (34)$$

with

$$f(y) = \frac{a L_B}{2} \frac{\partial_\psi B}{B^2}, \quad (35)$$

where B and g are also periodic.

We propose Eq. (32) as a minimal model that encapsulates the field-line-label dependence of the drive of the toroidal branch of the electrostatic ITG mode. Equation (34) is the simplest model for studying surface-global effects on ITG modes in the strong interchange limit. The function $f = f(y)$ can then be modelled in several ways, all with the common feature that “deviation from axisymmetry” is measured imposing a variation in the y -variable on the drive of the toroidal branch of the local ITG instability, e.g.

$$f(y) = f_0 + \epsilon_h f_1(y), \quad (36)$$

where $f_1 \sim \mathcal{O}(f_0)$, f_0 and ϵ_h are positive constants, and the origin of y is defined so that $f_1(y)$ has a maximum at $y = y_M$ with $f_0 f_1(y_M) > 0$. By construction $\overline{f(y)} = f_0$. Higher order terms could be kept in the definition of f . We content ourselves with a first order correction.

Then, the maximum growth rate of the local non-axisymmetric mode, with no FLR effects, is determined by

$$\tau \omega_{NAS}^2 = -k_y^2 \varrho_i^2 \frac{v_{thi}^2}{L_B L_T} [f_0 + \epsilon_h f_1(y_M)] \quad (37)$$

as opposed to

$$\tau \omega_{AS}^2 = -k_y^2 \varrho_i^2 \frac{v_{thi}^2}{L_B L_T} f_0 \quad (38)$$

in axisymmetric geometry: $\epsilon_h \rightarrow 0$. When calculating global effects associated with the y -variation of the strength of the magnetic drift, we will compare our results to both Eq. (37) and (38), and to their equivalent that retain FLR effects.

III. GEOMETRIC STABILIZATION IN THE LIMIT $l_\perp \ll a$

In the limit $l_\perp \ll a$, the differential operators in Eq. (34) only act on the electrostatic potential φ .

Then, after introducing the ansatz

$$\varphi(y) = e^{s(y)/\varrho_i}, \quad (39)$$

we obtain

$$\tau = \frac{v_{thi}^2 f(y)}{\omega^2 L_B L_T} \left[\varrho_i s'' + (s')^2 \right] + i \frac{v_{thi}}{4 L_T \omega} g(y) \left[\varrho_i^2 s''' + 3 \varrho_i s' s'' + (s')^3 \right], \quad (40)$$

which, for $\varrho_i \rightarrow 0$, yields

$$\tau = \frac{v_{thi}^2 f(y)}{\omega^2 L_B L_T} (s')^2 + i \frac{v_{thi}}{4L_T \omega} g(y) (s')^3. \quad (41)$$

We now perform an expansion in ϵ_h , which was defined in Eq. (36). We will have

$$\omega = \omega_0 + \epsilon_h \omega_1 + \dots, \quad (42)$$

$$g(y) = g_0 + \epsilon_h g_1(y) + \dots, \quad (43)$$

and

$$s' = s'_0 + \epsilon_h s'_1 + \dots \quad (44)$$

We notice that the eigenvalue ω in Eq. (34) is not a function of y and the same must be true for each term of its ϵ_h -expansion. The function s' , on the contrary, is y -dependent.

To zeroth order we then find

$$\tau = \frac{v_{thi}^2 f_0}{\omega_0^2 L_B L_T} (s'_0)^2 + i \frac{v_{thi} g_0}{4L_T \omega_0} (s'_0)^3, \quad (45)$$

which can be matched to the local eigenvalue equation for an axisymmetric system, Eq. (38), if

$$s'_0 = ik_y \varrho_i = \text{const.} \quad (46)$$

Periodicity in y implies quantization, in the sense that $k_y \varrho_i$ must be equal to $2\pi n$ for some integer n which can be chosen so that $\bar{s}'_1 = 0$. Notice that $k_y = 0$ maximizes the growth rate, since it minimizes the FLR stabilizing effect. This justifies our earlier assumption and it is important since we will be comparing maximum growth rates to determine whether the global effects are stabilizing.

To next order we have

$$2\tau\omega_0\omega_1 = 2\frac{v_{thi}^2 f_0}{L_B L_T} s'_0 s'_1 + 3i\frac{v_{thi}\omega_0}{4L_T} (s'_0)^2 s'_1 + i\frac{v_{thi}\omega_0}{4L_T} (s'_0)^3 g_1 + \frac{v_{thi}^2}{L_B L_T} (s'_0)^2 f_1 + i\frac{v_{thi}g_0}{4L_T} (s'_0)^3 \omega_1. \quad (47)$$

The periodic function $f(y) = f_0 + \epsilon_h f_1(y)$ averages to $\overline{f(y)} = f_0$, while $g(y)$ to $\overline{g(y)} \equiv g_0$. The function s'_1 averages to zero. Thus, the y -average of Eq. (47) yields

$$\omega_1 \propto \overline{s'_1} = 0, \quad (48)$$

Since the local non-axisymmetric mode is unstable to first order [see Eq. (37)], Eq. (48) proves that, to the relevant order, the global mode is less unstable than the most unstable non-axisymmetric local one, whose imaginary part has a non-zero ϵ_h contribution.

When comparing growth rates to an axisymmetric system, to evaluate whether the global correction is destabilizing or not, we need to go to second order in ϵ_h . The analysis is particularly clear if we neglect FLR effects, i.e. the second term in Eq. (45). This is possible for $k_y^3 \varrho_i^3 v_{thi} / (L_T \omega_0) \ll 1$, which gives $k_y \varrho_i \approx \varrho_i / l_\perp \ll (L_T / L_B)^{1/4} \ll 1$.

We obtain

$$2\tau\omega_0\omega_2 = \frac{v_{thi}^2}{L_B L_T} \left\{ f_0 (s'_1)^2 + 2f_1(y) s'_0 s'_1 + f_2(y) (s'_0)^2 + 2f_0 s'_0 s'_2 \right\}. \quad (49)$$

Again, we average this result, and get

$$2\tau\omega_2\omega_0 = \frac{v_{thi}^2}{L_B L_T} \left\{ 2s'_0 \overline{(s'_1)} f_1 + \overline{f_0 (s'_1)^2} \right\}, \quad (50)$$

By setting $\omega_1 = 0$ in Eq. (47), we find

$$s'_1 = -\frac{1}{2} \frac{f_1(y)}{f_0} s'_0. \quad (51)$$

After using Eq. (51), we obtain

$$\omega_2 = -i \frac{3}{8} \frac{\overline{f_1^2}}{f_0^2} \frac{v_{thi} f_0^{1/2}}{\sqrt{L_B L_T \tau}} k_y \varrho_i. \quad (52)$$

We therefore have

$$\frac{\omega}{|\omega_0|} \approx i \left\{ 1 - \epsilon_h^2 \frac{3}{8} \frac{\overline{f_1^2}}{f_0^2} \right\}. \quad (53)$$

The global correction is stabilizing through an average of the curvature square [the term $\overline{f_1^2}$ in Eq. (53)]. It is known, from the analysis of numerical solutions [17], that FLR corrections generally have a stabilizing effect that adds to the stabilization here derived.

IV. GEOMETRIC STABILIZATION FOR $l_\perp \sim a$

When the scale of the mode is comparable to the scale of the equilibrium magnetic field ($l_\perp \sim a$), the WKB approach of the previous section is not applicable. However, this case is very important because numerical simulations demonstrate that large-scale turbulent fluctuations are suppressed by their interaction with the magnetic field variation [3, 17].

This motivates us to try to understand the suppression linearly. The appropriate limit to describe such modes is $k_{\perp}\varrho_i \ll 1$, so we neglect the FLR terms in what follows. Again, using $f(y) = f_0 + \epsilon_h f_1(y)$, we obtain the equation

$$\tau\omega^2\varphi = \frac{v_{thi}^2}{L_B L_T} (f_0 + \epsilon_h f_1) \varrho_i^2 \frac{d^2\varphi}{dy^2}. \quad (54)$$

We expand

$$\varphi = \varphi_0 + \epsilon_h \varphi_1 + \dots, \quad (55)$$

$$\omega = \omega_0 + \epsilon_h \omega_1 + \epsilon_h^2 \omega_2 + \dots, \quad (56)$$

$$f_1(y) = \sum_m e^{imy} \hat{f}_1(m), \quad (57)$$

where the reality condition $\hat{f}(-m) = \hat{f}^*(m)$ applies, and we again assume $\bar{f}_1 = \hat{f}_1(0) = 0$. The method of solving this system involves identifying and eliminating secular growth at each order by imposing solubility constraints; see [30] Sec. 11.4. To zeroth order in ϵ_h , we find

$$\varrho_i^2 \frac{d^2\varphi_0}{dy^2} = \frac{\tau\omega_0^2}{f_0 v_{thi}^2 / (L_B L_T)} \varphi_0, \quad (58)$$

so we take $\varphi_0 = c_0 \exp(in_0 y) + \text{c.c.}$, with $n_0 \in \mathbb{Z}$ and c_0 a complex constant. Eq. (58) gives $\omega_0 = i\varrho_i n_0 f_0^{1/2} v_{thi} / \sqrt{L_B L_T \tau}$. This implies that $n_0 = \partial_y \ln \varphi_0 \sim L/a \gg 1$. To first order in ϵ_h , we have

$$\left(1 + \frac{1}{n_0^2} \frac{d^2}{dy^2}\right) \varphi_1 = - \left[2 \frac{\omega_1}{\omega_0} - \frac{f_1(y)}{f_0}\right] \varphi_0(y). \quad (59)$$

Both terms on the right hand side can induce secular growth in φ_1 which is incompatible with periodicity. To avoid this we impose two conditions, namely that the $\exp(\pm in_0 y)$ components of the right hand side are both zero. Multiplying by $\exp(\pm in_0 y)$ and integrating over a period, we find

$$2 \frac{\omega_1}{\omega_0} = \frac{c_0^* \hat{f}_1(2n_0)}{c_0 f_0}, \quad (60)$$

$$2 \frac{\omega_1}{\omega_0} = \frac{c_0 \hat{f}_1^*(2n_0)}{c_0^* f_0}. \quad (61)$$

Eliminating c_0^*/c_0 from (60), and using the result found in Eq. (61), we obtain

$$\omega_1 = \pm i n_0 \rho_i \frac{v_{thi} f_0^{1/2}}{\sqrt{L_B L_T \tau}} \frac{|\hat{f}_1(2n_0)|}{2f_0}. \quad (62)$$

Thus we find a first order correction to the mode frequency that can be either stabilizing or destabilizing. At first this could seem surprising. Let us consider the consequences of the first order result on the zeroth order eigenfunction φ_0 . The quantity c_0^*/c_0 is also determined by Eqs. (60)-(61). Thus we can express φ_0 as follows

$$\varphi_0 = c_0 \left\{ \exp(in_0 y) \pm \frac{|\hat{f}_1(2n_0)|}{\hat{f}_1(2n_0)} \exp(-in_0 y) \right\}, \quad (63)$$

where the sign \pm comes from Eq. (62). Without loss of generality, we assume that \hat{f}_1 is real. (This is ensured by a constant shift in y .) The two φ_0 solutions then have even and odd parities, respectively. The double sign in Eq. (62), therefore, takes into account of this fact. We now quantify how much instability is caused by the global first order correction of Eq. (62). The result of the asymptotic expansion in ϵ_h , to first order, is

$$\omega = \omega_0 \left\{ 1 + \frac{\epsilon_h}{2} \frac{|\hat{f}_1(2n_0)|}{f_0} \right\} + \mathcal{O}(\epsilon_h^2), \quad (64)$$

for an even zeroth order solution [e.g. we choose the “+” sign in Eq. (62) and (63)]. We shall not forget that the local maximum non-axisymmetric mode is

$$\omega_{NAS} = \omega_0 \left\{ 1 + \epsilon_h \sum_{m \neq 0} e^{iy_M m} \frac{\hat{f}_1(m)}{f_0} \right\}^{1/2}, \quad (65)$$

where we replaced $f_1(y) = \sum_m e^{iy m} \hat{f}_1(m)$ in Eq. (37). When the function f_1 is dominated by the resonance, $\sum_m e^{iy_M m} \hat{f}_1(m) \approx e^{iy_M 2n_0} \hat{f}_1(2n_0) + e^{-iy_M 2n_0} \hat{f}_1^*(2n_0)$. For an even φ_0 , $\hat{f}_1(2n_0) \in \mathbb{R}^+ \Rightarrow \hat{f}_1(2n_0) = |\hat{f}_1(2n_0)|$, thus $\sum_m e^{iy_M m} \hat{f}_1(m) = 2 \cos(2n_0 y_M) |\hat{f}_1(2n_0)|$. Then, Eq. (65) becomes

$$\omega_{NAS} = \omega_0 \left\{ 1 + \epsilon_h \frac{|\hat{f}_1(2n_0)|}{f_0} \cos(2n_0 y_M) \right\} + \mathcal{O}(\epsilon_h^2). \quad (66)$$

Since $-1 \leq \cos(2n_0 y_M) \leq 1$, we conclude that the destabilizing global effect on the local axisymmetric mode [the ϵ_h correction in Eq. (64)] is smaller than the ϵ_h contribution to the local maximum non-axisymmetric mode of Eq. (66) for the case $\hat{f}_1(m) = 0, \forall m \neq 2n_0$. The result could hold generally but a proof could not be found.

When $\hat{f}_1(\pm 2n_0) = 0$, the only way to avoid secular growth in φ_1 is to have $\omega_1 = 0$. We must then proceed to the next order to obtain the frequency correction ω_2 .

To second order in ϵ_h , we have

$$\left(1 + \frac{1}{n_0^2} \frac{d^2}{dy^2}\right) \varphi_2 = - \left[2 \frac{\omega_2}{\omega_0} \varphi_0 + \frac{f_1(y)}{f_0} \frac{1}{n_0^2} \frac{d^2 \varphi_1}{dy^2} \right], \quad (67)$$

with

$$\varphi_1 = c_1 e^{in_0 y} + c_0 n_0^2 e^{in_0 y} \int_{y_0}^y e^{-2in_0 y'} dy' \int_{y'_0}^{y'} \frac{f_1(y'')}{f_0} e^{2in_0 y''} dy'' + \text{c.c.}, \quad (68)$$

for $\omega_1 = 0$, where c_1 and y_0 are arbitrary constants.

As with the previous order, we must impose the solubility conditions

$$2 \frac{\omega_2}{\omega_0} c_0 = - \frac{1}{2\pi} \int_0^{2\pi} dy \exp(-in_0 y) \frac{f_1(y)}{f_0} \frac{1}{n_0^2} \frac{d^2 \varphi_1}{dy^2}, \quad (69)$$

$$2 \frac{\omega_2}{\omega_0} c_0^* = - \frac{1}{2\pi} \int_0^{2\pi} dy \exp(in_0 y) \frac{f_1(y)}{f_0} \frac{1}{n_0^2} \frac{d^2 \varphi_1}{dy^2}. \quad (70)$$

There are two terms from Eqn. (68) that must be calculated, the one proportional to c_0 and the other proportional to c_0^* . The constraints can thus be rewritten as

$$2 \frac{\omega_2}{\omega_0} c_0 = c_0 I_1 + c_0^* I_2^*, \quad (71)$$

$$2 \frac{\omega_2}{\omega_0} c_0^* = c_0^* I_1 + c_0 I_2. \quad (72)$$

where we have introduced the following integral quantities

$$I_1 = - \sum_{m=1}^{\infty} \frac{|\hat{f}_1(m)|^2}{f_0^2} \frac{2 - 6n_0^2/m^2}{1 - 4n_0^2/m^2}, \quad (73)$$

$$I_2 = - \frac{1}{f_0^2} \sum_{m=1}^{\infty} \hat{f}_1(m) \hat{f}_1(2n_0 - m) \left(\frac{n_0}{m} - 1 \right)^2. \quad (74)$$

Note that we have used $\hat{f}_1(0) = \hat{f}_1(\pm 2n_0) = 0$. Equations (71)-(72) yield the second order correction

$$2 \frac{\omega_2}{\omega_0} = I_1 \pm |I_2|. \quad (75)$$

If $\hat{f}_1(m)$ decays quickly, so that $|\hat{f}_1(m)| \approx 0$ for $m \geq n_0$ then $I_2 \rightarrow 0$, and we are left with

$$\omega_2 = -\frac{\omega_0}{2} \sum_{m=1}^{\infty} \frac{|\hat{f}_1(m)|^2}{f_0^2} \left(\frac{2 - 6n_0^2/m^2}{1 - 4n_0^2/m^2} \right). \quad (76)$$

It can be verified that the quantity in parentheses is positive unless $\sqrt{3}n_0 < m < 2n_0$, *i.e.* there is a window of **small** scales ($m \sim n_0 \sim L/a \gg 1$) that can act to destabilize the mode. However, large scale components of f_1 give negative contribution to the growth rate. If $\hat{f}_1(m)$ is a decaying function, such large scales will be dominant in the summation, resulting in an overall stabilizing effect. This result resonates with the recent numerical finding that large-scale turbulent structures are suppressed by the interaction with small equilibrium scales [17]. A simple case in which ω_2 is always stabilizing is when $\hat{f}_1(m) \equiv 0$ for $m \geq n_0$, then $I_2 \equiv 0$, and $\Im[\omega_2] < 0$. The obvious case in which global destabilization can occur is when the function \hat{f}_1 peaks at a particular m_{res} for which $\sqrt{3}n_0 < m_{res} < 2n_0$.

For the sake of completeness, we now consider $n_0 \gg m$, *i.e.* that the mode is at a scale much smaller than those represented in $f_1(y)$. Then we obtain

$$\omega_2 = -\frac{3\omega_0}{8} \frac{1}{2\pi} \int_0^{2\pi} \frac{f_1^2}{f_0^2}. \quad (77)$$

Happily, this expression matches Eqn. (52), which however was derived by first taking the $n_0 \gg m$, and then the $\epsilon_h \ll 1$ limit. Eq. (77) was derived by taking the same limits in inverse order. The two limits therefore commute.

V. RADIALLY VERSUS SURFACE GLOBAL EFFECTS

As previously mentioned in the Introduction, we can consider two types of global effects associated with the variation of equilibrium quantities: radial and poloidal. We now discuss in which circumstances we might expect each effect to be predominant.

Radially global structures of drift-wave-like linear modes [31, 32] (as the ITG mode) generally form on time scales too long to be observed before turbulent radial decorrelation would suppress them. On the other hand, fast variations of equilibrium quantities in y (α) are mostly given by the machine design, they are externally imposed, and they are in fact likely to manifest even nonlinearly [17, 33]. Therefore, it is reasonable to expect a clear separation between radial and surface global structures of the mode under consideration.

That said, it is still legitimate to ask the question which global effect is quantitatively

more important. The relation between the radially-global, Γ , and local, ω_0 , eigenvalue is well known in the theory of ballooning modes. For the case of ideal MHD, it can be found in Eq. (38) of the seminal paper of Connor Hastie and Taylor [34]

$$\Gamma^2 = \omega_0^2 + \frac{1}{2n |\nu'(\theta_0)|} \left(\frac{\partial^2 \omega_0^2}{\partial \psi^2} \frac{\partial^2 \omega_0^2}{\partial \theta_0^2} \right)^{1/2}, \quad (78)$$

where $n \gg 1$ is the mode number, ω_0 is the local eigenvalue maximized over the ballooning parameter θ_0 , and ν is so that $q = (2\pi)^{-1} \oint dz \nu$. Equation (78), for large $n \gg 1$, tells us that

$$\Gamma \approx \omega_0 \left\{ 1 + \mathcal{O} \left(\frac{1}{n} \right) \right\}, \quad (79)$$

as discussed by Hastie and Taylor in Ref. [35]. Similar results apply in the context of drift-wave turbulence [32, 36, 37], see Fig. (6) of Ref [37]. Equation (79) needs to be compared to the expression that takes into account of global (on the surface) correction of Eq. (52)

$$\Omega \approx i \frac{v_{thi}}{\sqrt{L_B L_T \tau}} \left\{ 1 - \epsilon_h^2 \frac{f_1^2}{4f_0^2} \right\}. \quad (80)$$

Thus, the radially-global correction of Eq. (79) can compete with the surface global correction of Eq. (80) when

$$\frac{\rho_i}{R} \sim \epsilon_h^2, \quad (81)$$

if $n^{-1} \sim \rho_i/R$.

For typical values we have $\rho_i/R \approx 10^{-3}$, and for machines such as Wendelstein 7-X [38, 39] and LHD [40, 41] we have $\epsilon_h \gtrsim 0.1$, thus surface global effects seem to be dominant in stellarators, but could also be as important as radially-global effects in large tokamaks. We conclude this Section by noting that we have considered global corrections to what are known as “isolated” [36] of “trapped” [42] modes. There is another class of modes known as “passing” [42] or “general” [36]. The two types of modes generally peak at different poloidal locations and their relative strength depends upon the machine geometry. Surface-global effects are likely to influence passing (general) modes as well.

VI. SUMMARY AND DISCUSSION

In this article we have reported on a gyrokinetic study of global (on the magnetic surface) effects on the linear, strongly driven, toroidal ITG instability. A minimal model for the

study of this instability in a surface-global setting has been derived [Eq. (34)] and analysed perturbatively. This analysis is based on the smallness of the parameter that measures deviation from axisymmetry, ϵ_h , [see Eqs. (36) and (37)] and on $l_\perp \ll a$, where l_\perp^{-1} is the characteristic perpendicular wave-length of the mode and a is an intermediate characteristic scale of variation of the equilibrium magnetic field discussed in Eqs. (7)-(8).

It has been found that, for systems close to axisymmetry, $\epsilon_h \ll 1$, surface-global corrections are stabilizing. The mechanism for this stabilization is a reduction of the field curvature drive caused by its averaging over the flux surface [Eq. (52)].

For $l_\perp \sim a$, we observe the presence of a resonance between the magnetic drift and the perturbed electrostatic potential. Such resonance can cause global destabilization [Eq. (62)], but below the level set by the maximum local non-axisymmetric mode [see Eq. (64) and (66)]. This is proven in the case in which the resonance is dominant. In the absence of the resonance, stabilization remains the most important effect if the spectrum of the non-axisymmetric component of the magnetic drift is a decaying function [Eq. (76)-(77)]. We also find that surface-global effects can compete with radially global ones for $\epsilon_h^2 \sim \rho_*$, where ρ_* is the small expansion parameter of gyrokinetic theory. This suggests that surface-global effects might be dominant in stellarators (for which $\epsilon_h \sim 0.1$, and $\rho_* \sim 10^{-3}$), and can compete with radially global ones in large tokamaks. In the latter case, non-axisymmetric effects are expected to be non-negligible towards the plasma pedestal. Our results pertain to the physics of the ITG mode. However, we might expect the approach to be applicable also to other modes, in any context that requires the breaking of axisymmetry, as in ELMs mitigation, for instance [43]. The geometric stabilization we identified does not depend on the long time evolution of macroscopic quantities such as plasma density and temperature radial profiles, as in the case of finite ρ_* effects. It stems from the geometric properties of the device under consideration. In this sense it is also intrinsic [17].

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