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Two Stories for Beyond the Standard Model

Master thesis in

Physics

submitted by

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Heidelberg,

October, 2016

Zwei Geschichten von jenseits des Standardmodells:

In dieser Arbeit werden zwei Erweiterungen des Standardmodells prsentiert: Zee-SU(5) und Zee-LR. In beiden Theorien erhalten die Neutrinos aufgrund des Zee-Mechanismus durch Quanteneffekte eine Masse. Zee-SU(5) basiert auf der SU(5) groen vereinheitlichten Theorie mit dem skalaren Sektor 5_H, 45_H und 10_H. Die Rolle des zweiten Higgs-Dubletts in 45_H ist entscheidend fr die Korrektur der Massenrelation der down-artigen Quarks und geladenen Leptonen und die Erzeugung der Neutrinomassen. Zur berprfung der Testbarkeit dieser Theorie werden Bedingungen fr die Vereinheitlichung detailliert untersucht und Beschrnkungen fr die Beobachtung des Zerfalls des Protons an aktuellen und zuknftigen Experimenten wie Hyper-Kamiokande betrachtet. Die Theorie sagt auf natrliche Weise eine Relation zwischen den Massen der Neutrinos und der geladenen Leptonen vorher sowie das Auftreten eines leichten, farbgeladenen Oktetts, das zu exotischen Signaturen am LHC fhren knnte. Das Zee-LR-Modell hingegen ist ein simples links-rechts-symmetrisches Modell, dessen skalarer Sektor aus zwei Higgs-Dubletts besteht, die die links-rechts-Symmetrie brechen, sowie einem geladenen Singlett, durch das Neutrinomassen generiert werden und leptonenzahlverletzende Prozesse impliziert werden. Dieses Modell sagt leichte sterile Neutrinos voraus. Um die Phnomenologie der Theorie zu untersuchen, werden die Signaturen am LHC mit zwei geladenen Leptonen unterschiedlichen Flavors und fehlender Energie sowie Zerflle der schweren Eichbosonen mit dem leichten, rechtshndigen Neutrino untersucht. Es wird angemerkt, dass beide Theorien die fr Symmetriebrechung und Massenerzeugung minimal bentigte Anzahl an Freiheitsgraden im skalaren Sektor einfhren, unter der die Renormierbarkeit der Theorie erhalten ist (im Fall von Zee-SU(5) ohne zustzliche Fermionensingletts).

Two stories for beyond the Standard Model:

We propose two extensions of the Standard Model which predict Majorana neutrinos: Zee-SU(5) and Zee-LR. In both theories neutrinos get mass at the quantum level through the Zee mechanism. The Zee-SU(5) is based on the SU(5) grand unified theory with 5_H, 45_H and 10_H composing the scalar sector. The role of the second Higgs doublet in the 45_H is crucial since it is responsible of correcting the down-type quarks and charged leptons mass relation and generating neutrino masses. In order to understand the testability of the theory, unification constraints are studied in detail and proton decay bounds are discussed at current and future experiments such as Hyper-Kamiokande. The theory predicts as a natural outcome a beautiful relation between neutrino masses and charged fermion masses and a light colored octet which could give rise to exotic signatures at the LHC. On the other hand, the Zee-LR is a simple left-right symmetric model whose scalar sector is composed of two Higgs doublets, responsible of breaking the left-right symmetry, and one charged singlet, responsible of the neutrino mass generation and lepton number violation processes. This model predicts light sterile neutrinos. In order to understand the testability of the theory, we study the signatures with two charged leptons of different flavor and missing energy at the LHC and the decays of the heavy gauge bosons involving the light right handed neutrino.We remark that both theories have the minimal degrees of freedom in the scalar sector needed for symmetry breaking and mass generation such that the renormalizability of the theory is preserved (without extra fermion singlets in the case of Zee-SU(5)).

Contents

I. INTRODUCTION

The Standard Model (SM) is currently the most satisfying theory to describe physics at the electroweak scale. Its agreement with experimental data is astonishing, in particular after the discovery of the W and Z bosons, being the discovery of the Higgs boson the cherry on the top of the cake. Up to now, there is no phenomenological disagreement with the SM predictions except for the fact that neutrinos are massive in nature, whereas the SM predicts them to be massless. Apart from this experimental issue, there are open theoretical aspects which strongly suggest that the SM is only an intermediate step in our understanding of physics. For instance, the SM does not provide a unified description of the strong, weak and electromagnetic forces. In the context of the SM, charge quantization is not explained (hypercharge is put in by hand), the fermion assignment is made ad hoc, the number of families and the hierarchy in the fermion masses is completely arbitrary, not to mention the arbitrariness in the Higgs sector and the apparent violation of the naturalness principle. All of these issues encourage us to expect a more fundamental theory at higher energies. Nevertheless, since the SM fits the observed data so perfectly well, one expects any new theory to match the SM at low energies.

A fundamental tool to explore beyond the electroweak scale is the concept of an effective field theory. To start with, we all know that gravity is there and it is unfortunately not contemplated by the SM, so that we are aware that the SM breaks down, at latest, at the energy where gravitational effects start to matter, i.e. the Plank scale $M_{Pl} \sim 10^{19}$ GeV. In that sense, the SM is indeed an effective field theory so far, so that its Lagrangian can be written as:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{n=5}^{\infty} \sum_{i} \left(\frac{C_i^n}{\Lambda^{n-4}} \mathcal{O}_i^n + \text{h.c.} \right) , \tag{I.1}$$

Looking at the above expression, one may wonder which would be the next effective operator O in order of relevance. Since higher dimensional operators are suppressed by powers of the mass scale of the new theory Λ, we look at the next higher dimension operator (dimension five) and then we find that there is only one possible operator according to the SM symmetries, the so-called Weinberg operator, which (a) violates lepton number, (b) predicts massive neutrinos. Indeed, as predicted by this effective approach, neutrinos have been found to be massive (surprise!), which sets up a scale of $\sim 10^{14}$ GeV for the next scenario of new physics.

One may think about the effective field theories as layers of an onion, where each layer corresponds to an effective theory at some energy scale. Under the perspective of the SM being an effective field theory, we could ask ourselves about the way to reach the next layer (i.e. SM UV-completion), taking into account that any attempt to build a Beyond the Standard Model (BSM) theory will be strongly constrained since it must reproduce the SM at low energies.

We know that Yang-Mills theories seem to work pretty well. But why the universe is described by three different groups? Why do we have three couplings (i.e. three inputs in our theory) to describe the observables? Since early times, there has always been a trend among philosophers and scientists to reduce the multiple phenomena in Nature to the minimal number of laws and principles. Perfection, beauty and simplicity are concepts intimately bounded when qualifying a certain physical theory. Therefore, we may find attractive the idea of a unified group describing Nature. Theories of this kind are known as GUTs (Grand Unified Theories). The simplest GUT, based on SU(5), was proposed in 1974 by H. Georgi and S.

Glashow [1] and as we will show later, it comes with some striking predictions such as the decay of the proton [2]. Indeed, if next dimension operators in Eq. (I.1), i.e dimension six, are taken into account, one realizes that baryon number is violated, so that they predict the proton will decay at some point. The reader may notice that this process can be really suppressed at the EW scale (although it really depends on the magnitude of the new physics scale) but give to the proton enough time and for sure it will decay (unless there is a special hidden symmetry forbidding these dimension 6 operators). This is also a good motivation for GUTs. This theory also predicts a great dessert between the electroweak and the GUT scale. However, this simplest GUT model has been ruled out since it cannot reproduce the experimental data at low energies.

On the other hand, one can also find unsightly to have the parity asymmetry in the electroweak interactions. In the SM there is no explanation about parity violation in the electroweak sector. To make the SM look nicer, one may attempt to correct this asymmetry, adding the correspondent right partners ($SU(2)_R$) and ending up with a left-right symmetric model [3–6] which, surprise, predicts massive Dirac neutrinos as a natural outcome. One of the main attractive points on this theories is the existent connection between spontaneous parity violation and neutrino masses [7].

We will follow these two tendencies (GUT theories and left-right symmetric models) and will introduce in the following two different simple and realistic extensions of the Standard Model in which neutrinos get mass at the quantum level through the Zee mechanism [8]. Regarding the structure of the presented material, this master thesis is organized as follows: first, in section II, we introduce the simplest possible GUT theory [1], based on SU(5), and discuss its advantages and limitations. In section III, the simplest mechanisms to generate neutrino masses are discussed along with their application in the context of SU(5) theories. We end the topic of GUTs, in section IV, by introducing a simple renormalizable extension based on SU(5) (Zee-SU(5) [9]) which corrects the main problems contained in the simplest SU(5) theory. This model contains, apart from the usual $\bar{5}$, 10 fermionic and 5_H, 24_H scalar representations, an extra 10_H and 45_H, responsibles for the Zee mechanism. We show the consistency of the theory according to experimental constraints such as unification and proton decay in current and future experiments such as Hyper-Kamiokande, which predicts a light scalar colored octet. An interesting relation between neutrino and charged fermion masses appears for the first time in the context of GUT theories.

The second block the thesis is based on left-right symmetric models. We start in section V revisiting the simplest left-right symmetric theory [6], with the minimal Higgs sector to break the LR-symmetry, which predicts Dirac neutrinos. In section VI we list some extensions which allow neutrinos to have Majorana masses though the seesaw mechanisms. In section VII, we propose a new left-right symmetric model, LR-Zee [10], in which neutrinos get mass through the Zee mechanism. This is the most economic model regarding the field content of the theory which contemplates massive Majorana neutrinos and it comes along with some new interesting predictions which are also studied in this section such as a light sterile neutrino in which new gauge bosons could decay and lepton number violation signals such as decays into two charged leptons of different families and missing energy, which could be tested in the near future by experiments such as MEG2 or Mu2e. Details regarding calculations, Feynman rules, field content and properties of the new two BSM-theories proposed here can be found in the appendix.

The results of this master thesis are summarized in the following publications:

- P. Fileviez Perez and C. Murgui, 'Renormalizable SU(5) Unification," Phys. Rev. D **94** (2016) no.7, 075014 doi:10.1103/PhysRevD.94.075014 [arXiv:1604.03377 [hep-ph]].
- P. Fileviez Perez, C. Murgui and S. Ohmer, "Simple Left-Right Theory: Lepton Number Violation at the LHC," Phys. Rev. D **94** (2016) no.5, 051701 doi:10.1103/PhysRevD.94.051701 [arXiv:1607.00246 [hep-ph]].

II. GRAND UNIFIED SU(5) THEORY

In this section we present a brief review of the simplest unified theory, SU(5)-GG, proposed by Georgi and Glashow in 1979 [1]. We will remark the crucial points of the theory without getting to deep into details, since our interest lies mostly in the new variant of the model that we propose as an improvement of the original theory [9]. An advanced reader could skip this part, although he or she may find it interesting in the sense that this has been written as a summary of the theory keeping an advanced level. A completely beginner in this topic is also encouraged to read it since, even thought it may seem too technical, this summary has been filled up with pedagogical references that hopefully will help its understanding.

A. Field content and SU(5) structure

The rank of a symmetry group is given by the total number of diagonal generators. For a given SU(N), the number of diagonal generators corresponds to $N-1$. In the SM one has two for SU(3) (λ_3, λ_8), one for SU(2) (T_3) and one for $U(1)_Y$ (i.e. the identity). Thus, any gauge group embedding the SM must have at least rank 4. Besides, in order to reach unification, the candidate group must be simple or else the product of identical simple factors (in this case a common coupling constant could be obtained by imposing certain discrete symmetries). There are many groups with rank 4 which could be in principle candidates to embed the SM: $[SU(2)]^4$, $[O(5)]^2$, $[SU(3)]^2$, $[G_2]^2$, $O(8)$, $O(9)$, $Sp(8)$, F_4 and $SU(5)$. From the above list, the first two candidates are ruled out since they do not contain $SU(3)$ as a subgroup. In order to avoid arbitrary super-heavy fermion masses in the Lagrangian, the candidate group must accommodate complex representations (otherwise an explicit mass term for the fermions would be allowed by the symmetry). The groups $[G_2]^2$, $O(8)$, $O(9)$, $Sp(8)$ and F_4 have only real representations whereas the candidate $[SU(3)]^2$ is not suitable since the charge operator would be a generator of SU(3) and its traceless condition would notably violate the generation structure of the quarks [11]. Therefore, we are only left with the candidate $SU(5)$ as the minimal group (rank 4) in which the SM can be embedded.

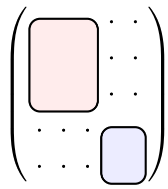
SU(5) is a non-abelian, special unitary group. The corresponding restrictions of its nature, i.e. traceless and determinant equal to the unity, determines the number of generators (degrees of freedom $\equiv$ d.o.f.) of the group: $N^2-1 \overset{N=5}{\to} 24$. From here we already see that the number of gauge bosons doubles the SM one, which turns into a prediction of new heavy gauge bosons to which the SM is completely blind.

Moreover, SU(5) is a simple group, which means that above the scale M_{GUT} (in the regime where it is a good symmetry), where M_{GUT} refers to the scale where SU(5) breaks to the SM, the couplings are unified

$$g_1(M_{GUT}) = g_2(M_{GUT}) = g_3(M_{GUT}) = g_5. \tag{II.1}$$

where g_5 refers to the coupling of SU(5).

Let us now focus in the structure of SU(5). One has to take into account that the electroweak interactions are blind to color and vice versa, which basically implies that the groups SU(3) and SU(2) need to commute with each other. Furthermore, leptons are color singlets, so that the SU(3) generators must have zero eigenvalues for these components. These facts tell us about the structure of the representations for the generators in SU(5): we reserve the first three rows and columns for the color group SU(3) (red area) and the last two rows and columns for the weak group SU(2) (blue area):

$24 \sim$  (II.2)

The generators of SU(5) can be built by taking the generators of U(5), imposing the traceless condition and normalizing them to 1/2 in the usual manner (see comments in section -unification constraints- about the normalization of a non-abelian gauge group and the Dynkin index). The complete list of the generators can be found in the appendix (see reference [12] chapter 9 for a detailed construction of the generators from a group theory perspective). Here we will briefly stress the diagonal generators of SU(5) since they are of special importance in the spontaneous symmetry breaking context. There are a total of four diagonal generators λ_i (where a normalization of $1/2$ is understood):

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \lambda_{24} = \frac{2}{15} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}.$$

The first two correspond to the diagonal generators of SU(3), whereas λ_{23} corresponds to the diagonal generator T_3 of SU(2). The last one corresponds to the hypercharge operator, generator of $U(1)_Y$. Thus, the symmetry must be broken in such a way that the hypercharge remains unbroken if the SM wants to be recovered, so that the breaking has to occur in the direction of λ_{24}. Notice that the generator of $U(1)_Y$ has been built such that it commutes with SU(2) and SU(3) groups. The traceless condition of SU(N) generators and the normalization criterion has fixed the rest of the details.

As it will be discussed in the following, the breaking of SU(5) to the SM is realized by an adjoint representation of scalars, 24_H, getting a vev in the direction of the hypercharge operator. Let us now deal with the breaking of a general SU(N) by an adjoint representation of scalars. The adjoint representation transforms as $\Sigma \to U\Sigma U^\dagger$, where U is a unitary matrix which can be generally expressed as $U = e^{i\Theta_a T^a}$, being T_a the generators of the group. Therefore, an infinitesimal transformation of the field Σ is given by $\delta\Sigma = i\Theta_a[T^a, \Sigma]$, which after symmetry breaking reads as $\delta \langle\Sigma\rangle = i\Theta_a[T^a, \langle\Sigma\rangle]$. Thus, broken generators do not commute with $\langle\Sigma\rangle$ whereas generators of the unbroken symmetry do. Due to symmetry transformation invariance of the potential, one always have the freedom to bring $\langle\Sigma\rangle$ to a diagonal form. Taking into account that the identity matrix commutes with all matrices, it is straightforward to realize that, if $\langle\Sigma\rangle$ has a $i \times i$-th block of equal diagonal entries, all generators whose non-zero entries lie entirely within the $i \times i$-th block will commute with $\langle\Sigma\rangle$. Furthermore, any combination of the diagonal operators proportional to $\langle\Sigma\rangle$ also commutes with Σ. This explains why if the vev of $\langle\Sigma\rangle$ in the SU(5) case is taken in the direction of λ_{24}, i.e. $\langle\Sigma\rangle \propto \lambda_{24}$, the SM is recovered and $Y = \#\frac{1}{2}\lambda_{24}$ where $\#$ is an arbitrary constant which we choose to be $\# \equiv \sqrt{\frac{5}{3}}$ in order to match it with the SM hypercharge. Notice that, since

$$\mathrm{Tr}\{(\frac{1}{2}\lambda_{24})^2\} = \frac{3}{5}\mathrm{Tr}\{Y^2\} = \frac{1}{2}, \tag{II.3}$$

then $g_{24} \equiv \frac{1}{2}\lambda_{24} = \sqrt{\frac{3}{5}}Y$. The further breaking of the SM to $U(1)_{em}$ hypercharge is performed by the usual Higgs complex doublet which lives in the fundamental representation of SU(5). Therefore, the remaining unbroken generator, the charge operator, is given by

$$Q = \frac{1}{2}\lambda_{23} + \sqrt{\frac{5}{3}}\frac{1}{2}\lambda_{24} \tag{II.4}$$

$$= T_3^L + Y. \tag{II.5}$$

Here, one can see how the quantization of the electric charge arises naturally from the intrinsic properties of SU(5); the traceless condition imposes the relation between quark and lepton charges which in the SM, in contrast, are considered an input of the theory. This is one of the main successful predictions of the SU(5) grand unified theory, which shows the power of having all forces participating in the SM unified in only one group.

In SU(5) we have the fundamental irreducible representation 5 which splits, according to the convention we have chosen for the distribution of the SM groups in SU(5), as follows:

$$5 \sim \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \end{pmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} SU(3) \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} SU(2) \end{array}$$

According to the above convention, this representation can be written from the SM point of view: $5 \sim (3,1) \oplus (1,2)$ where the first quantum number represents the multiplet under SU(3) and the second one, the multiplet under SU(2). By taking combinations of the fundamental 5 and anti-fundamental $\bar{5}$ representations, one can build the rest of irreducible representations of $SU(5)$. We show here explicitly the construction of the ones we will use to accommodate the field content of the SM. The quantum numbers of the representations along with the location of the fields can be found in detail in the appendix. Composite representations can be built through the tensor product in the following way:

$$5 \otimes 5 = \{(3,1,-1/3) \oplus (1,2,1/2)\} \otimes \{(3,1,-1/3) \oplus (1,2,1/2)\} \sim \tag{II.6}$$

$$\overbrace{\underbrace{(\bar{3},1,-2/3)}_{(u^c)_L} \oplus \underbrace{(3,2,1/6)}_{q_L} \oplus \underbrace{(1,1,1)}_{(e^c)_L}}^{\mathbf{10}} \oplus \overbrace{(6,1,-2/3) \oplus (3,2,1/6) \oplus (1,3,1)}^{\mathbf{15}}, \tag{II.7}$$

$$5 \otimes \bar{5} = \{(3,1,-1/3) \oplus (1,2,1/2)\} \otimes \{(\bar{3},1,1/3) \oplus (1,2,-1/2)\} \sim \tag{II.8}$$

$$\overbrace{\underbrace{(8,1,0)}_{G_\mu} \oplus \underbrace{(1,3,0)}_{W_\mu} \oplus \underbrace{(3,2,-5/6)}_{X_\mu} \oplus \underbrace{(\bar{3},2,5/6)}_{Y_\mu} \oplus \underbrace{(1,1,0)}_{\gamma_\mu}}^{\mathbf{24}} \oplus \overbrace{(1,1,0)}^{\mathbf{1}}. \tag{II.9}$$

where the third quantum number corresponds to the hypercharge. These representations have two indices since they are constructed from a tensor product. The 10 representation is antisymmetric ($\frac{N(N-1)}{2}$ d.o.f) and the 15 is symmetric ($\frac{N(N+1)}{2}$ d.o.f.). The 24 corresponds to the adjoint representation, which is a bit special since it is a real representation and is traceless, i.e. $24^i_i = 0$ (Einstein convention is assumed). This representation accommodates the gauge bosons of the theory. Explicitly,

$$V_\mu = \sum_{a=1}^{24} V_\mu^a \frac{\lambda^a}{2} = \tag{II.10}$$
$$= \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} G_{1\mu} + \frac{2B_\mu}{\sqrt{30}} & G_{2\mu}^1 & G_{3\mu}^1 & X_\mu^{C1} & Y_\mu^{C1} \\ G_{1\mu}^2 & G_{2\mu}^2 + \frac{2B_\mu}{\sqrt{30}} & G_{3\mu}^2 & X_\mu^{C2} & Y_\mu^{C2} \\ G_{1\mu}^3 & G_{2\mu}^3 & G_{3\mu}^3 + \frac{2B_\mu}{\sqrt{30}} & X_\mu^{C3} & Y_\mu^{C3} \\ \hline X_\mu^1 & X_\mu^2 & X_\mu^3 & \frac{W_\mu^3}{\sqrt{2}} - \sqrt{\frac{3}{10}} B_\mu & W_\mu^+ \\ Y_\mu^1 & Y_\mu^2 & Y_\mu^3 & W_\mu^- & -\frac{W_\mu^3}{\sqrt{2}} - \sqrt{\frac{3}{10}} B_\mu \end{array} \right).$$

Notice that the theory predicts 12 new gauge bosons which were not present in the SM (off-diagonal blocks above). More about these new fields will be discussed in the next sections.

The matter component (fermions) fits perfectly (regarding quantum numbers of the multiplets components) in the $\bar{5}$ and 10 representations (see appendix for detailed description of fields location inside the representations):

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}. \tag{II.11}$$

It is remarkable how the 15 d.o.f. of the SM can be embedded by using the two simplest representations in SU(5) in such a way that quarks and leptons are unified. This is one of the highest motivations to believe that the SU(5) is actually the UV-completion of the SM.

The scalar content of the SU(5)-GG is composed of the minimal amount of scalar Higgses needed to break SU(5) to $U(1)_{em}$ by stepping into the SM. In order to first break SU(5) to the SM, a 24 Higgses are needed, as we will discuss in next section. The 24 scalars can be distributed as follows:

$$24 = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_3 \end{pmatrix} + \Sigma_S \, \lambda_{24}, \tag{II.12}$$

where $\Sigma_{(3,2)}$ and $\Sigma_{(\bar{3},2)}$ will play the role of Goldstone bosons once $SU(5)$ is broken.

An extra 5_H representation (the subindex H refers to the scalar representation, i.e. “Higgses”) is needed in order to break the SM to $U(1)_{em}$. This fundamental scalar representation is composed of a colored scalar triplet T and a complex Higgs doublet H which will play the same role as the well-known Higgs in the SM:

$$5_H = \left(\begin{array}{c} T^1 \\ T^2 \\ T^3 \\ \hline H^+ \\ H^0 \end{array} \right). \tag{II.13}$$

For completion, we write here the Lagrangian of the minimal SU(5) unified theory (the definition of the covariant derivative and the transformation properties under SU(5) for each representation can be found in the appendix, along with more details about the Lagrangian),

$$\mathcal{L} = \mathcal{L}_K^F + \mathcal{L}_Y + \mathcal{L}_K^S - V, \tag{II.14}$$

where $\mathcal{L}_K^F$ refers to the kinetic Lagrangian of the fermions [1],

$$\mathcal{L}_K^F = i\frac{1}{2}\mathrm{Tr}\{\overline{10}\not{D}10\} + i\bar{5}\not{D}5, \tag{II.15}$$

which is responsible of the phenomenology regarding interactions between fermions and gauge bosons, and $\mathcal{L}_K^S$ is the kinetic Lagrangian of the scalar sector,

$$\mathcal{L}_K^S = \frac{1}{2}\mathrm{Tr}\{(D_\mu 24_H)^\dagger (D^\mu 24_H)\} + (D_\mu 5_H)^\dagger (D^\mu 5_H), \tag{II.16}$$

responsible of the gauge bosons masses plus gauge bosons - Higgs interactions. The Yukawa Lagrangian $\mathcal{L}_Y$, as we will see in the section "Fermion masses" will generate masses for the fermions once the symmetry is spontaneously broken, along with some interactions Higgses-fermions. Finally, the scalar potential is addressed in the next section and it is responsible of the spontaneous symmetry breaking taking place in the theory.

B. Symmetry breaking and scalar potential

As far as we are aware from our energy range availability, quarks and leptons are different particles which cannot transform among each other. Therefore, the SU(5) symmetry must be strongly broken at some energy scale to the well-known SM. Besides, the SM will further break to $U(1)_{em}$ at the electroweak (EW) scale. Thus, the symmetry breaking takes part in two steps according to the following patterns:

(i) $SU(5) \overset{M_{GUT}}{\rightarrow} SU(3) \otimes SU(2) \otimes U(1)_Y$,

(ii) $SU(3) \otimes SU(2) \otimes U(1)_Y \overset{M_Z}{\rightarrow} U(1)_{em}$.

In the first step, a total of 12 d.o.f. (24 d.o.f. from SU(5) - 12 d.o.f. from the SM) are broken, so that 12 Goldstones will arise. Those will be eaten by an equal number of gauge bosons which therefore will become massive. These bosons are the leptoquark gauge fields which mediate proton decay. The non-observation of this phenomenon forces the mass of these new gauge bosons to be heavy. Thus, so does the breaking scale of $SU(5)$, fact that is completely in agreement with unification constraints (see upcoming sections). For this breaking to occur, one needs to introduce a set of scalars in the adjoint representation, i.e. 24 Higgses.

In the second step, the SM gauge symmetry group breaks to the $U(1)_{em}$. In the process, three d.o.f. are broken, which turn into the three massive gauge bosons $W_\mu^\pm$ and Z_μ, leaving only nine massless gauge fields (9 unbroken d.o.f.), which correspond to the photon and the gluons. This second breaking-step requires to introduce a fundamental scalar representation 5_H.

[1] The normalization factor $1/2$ appears as a consequence of the explicit form of the covariant derivative for the antisymmetric representation. See appendix for the explicit expression.

Breaking step I: $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)_Y$

In order to break the SU(5) symmetry to the SM, which is a maximal group (same rank = 4), the scalar Higgs whose vev is responsible for the breaking must be a singlet under the SM plus it must be real. Apart from 1, the smallest real representation of $SU(5)$ is the adjoint representation 24. In general, a scalar field in the adjoint representation has the property that does not break the rank of the group, due to the fact that under a unitary transformation $\langle 24\rangle \to U \langle 24\rangle U^\dagger$, one can bring $\langle 24\rangle$ to a diagonal form, which in turn implies

$$[\langle 24\rangle, T_a \text{diagonal}] = 0, \tag{II.17}$$

i.e. the diagonal generators are preserved [13]. Thus, the 24_H is our candidate.

The most general scalar potential for the 24_H which is invariant under a given SU(N) gauge symmetry is given by

$$V(24_H) = \frac{\mu^2}{2}\text{Tr}\{24_H^2\} + \frac{\lambda_1}{4}(\text{Tr}\{24_H^2\})^2 + \frac{\lambda_2}{4}\text{Tr}\{24_H^4\}, \tag{II.18}$$

where $\mu^2 < 0$ in order to ensure spontaneous symmetry breaking. Here, a discrete Z_2 symmetry has been assumed for simplicity [2]. The gauge invariance of the potential allow us to bring the vev of the 24_H in a suitable diagonal form through a SU(N) unitary transformation (since fields in 24_H are hermitian), i.e. $\langle 24_H\rangle^i_j = \delta_{ij}\Phi^i$ where Φ^i are real. After rotating the fields to its diagonal form the potential reads as

$$V(\langle 24_H\rangle) = \frac{\mu^2}{2}\sum_{i=1}^{N}\Phi_i^2 + \frac{\lambda_1}{4}\left(\sum_{i=1}^{N}\Phi_i^2\right)^2 + \frac{\lambda_2}{4}\sum_{i=1}^{N}\Phi_i^4. \tag{II.19}$$

Notice that the traceless condition must be satisfied, i.e. $\sum_{i=1}^N \Phi_i = 0$. The fact that the potential can be written as a function of the diagonal entries constraints the possibilities for the breaking pattern of $SU(5)$. By taking

$$\begin{aligned}\left(\text{Tr}\{\Phi^2\}\right)^2 - \text{Tr}\{\Phi^4\} &= \left(\sum_{i=1}^{N}\Phi_i^2\right)^2 - \sum_{i=1}^{N}\Phi_i^4 = 2\left[\sum_{1\le i<j\le N-1}\Phi_i^2\Phi_j^2 + \left(\sum_{i=1}^{N-1}\Phi_i^2\right)\left(\sum_{j=1}^{N-1}\Phi_j\right)^2\right] \ge 0,\\ &\Rightarrow (\text{Tr}\{\Phi^2\})^2 - \text{Tr}\{\Phi^4\} \ge 0,\end{aligned} \tag{II.20}$$

one realizes that the dominant term in the above potential is the one leaded by λ_1. Thus, the requirement that V is bounded from below implies that $\lambda_1 > 0$. Now, one must distinguish between two possibilities:

(a) For $\lambda_2 < 0$ (the parameter λ_2 may be negative for a restricted range of values $\left(\frac{N(1-N)}{N^2-3N+3}\right)\lambda_1 < \lambda_2 < 0$) the vev which minimizes the potential V is given by [15],

$$\langle\Phi_i\rangle = \frac{v_{24}}{\sqrt{2N(N-1)}} \times \begin{cases} 1 \text{ for } 1 \le i \le N-1, \\ -(N-1) \text{ for } i = N. \end{cases} \tag{II.21}$$

[2] See reference [14] for the potential without any global symmetry assumption.

Hence, in our case $N = 5$,

$$\left.\begin{array}{l}\langle\Phi_1\rangle = \langle\Phi_2\rangle = \langle\Phi_3\rangle = \langle\Phi_4\rangle = \dfrac{v_{24}}{2\sqrt{10}} \\ \langle\Phi_5\rangle = \dfrac{v_{24}}{2\sqrt{10}}(-4)\end{array}\right\} \Rightarrow \langle\Phi\rangle = \frac{v_{24}}{2\sqrt{10}}\begin{pmatrix}1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4\end{pmatrix}. \tag{II.22}$$

This would break $SU(5) \to SU(4) \otimes U(1)$.

(b) For $\lambda_2 > 0$ and N odd, i.e. $N = 2k+1$, the potential is minimized by [15],

$$\langle\Phi_i\rangle = v_{24}\left[\frac{k}{2(k+1)(2k+1)}\right]^{1/2} \times \begin{cases} 1 \text{ for } 1 \leq i \leq k+1, \\ -\dfrac{k+1}{k} \text{ for } k+2 \leq i \leq 2k+1. \end{cases} \tag{II.23}$$

For $N = 5$ (i.e. $k = 2$),

$$\left.\begin{array}{l}\langle\Phi_1\rangle = \langle\Phi_2\rangle = \langle\Phi_3\rangle = \dfrac{v_{24}}{\sqrt{15}} \\ \langle\Phi_4\rangle = \langle\Phi_5\rangle = \dfrac{v_{24}}{\sqrt{15}}\left(-\dfrac{3}{2}\right)\end{array}\right\} \Rightarrow \langle\Phi\rangle = \frac{v_{24}}{2\sqrt{15}}\begin{pmatrix}2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3\end{pmatrix}. \tag{II.24}$$

The last step corresponds to the breaking of SU(5) to the SM since the group is broken in the direction of λ_{24} (i.e. hypercharge operator). The minimum condition,

$$\frac{\partial V}{\partial \Phi_i} = \Phi_i\left(\mu^2 + \lambda_1\sum_{i=1}^{N}\Phi_i^2 + \lambda_2\Phi_i^2\right) \overset{!}{=} 0, \tag{II.25}$$

yields to the following expression for the vev:

$$v_{24}^2 = \frac{-2\mu}{\lambda_1 + 2\lambda_2}. \tag{II.26}$$

From the kinetic Lagrangian for the scalar fields,

$$\begin{aligned}\mathcal{L}_K^S &\supset \frac{1}{2}\mathrm{Tr}\{(D_\mu 24_H)^\dagger(D^\mu 24_H)\} \\ &\overset{SSB}{\to} \frac{1}{2}\mathrm{Tr}\left\{(ig_5[A_\mu,\, \langle 24_H\rangle])^\dagger(ig_5[A^\mu,\, \langle 24_H\rangle])\right\} = \frac{5}{6}g_5^2 v_{24}^2\left(\bar{X}_\mu^i X^{i\mu} + \bar{Y}_\mu^i Y^{i\mu}\right) + \text{h.c.}\end{aligned} \tag{II.27}$$

one can read the mass of the new gauge bosons, which is given by $M_X^2 = M_Y^2 = \dfrac{5}{6}g_5^2 v_{24}^2$.

Breaking step 2: $SU(3) \otimes SU(2) \otimes U(1)_Y \to U(1)_{em}$

We still need to break the SM down to $U(1)_{em}$. It turns out that the easiest way to do it is by introducing a fundamental scalar representation 5_H which decomposes, according to our organization criterion, into a colored triplet under SU(3) (three first components) and a scalar doublet under SU(2) (last two components) which plays exactly the same role as the SM Higgs. In any SU(N) symmetric group, the breaking of the symmetry by using a vector representation implies the reduction of the rank of the group by two units (if the symmetry of a certain SU(N) group is broken by a vector representation, the breaking pattern goes as

$SU(N) \to SU(N-2)$. Since the rank of a special unitary group of dimension N is given by N-1, the resulting group after the SSB will have two diagonal generators less than the original one) [16].

The most general potential for a 5_H representation reads as

$$V(5_H) = -\mu_5^2 5_H^\dagger 5_H + \lambda(5_H^\dagger 5_H)^2 \tag{II.28}$$

We assume the vev to be in the neutral direction,

$$\langle 5_H \rangle = \text{diag}(0,0,0,0,v_5) \tag{II.29}$$

in order to induce the desired pattern (keep the electric charge unbroken). According to this breaking, the following relation between the gauge masses is obtained as expected:

$$M_W^2 = \cos^2\theta_W M_Z^2 = \frac{1}{4} g_2^2 v_5^2. \tag{II.30}$$

However, since the 24 real scalars are also needed for performing the two breaking steps, one has to take into account the cross terms in the potential which are also gauge invariant,

$$V(24_H, 5_H) = \alpha\, 5_H^\dagger 5_H \text{Tr}\{24_H^2\} + \beta\, 5_H^\dagger 24_H^2 5_H. \tag{II.31}$$

Hence, the whole scalar potential is composed by the following three contributions: $V = V(24_H) + V(5_H) + V(24_H, 5_H)$. After the breaking of $SU(5)$, we can write the total potential as an "effective" potential as a function of the triplet and the doublet Higgses sitting in 5_H (from a SM point of view) [3].

$$V(24_H, 5_H) \overset{SSB}{\to} V(\langle 24_H \rangle, 5_H) = \alpha\, \frac{1}{2} v_{24}^2 (T^\dagger T + H^\dagger H) + \beta\, v_{24}^2 \left(\frac{1}{15} T^\dagger T + \frac{3}{20} H^\dagger H \right). \tag{II.32}$$

Thus, the masses of the colored triplet and the Higgs doublet read as,

$$M_H^2 \;=\; -\mu_5^2 + \frac{1}{60}(30\alpha + 9\beta) v_{24}^2, \tag{II.33}$$

$$M_T^2 \;=\; -\mu_5^2 + \frac{1}{60}(30\alpha + 4\beta) v_{24}^2. \tag{II.34}$$

The Higgs doublet is required to live at the EW scale, owing to the fact that it contains the Goldstones which will be eaten by the W and Z. In order to achieve that, a delicate cancellation has to occur since $M_X \gg M_W$. However, if some fine-tuning is applied in Eq. (II.33), the mass of the triplet will be heavy. This huge splitting in the mass scale of the 5_H representation is known as the doublet-triplet splitting problem and, whereas it is not in contradiction with the theory, it is pretty anti-aesthetic though. This unnatural cancellation can be arranged at tree level, but radiative corrections will re-introduce this problem. This is known as the Higgs hierarchy problem and unfortunately we cannot get rid of it in the SM, neither in the context of this grand unified theory.

[3] Strictly speaking, the vev of the 24_H also contributes to the breaking of SU(2) due to the crossed terms in $V(24_H, 5_H)$, since $\langle 24_H \rangle = \text{Diagonal}(2,2,2,-3-\epsilon,-3+\epsilon)$. Here, $\epsilon \sim O(M_W^2/M_{GUT}^2)$. Indeed, this means that the 24_H contribution to the breaking of SU(5) is much smaller than 5_H, which is highly encouraged by the electroweak precision test ($\rho \sim 1$).

C. Fermion masses

The most general Yukawa interactions which are renormalizable and SU(5) gauge invariant are

$$\mathcal{L}_Y \supset Y_1 \bar{5}\; 10\; 5_H^* \; + \; Y_3 10\; 10\; 5_H\; \epsilon_5, \tag{II.35}$$

where ϵ_5 refers to the dimension 5 Levi-Civita tensor. The nomenclature of the Yukawa couplings will be justified in the following sections. The above Yukawa Lagrangian, apart of generating part of the phenomenology of the theory, is the responsible of the fermion masses after the symmetry $SM \to U(1)_{em}$ is spontaneously broken, i.e. the neutral component of 5_H gets a vev. Explicitly written in terms of the color ($i, j, k = 1, 2, 3$) and weak isospin ($\alpha, \beta = 4, 5$) indices, the above equation reads as

$$\mathcal{L}_Y \supset Y_1(\bar{5}_i 10^{i\alpha} + \bar{5}_\beta 10^{\beta\alpha}) 5^*_{H\alpha} + Y_3(10^{ij}10^{k\alpha} + 10^{i\alpha}10^{jk}) 5^\beta_H \epsilon_{ijk\alpha\beta}. \tag{II.36}$$

Now, the Levi-Civita symbol can be split in a 3-d tensor with color indices times 2-d tensor with SU(2) indices as follows, $\epsilon_{ijk\alpha\beta} = \epsilon_{ijk}\epsilon_{\alpha\beta}$. But one has to be careful because once the tensor is split, the antisymmetric contractions with the other representations which involve $SU(2)$ and color indices mixed are lost. Then, we have to consider them when breaking the 5-dim tensor:

$$\mathcal{L}_Y \supset Y_1\{d_i^C q^{i\alpha} + \ell_\beta \epsilon^{\beta\alpha} e^C\} H^*_\alpha + 2Y_3(u_l^C \epsilon^{ijl} q^{k\beta} + q^{i\beta} u_m^C \epsilon^{jkm}) H^\alpha \epsilon_{ijk}\epsilon_{\beta\alpha}. \tag{II.37}$$

By performing the contraction $\epsilon_{ija}\epsilon^{ijb} = 2\delta_a^b$,

$$\mathcal{L}_Y \supset Y_1\{d_i^C q^{i\alpha} + \ell_\beta \epsilon^{\beta\alpha} e^C\} H^*_\alpha + 4Y_3(u_k^C q^{k\beta} + q^{i\beta} u_i^C) H^\alpha \epsilon_{\beta\alpha}. \tag{II.38}$$

When the Higgs gets the vev, i.e. $\langle 5_H \rangle^5 = v_5$ and $\langle 5_H \rangle^i = 0$ for $i \neq 5$ (spontaneous symmetry breaking), so that

$$\mathcal{L}_Y \supset Y_1 \frac{v^*}{\sqrt{2}}(d_i^C d^i + e e^C) + 4(Y_3 + Y_3^T)\frac{v}{\sqrt{2}} u_i^C u^i. \tag{II.39}$$

Therefore, the masses of the fermions are given by

$$M_d \;=\; M_e^T = Y_1 \frac{v^*}{\sqrt{2}}, \tag{II.40}$$

$$M_u \;=\; 4(Y_3 + Y_3^T)\frac{v}{\sqrt{2}}. \tag{II.41}$$

Notice that the Yukawa coupling of the up-type quarks, $Y_u \equiv Y_3 + Y_3^T$, is symmetric. Here we have another strong prediction from SU(5): independently of the freedom of the unfixed Yukawa couplings Y_1 and Y_3, the model predicts exact masses for the down-type quarks and the charged leptons at an energy scale in which SU(5) is a good symmetry. This prediction, when running down to low energies through the RGE equations, turns to be wrong according to the current experimental values of the measured fermion masses at the EW scale. This is one of the main reasons why this simple theory of SU(5) has been ruled out. But, as we will see soon, it can become realistic again by introducing some extra representations, although it looses a unique prediction like the above one coming naturally from the model itself.

D. Unification of the gauge couplings

SU(5) is an unified theory, parametrized by only one coupling. In order to test the viability of the theory, one needs to compare the GUT predictions with the low energies experimental values. The running group equations (RGE), which are a direct consequence of the Callan-Symanzik equation (any physical observable must not depend on the renormalizable scale, which is unphysical), allow us to connect different energy scales, according to

$$\mu \frac{d}{d\mu} g_i(\mu) = \beta_i[g_i(\mu)], \tag{II.42}$$

where β_i refers to the β-function of a Young-Mills theory, which is given by

$$\beta_i = \frac{g_i^3}{16\pi^2} b_i \text{ where } b_i = \left[\frac{1}{3}\sum_R S(R) T_i(R) \prod_{j\neq i} \text{dim}_j(R)\right]. \tag{II.43}$$

The β-function encodes the dependence of the coupling g_i on the renormalization scale. Here, R is the representation according to which a complex field transforms [4], $T_i(R)$ is the Dynkin index of the representation which is given by

$$T_i(R)\delta^{ab} = \text{Tr}\{T_i^a(R) T_i^b(R)\}, \tag{II.44}$$

and it depends on the choice of the gauge group and on the representation R. By contracting the above expression with δ_{ab} the following relation arises:

$$d(G) T_i(R) = d(r) C_2(r), \tag{II.45}$$

where $d(G)$ is the dimension of the group and $d(r)$ the dimension of the chosen representation. The $\mathbb{1} C_2(r) = T^2$ is called quadratic Casimir operator. [5] For a SU(N), the fundamental representation is usually normalized to 1/2. It is important to remark that, whereas the above normalization is chosen for the non-abelian groups SU(2), SU(3) and SU(5), the abelian U(1) normalization may be arbitrarily normalized (for the SM case, $U(1)_Y$ is normalized in order to obtain $Q(e) = 1$). Notice that the hypercharge operator in SU(5), however, is normalized as a non-abelian generator, i.e. $C(r) = 1/2$, which then differs from the old SM U(1) coupling:

$$\left.\begin{array}{l}\text{Old SM coupling: } g' \frac{Y}{2} \\ \text{New SM coupling: } g_1 T_Y\end{array}\right\} \Rightarrow \left(g' \frac{Y}{2}\right)^2 \stackrel{!}{=} (g_1 T_Y)^2 \tag{II.46}$$

$$\Rightarrow g'^2 \left(3\left(\frac{1}{3}\right)^2 + 2\left(-\frac{1}{2}\right)^2\right) \stackrel{!}{=} g_1^2 \frac{1}{2}, \tag{II.47}$$

Hence,

$$g' = \sqrt{\frac{3}{5}} g_1 \tag{II.48}$$

[4] In case of real fields (i.e. the adjoint representation) it should be taken into account that complex fields have the double number of degrees of freedom, so that expression (II.43) must be divided by a factor of 2.

[5] Since $[T^b, T^a T^a] = 0$ for all T^c of SU(N), $C_2(r)$ is an invariant of the algebra, so that it characterizes each irreducible representation of the group. Hence, $T^2 \stackrel{!}{\propto} \mathbb{1}$. The constant of proportionality is indeed the quadratic Casimir operator so that it corresponds to the normalization of the representation.

This is an important fact that one must take into account in order to compare both theories, SM and SU(5). The term dim(R) in equation (II.43) refers to the dimension of the field concerning the other gauge groups (i.e. multiplicity due to other gauge symmetries). The $S(R)$ is given by

$$S(R) \begin{cases} 1 \text{ for } R \text{ scalar}, \\ 2 \text{ for } R \text{ chiral fermion}, \\ -11 \text{ for } R \text{ gauge boson}. \end{cases} \tag{II.49}$$

In Table I are listed the beta contributions of the SM fields plus the extra fields predicted by the SU(5). As

TABLE I: Contributions to the B_{ij} coefficients for SU(5).

	$\bar{5}$		10			V_{24}		5_H		24_H	
b_i/B_{ij}	l_L	$(d^c)_L$	$(u^c)_L$	q_L	$(e^c)_L$	G_μ	W_μ	H_1	T	Σ_8	Σ_3
b_1	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{8}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	0	0	$\frac{1}{10}$	$\frac{1}{15}r_T$	0	0
b_2	1	0	0	3	0	0	$-\frac{22}{3}$	$\frac{1}{6}$	0	0	$\frac{1}{3}r_{\Sigma_3}$
b_3	0	1	1	2	0	-11	0	0	$\frac{1}{6}r_T$	$\frac{1}{2}r_{\Sigma_8}$	0
B_{12}	$-\frac{4}{5}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{44}{15}$	$-\frac{2}{5}$	0	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}r_T$	0	$-\frac{1}{3}r_{\Sigma_3}$
B_{23}	1	-1	-1	1	0	11	$-\frac{22}{3}$	$\frac{1}{6}$	$-\frac{1}{6}r_T$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$

one may already have noticed, unlike in the SM in which the parameters g_2 and $g^{'}$ are independent, in SU(5) there is only one coupling, i.e. g_5, so that the Weinberg angle in the context of SU(5) is totally fixed. In the energy range where the SU(5) symmetry holds, $g_5 = g_1 = g_2 = g_3$,

$$\sin^2\theta_W = \frac{\alpha_{em}}{\alpha_s} = \frac{g^{'2}}{g^{'2} + g_2^2} = \frac{\frac{3}{5}g_1^2}{\frac{3}{5}g_1^2 + g_2^2} = \frac{\frac{3}{5}g_5^2}{\frac{3}{5}g_5^2 + g_5^2} = \frac{3}{8}. \tag{II.50}$$

This prediction is only valid when $SU(5)$ is a good symmetry, i.e. at scales $> M_X$. In order to compare this number with the experimental value, one needs to consider the radiative corrections to continue the couplings and masses to the low scale at which experiments are made (see last part of this section).

The running of the gauge couplings at 1-loop level (RGE) is given by,

$$\alpha_i^{-1}(\mu^*) = \alpha_i^{-1}(\mu) + \frac{b_i}{2\pi}\text{Log}\left(\frac{\mu}{\mu^*}\right), \tag{II.51}$$

where $\alpha_i = g_i^2/(4\pi)$ and μ^* is a given energy. For the Standard Model particle content, the beta functions acquire the following values, according to equation (II.43):

$$b_1^{SM} = \frac{41}{10}, \quad b_2^{SM} = \frac{-19}{6}, \quad b_3^{SM} = -7. \tag{II.52}$$

Fig. 1 shows how the couplings do not match altogether at a certain point, for any energy scale. Thus, the SM alone cannot yield to unification when we let its couplings run to high energies.

To proceed in the study of unification, let us consider now the overall contribution

$$B_i = b_i + \sum_I b_i^I r^I, \text{ where } r^I = \frac{\text{Log}(\Lambda_{GUT}/M_I)}{\text{Log}(\Lambda_{GUT}/M_Z)}, \tag{II.53}$$

being I an intermediate particle between $M_Z < M_I < M_{GUT}$. Following Giveon et al. [17], one may

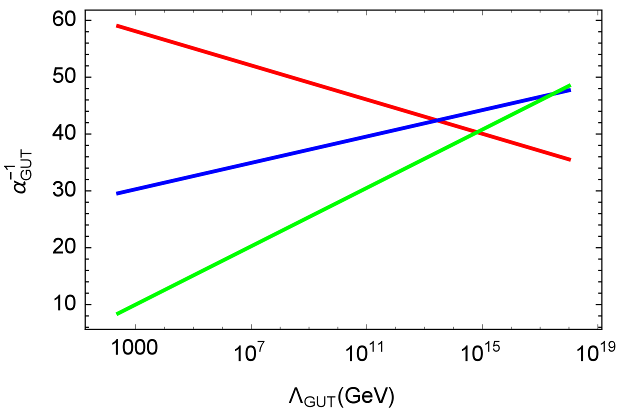


FIG. 1: Running of the couplings of the SM. α_1, α_2 and α_3 are plotted in red, blue and green, respectively. Couplings do not match at any energy scale. In the present plot only the particle content of the Standard Model was taken into account.

express the running shown in Eq. (II.51) in a more suitable way, i.e. in terms of the differences $B_{ij} = B_i - B_j$ and the low energy scale observables:

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\,\frac{\sin^2\theta_W - \alpha/\alpha_3}{3/8 - \sin^2\theta_W} \tag{II.54}$$

$$\text{Log}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{16\pi}{5\alpha}\,\frac{3/8 - \sin^2\theta_W}{B_{12}} \tag{II.55}$$

where unification, i.e. $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}) = \alpha_{GUT}$, has been assumed. Here $\alpha^{-1}(M_Z) = 127.94$, $\sin^2\theta_W(M_Z) = 0.231$ and $\alpha_s(M_Z) = 0.1185$ [18]. Adopting the experimental values mentioned above,

$$\frac{B_{23}}{B_{12}} = 0.718, \tag{II.56}$$

$$\text{Log}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}. \tag{II.57}$$

Thus, this values must be reached in order to achieve unification of the couplings (i.e. to accomplish equation (II.51) where unification is already assumed). From now on we will refer to these equations as the unification constraints.

For the minimal SU(5) model (SU(5)-GG) the unification ratio takes the form:

$$\frac{B_{23}^{SU(5)}}{B_{12}^{SU(5)}} = \frac{B_{23}^{SM} + \frac{1}{3}r_{\Sigma_3} - \frac{1}{6}r_T - \frac{1}{2}r_{\Sigma_8}}{B_{12}^{SM} - \frac{1}{3}r_{\Sigma_3} + \frac{1}{15}r_T} \tag{II.58}$$

where the leptoquark bosons X_μ and Y_μ are, by definition, at the GUT scale (where the breaking from SU(5) to the SM gauge group occurs), so that $r = 0$ for them. As we have already shown before, only with the SM field content unification cannot be achieved. The question is "Could unification be reached by the influence of the extra fields that SU(5) includes?"

Let us assume the most optimistic model, where Σ_3 is super light ($r_T = 1$), Σ_8 is super heavy ($r_{\Sigma_8} = 0$)

and the triplet is super heavy too ($r_T = 0$), since Σ_3 help to increase the unification ratio whereas Σ_8 and the triplet low it. Under these optimistic assumptions, which reflect the most suitable situation to achieve unification in SU(5) i.e. the maximum value achievable, the value of the unification ratio reads as

$$\frac{B_{23}^{SU(5)}}{B_{12}^{SU(5)}} \lesssim 0.6 \tag{II.59}$$

Thus, unfortunately, unification cannot be reached for the SU(5)-GG. This is one of the reasons why this model, in spite of its beauty and its simplicity, has been ruled out.

To express it differently, if we leave the $\sin\theta_W$ unfixed and assume that unification is reached at some energy scale, the Weinberg angle at the EW scale predicted by the SU(5) can be computed. First, let us write explicitly and in a suitable way the equations in (II.51):

$$\alpha_1^{-1}(\mu) = \alpha^{-1}\cos^2\theta_W \frac{3}{5} = \alpha_{GUT}^{-1}(M_{GUT}) + \frac{b_1}{2\pi}\,\mathrm{Log}\left(\frac{M_{GUT}}{\mu}\right), \tag{II.60}$$

$$\alpha_2^{-1}(\mu) = \alpha^{-1}\sin^2\theta_W = \alpha_{GUT}^{-1}(M_{GUT}) + \frac{b_2}{2\pi}\,\mathrm{Log}\left(\frac{M_{GUT}}{\mu}\right), \tag{II.61}$$

$$\alpha_3^{-1}(\mu) = \alpha_{GUT}^{-1}(M_{GUT}) + \frac{b_3}{2\pi}\,\mathrm{Log}\left(\frac{M_{GUT}}{\mu}\right). \tag{II.62}$$

By manipulating the above equations in this way (II.61)+$\frac{5}{3}$(II.60)$-\frac{8}{3}$(II.62), one gets the following relation,

$$\mathrm{Log}\left(\frac{\Lambda_{GUT}}{\mu}\right) = \frac{6\pi}{3b_2 + 5b_1 - 8b_3}\left[\frac{1}{\alpha(\mu)} - \frac{8}{3}\frac{1}{\alpha_3(\mu)}\right]. \tag{II.63}$$

Playing with equations (II.60) and (II.61) and using the above equation, the GUT coupling can be expressed as a function of the values of α and α_3 at the SM scale,

$$\alpha_{GUT}^{-1} = \alpha^{-1}(\mu)\frac{1}{5b_1 + 3b_2 - 8b_3}\left[(5b_1 + 3b_2)\frac{\alpha(\mu)}{\alpha_3(\mu)}\right]. \tag{II.64}$$

Finally, by taking (II.61)+(II.62) and using the equation (II.63), the Weinberg angle is given by

$$\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1}\frac{\alpha}{\alpha_3}. \tag{II.65}$$

Adopting the experimental values $\alpha(M_Z)^{-1} = 129.94$ and $\alpha_3(M_Z) = 0.1185$ (at the EW scale) [18], the predictions from SU(5) for the GUT scale and the unified coupling are

$$\Lambda_{GUT} \sim 1.195 \times 10^{15}\mathrm{GeV}, \tag{II.66}$$

$$\alpha_{GUT}^{-1} \sim 40.747, \tag{II.67}$$

and the value of the Weinberg angle at the EW scale such that unification is achieved is given by,

$$\sin\theta_W \sim 0.207. \tag{II.68}$$

This value is surprisingly close to the experimental value $\sin^2\theta_W(M_Z) = 0.231$[18] but still it is outside the experimental error range, even considering two-loop corrections. Therefore, unification is not achieved in the context of the minimal SU(5) theory.

E. Anomaly cancellations

One of the most beautiful, magical we would dare to say, property of the minimal SU(5) theory is that is anomaly free. An anomaly is a breakdown of a classical symmetry by quantum effects. It spoils the Ward-Takahashi identities at any regularization of the full quantum theory which, in other words, means that any gauge theory with non-vanishing anomalies is non-renormalizable. The fermion content in the SM is just right to make the theory anomaly free. Since the presence of anomalies spoil the renormalizability of the theory, any extension of the SM will be constrained by the anomaly cancellation conditions. Anomaly cancellation will thus play a central role in building any beyond the SM theory.

It can be shown that in 4 space-time dimensions all anomalies involve the triangle anomaly shown in Fig. 2, with two vector and one axial couplings. Thus, eliminating this anomaly eliminates all of them [11]. The amplitude of the above process is given by

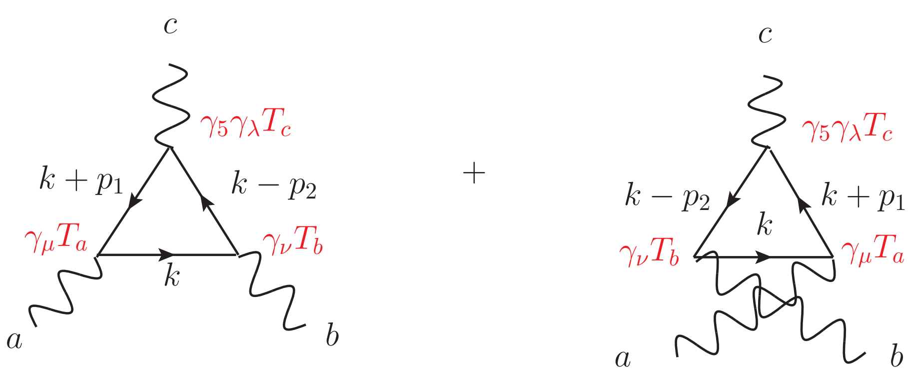


FIG. 2: Triangle anomaly graphs.

$$T_{\lambda\mu\nu} = \tag{II.69}$$
$$g^2 \int \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[T_c T_a T_b\right] \mathrm{Tr}\left[\gamma_5\gamma_\lambda(\not{k}+\not{p}_1)\gamma_\mu \not{k}\gamma_\nu(\not{k}-\not{p}_2)\right] - \mathrm{Tr}\left[T_c T_b T_a\right\}\mathrm{Tr}\{\gamma_5\gamma_\lambda(\not{k}-\not{p}_2)\gamma_\nu \not{k}\gamma_\mu(\not{k}+\not{p}_1)\right]}{k^2(k+p_1)^2(k-p_2)^2},$$

where g is the gauge coupling of a given gauge group. Notice that the trace involving the generators of the symmetry group T_k is taken over the charges of the group whereas the trace involving gamma matrices is taken over the Dirac space.

From the Ward-Takahashi identities, which are the generalization of the Noether's theorem at the quantum level, the vector and axial currents are expected to be free of divergences, so that the following contractions,

$$\begin{aligned} (p_1+p_2)^\lambda T_{\lambda\mu\nu} &= 0, \\ p_1^\mu T_{\lambda\mu\nu} &= 0, \\ p_2^\nu T_{\lambda\mu\nu} &= 0. \end{aligned} \tag{II.70}$$

are expected to be zero. Let us focus on the first equation of the set (II.70). Since the left-hand side corresponds to a pseudo-scalar quantity, the only combination proportional to momenta p_1 and p_2 that can be built is the following one:

$$(p_1+p_2)^\lambda T_{\lambda\mu\nu} = \frac{Ag^2}{16\pi^2}\, 4\,\epsilon_{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma, \tag{II.71}$$

where the factor 4 is just a matter of convention (although the choice of the normalization of A is in principle arbitrary, it will be understood at the end of this section).

Let us now take the derivative with respect to p_1^α in both sides of the above relation, setting $p_1 = -p_2 = p$ for simplicity,

$$\delta_\alpha^\lambda T_{\lambda\mu\nu} = -\frac{Ag^2}{16\pi^2} 4\epsilon_{\mu\nu\rho\sigma}\delta_\alpha^\rho p^\sigma \tag{II.72}$$

$$\Rightarrow T_{\alpha\mu\nu}(p,-p) = \frac{Ag^2}{16\pi^2} 4\epsilon_{\mu\nu\alpha\sigma} p^\sigma. \tag{II.73}$$

Here, the factor A parametrizes the anomaly in the sense that $A \neq 0$ corresponds to a violation of the conservation laws. In order to obtain an explicit expression of A, let us perform the amplitude of Eq. (II.70). By replacing $k^{'} \equiv k + p$, Eq. (II.70) reads as,

$$T^{\alpha\mu\nu}(p,-p) = g^2 \int \frac{d^n k'}{(2\pi)^n} \frac{\mathrm{Tr}\left[T^c T^a T^b\right] \mathrm{Tr}\left[\gamma^5\gamma^\alpha(\not{k}' - \not{p})\gamma_\nu \not{k}'\right] - \mathrm{Tr}\left[T^c T^b T^a\right] \mathrm{Tr}\left[\gamma^5\gamma^\alpha \not{k}' \gamma_\nu(\not{k}' - \not{p})\gamma_\mu \not{k}'\right]}{({k'}^2)^2 (k'-p)^2}. \tag{II.74}$$

By using some trace identities such as $\not{k}'\not{k}' = {k^{'}}^2$ and the properties of Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, the above integral can be rewritten as

$$T^{\alpha\mu\nu} = g^2 \int \frac{d^n k'}{(2\pi)^n} \frac{\#\mathrm{Tr}\left[T^c T^a T^b\right] + \#\mathrm{Tr}\left[T^c T^b T^a\right]}{({k'}^2)^2 (k'-p)^2} = g^2 \mathrm{Tr}\left[T_c\{T_a, T_b\}\right] \int \frac{d^n k}{(2\pi)^n} \frac{\#}{({k'}^2)^2 (k'-p)^2}, \tag{II.75}$$

where $\# \equiv {k^{'}}^2 k^{'}_\beta \mathrm{Tr}\left[\gamma^5\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\right] - {k'}^2 \mathrm{Tr}\left[\gamma^5\gamma^\alpha\gamma^\mu\not{p}\gamma^\nu\right] + 2{k'}^\alpha {k'}^\delta \mathrm{Tr}\left[\gamma^5\gamma_\delta\gamma^\mu\not{p}\gamma^\nu\right]$. The integrals with respect the momenta $k^{'}$ can be evaluated by using the tabulated integrals in the appendix. Once the momentum $k^{'}$ has been integrated out, the amplitude reads as

$$T^{\alpha\mu\nu}(p,-p) = \frac{g^2}{16\pi^2} \mathrm{Tr}\left[\{T^a, T^b\}, T^c\right] i\mathrm{Tr}\left[\gamma^5\gamma^\mu\gamma^\nu\gamma^\alpha\not{p}\right] \tag{II.76}$$

$$= \frac{g^2}{16\pi^2} \mathrm{Tr}\left[\{T^a, T^b\}, T^c\right]\; 4\,\epsilon_{\mu\nu\alpha\sigma} p^\sigma. \tag{II.77}$$

Hence, comparing the above equation with Eq. (II.73) we identify $A \equiv \mathrm{Tr}\left[\{T^a, T^b\}, T^c\right]$. Anomaly factors corresponding to different representations in SU(5) are listed below. From Table II one can see that in SU(5) anomalies in the fermion sector are beautifully canceled since

$$A(\bar{5}) + A(10) = 0. \tag{II.78}$$

Any extension of the SU(5)-GG which involves extra fermion representations should be added in such a way that the anomaly coefficients cancel out.

TABLE II: Anomaly coefficients for some representations in SU(N) and, concretely, SU(5) [19].

Irrep.	dim(r)		A(r)	
	SU(N)	SU(5)	SU(N)	SU(5)
$\ydiagram{1}$	N	5	1	1
Ad	N^2-1	24	0	0
$\ydiagram{1,1}$	$\frac{N(N-1)}{2}$	10	$N-4$	1
$\ydiagram{2}$	$\frac{N(N+1)}{2}$	15	$N+4$	9
$\ydiagram{1,1,1}$	$\frac{N(N-1)(N-2)}{6}$	$\overline{10}$	$\frac{(N-3)(N-6)}{2}$	-1
$\ydiagram{2,1,1}$	$\frac{N(N+1)(N-1)(N-2)}{8}$	45	$\frac{(N-4)(N^2-N-8)}{2}$	6
$\ydiagram{1,1,1,1}$	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\bar{5}$	$\frac{(N-4)(N-3)(N-8)}{6}$	-1

F. Proton decay

Last but not least, we would like to end this brief review about SU(5) with one of its most striking predictions. Unlike in the SM, where the proton is completely stable, in SU(5) there are indeed fields which can mediate proton decay. This process is strongly constrained by experiment, so that the GUT scale is pushed to be high. The discovery of proton decay signals would become a strong argument pro the SU(5) unified model as the candidate for the SM UV completion, but unfortunately no evidence on proton decay has been found yet.

In the minimal SU(5) theory, proton decay can be mediated by the new gauge bosons X and Y plus the colored Higgs triplet. We will study qualitatively the proton decay from an effective point of view. For that, we will construct all the effective operators which lead to proton decay processes, i.e. operators involving the leptoquark gauge bosons and the colored scalar triplet.

Lepto-quark gauge bosons contribution

The fermion-gauge boson interactions are encoded in the fermion kinetic Lagrangian $\mathcal{L}_K^F$:

$$\mathcal{L}_K^F \supset \frac{g_5}{\sqrt{2}}(\overbrace{\overline{d^C}^i\epsilon_{\alpha\beta}\ell^\beta}^{\bar{5}\not{A}^T 5} + \overbrace{\overline{e^C}\epsilon_{\alpha\beta}q^{i\beta} + \overline{q_{j\alpha}}\epsilon^{ijk}u_k^C}^{-\mathrm{Tr}\{\overline{10}\not{A}10\}})\not{X}_i^\alpha + \text{h.c.} \tag{II.79}$$

where $X_\alpha^{\mu i} = X^{\mu i}(= Y^{\mu i})$ for $\alpha = 4(=5)$. Here, left chirality of the fields is understood. By integrating out the leptoquark gauge boson fields we are left with a six dimensional effective Lagrangian that reads as

$$\mathcal{L}_{d=6}^X = \frac{g_5^2}{M_X^2}\epsilon_{ijk}(\overline{u^C})^i\gamma_\mu q^{\alpha j}\{\overline{e^C}\epsilon_{\alpha\beta}\gamma^\mu q^{k\beta} + (\overline{d^C})^k\gamma^\mu\epsilon_{\alpha\beta}\ell^\beta\} + \text{h.c.} \tag{II.80}$$

The above interactions can lead to two different channels of proton decay, $p \to \pi^0\ e^+(\mu^+)$ and $p \to \pi^+\bar{\nu}$ (see Fig. 3,4). The decay rate of these processes can be estimated qualitatively from the above Lagrangian. According to the effective theory, it will be proportional to the coupling of the effective interaction squared. Hence,

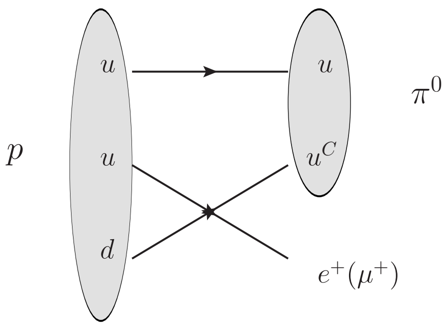


FIG. 3: Decay channel $p \to \pi^0\, e^+(\mu^+)$.

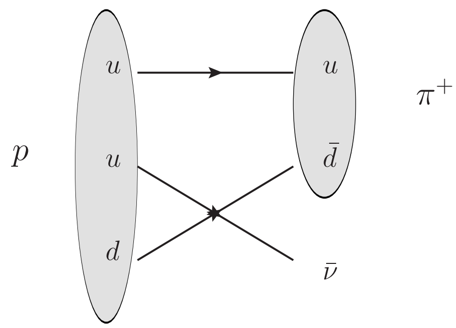


FIG. 4: Decay channel $p \to \pi^+ \bar{\nu}$.

$$\Gamma_X \sim \alpha^2_{GUT} \frac{m_p^5}{M_X^4}, \tag{II.81}$$

where the fifth power of the proton mass has been added to fulfill dimensional analysis (the decay rate has dimensions of energy).

From the experimental bound on the process $p \to \pi^0 e^+$, we have that the decay lifetime $\tau(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years [20]. We took this bound since it is the most restrictive one and therefore sets a lower bound for the mass-scale of the leptoquarks in the SU(5)-GG:

$$M_X \gtrsim 5 \times 10^{15}\text{GeV}, \tag{II.82}$$

where we used $m_p = 0.938$ GeV and α_{GUT} has been taken from the unification predictions, i.e. $(\alpha_{GUT})^{-1} \sim 40$. This result is in perfect agreement with the unification scale predicted by the unification constraints $\Lambda_{GUT} \sim 10^{15}$ GeV.

Colored triplet contribution

For the colored triplet case, the "dangerous" interactions which may give rise to proton decay live in the Yukawa Lagrangian (see appendix for more details):

$$\mathcal{L}_Y \supset Y_1\{q_L\, T^\dagger\, l_L + (d^c)\, T^\dagger\, (u^c)\} + Y_3\{q_L\, T\, q_L + (u^c)\, T\, (e^c)\}, \tag{II.83}$$
$$\Rightarrow \quad \mathcal{L}_Y \supset Y_1^{ab}(\nu_a d_b^i - e_a u_b^i) T_i^\dagger - Y_1^{ab}(d_a^c)_i (u_b^c)_j T_k^\dagger \epsilon^{ijk} - 8(Y_3^{ab} + Y_3^{ba}) u_a^i d_b^j T^k \epsilon_{ijk} + 4(Y_3^{ab} + Y_3^{ba}) e_a^c (u_b^c)_k T^k.$$

By integrating out the colored triplet the following dimension 6 effective Lagrangian is obtained,

$$\begin{aligned}\mathcal{L}^T_{d=6} \;\sim\; & \frac{1}{m_T^2}\, Y_1^{ab}\, (Y_3^{cd} + Y_3^{dc}) \Big\{ \nu_a d_b^i e_c^c (u_d^c)_i - e_a u_b^i e_c^c (u_d^c)_i - \mathbf{(d_a^c)_i (u_b^c)_j e_c^c (u_d^c)_k \epsilon^{ijk}} \\ & + \mathbf{2 e_a u_b^i u_c^j d_d^k \epsilon_{ijk}} - \mathbf{2 \nu_a d_b^i u_c^j d_d^k \epsilon_{ijk}} + 2 (d_a^c)_i (u_b^c)_j u_c^k d_d^l (\delta_k^i \delta_l^j - \delta_l^i \delta_k^j) \epsilon^{ijk} \Big\}. \end{aligned} \tag{II.84}$$

Therefore, the decay rate can be estimated qualitatively through the effective vertex of the interaction leading to proton decay (**bold ones**). The decay rate will be proportional to the feynman rule squared, so that

$$\Gamma_T \sim \big(Y_1 (Y_3 + Y_3^T)\big)^2\, \frac{m_p^5}{M_T^4}, \tag{II.85}$$

where, as before, the fifth power of the proton mass has been added to compensate dimensions.

In order to estimate the lower bound on the triplet mass induced by proton decay constraints, we will assume the Yukawa couplings to be of the order of ~ 0.01 GeV. Notice that the mass of the colored triplet is less constrained than the leptoquark gauge bosons mass, since the decay is proportional to the product of the two Yukawas which can be really small. Imposing again the most constraining bound from proton decay, $\tau(p \to \pi^0 e^+)$, we get

$$M_T \gtrsim 10^{13}\text{GeV}. \tag{II.86}$$

This result is also in agreement with the predictions coming from unification constraints.

III. MASSIVE NEUTRINOS

Probably one of the most known failures of the SM is the prediction of massless neutrinos. Even thought for long time it was thought that neutrinos had not mass at all (due to their actually pretty tiny mass), their massive nature has been proved through effects such as neutrino oscillations. Therefore, the SM needs to be extended in order to allow neutrinos to have mass. Different mechanisms has been introduced in literature to provide mass to neutrinos. Here we review the most common ones: the seesaw mechanism and radiative corrections.

Neutrino masses are not allowed in the SM due to mainly three reasons:

(a) The absence of a right handed neutrino.

(b) The scalar content of the SM is composed of a Higgs doublet.

(c) The renormalizability of the theory.

The condition (a) does not allow neutrinos to have Dirac masses in the SM. Reasons (b) and (c) do not allow the SM to have Majorana masses since they imply exact conservation of the lepton and baryon numbers. Therefore, massive neutrinos require to abandon one of those, i.e. require to go beyond the SM.

Let us start by abandoning reason (a), so that we add a right handed neutrino to our theory. In this case, neutrinos would get mass in an exactly analogous way as charged leptons do. After the symmetry breaking, the Dirac neutrino mass would read as

$$M_\nu = Y_\nu v \tag{III.1}$$

but it turns out that neutrino masses are a couple of orders of magnitude lower than the lightest charged lepton mass. Assuming that $M_\nu \sim 0.1$ eV, $Y_\nu \lesssim 10^{-12}$. This result does not look “nice” in the sense that there is no reason able to explain why the neutrino Yukawa is that small compared with the rest of the Yukawas. Apart from “aesthetic” issues, Dirac neutrinos are perfectly valid under a theoretical point of view and they are currently not ruled out (yet) [6].

On the other hand, neutrinos might be Majorana particles. There is an overall preference among the physicist community for Majorana neutrinos since they may explain the smallness of neutrino masses. We will contemplate the possibility of having Majorana neutrinos, so that the global symmetry B-L is broken. In the context of new physics, the SM can be regarded as an effective field theory of a higher energy fundamental theory. Assuming that there is new physics at a scale Λ (higher than the EW scale), it will manifest itself by non-renormalizable operators suppressed by powers of E/Λ where E refers to the energy scale of the interaction and Λ to the mass-scale of the new particles.

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{n=5}^{\infty} \sum_i \left(\frac{C_i^n}{\Lambda^{n-4}} \mathcal{O}_i^n + \text{h.c.} \right), \tag{III.2}$$

[6] Notice that we did not specify how this right-handed (RH) neutrino is introduced in the theory. If the RH-neutrino is introduced through a singlet, a Majorana mass term would automatically show up since it is allowed by the symmetries of the Lagrangian and therefore neutrinos would be Majorana. Nevertheless, if the RH-neutrino is introduced through some other representation such that the Majorana mass term is forbidden by the symmetries, one could enjoy Dirac neutrinos. For instance, the simplest left-right symmetric model predicts Dirac neutrinos, as we will see later.

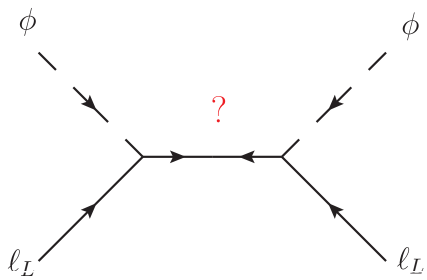


FIG. 5: Topology (a).

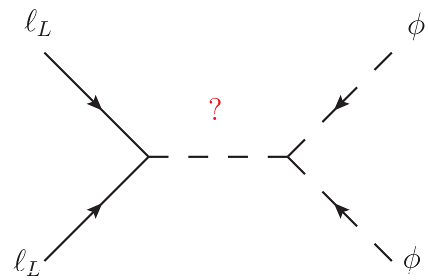


FIG. 6: Topology (b).

where C_i^n are the coupling constants. The heavier the new particles, the weaker their effect, so that the effect of new physics is dominated by the low dimensional operators. Therefore, we are interested in building the lowest dimensional operators which violate lepton number. It turns out that there is only one gauge and Lorenz invariant dimension 5 operator which does not conserve lepton number, and it is called the Weinberg operator, which is defined as,

$$\mathcal{O}_W^{d=5} = Y_{ab}^2 \frac{(\overline{\ell_{La}^C}\tilde{\phi}^*)(\tilde{\phi}^\dagger \ell_{Lb})}{\Lambda^2}, \tag{III.3}$$

where a and b are flavor indices. This operator will lead to a Majorana mass term for the neutrinos after the spontaneous symmetry breaking,

$$\mathcal{O}_W^{d=5} \overset{SSB}{\rightarrow} \frac{1}{2} Y_{ab}^2 \frac{v^2}{\Lambda} \overline{\nu_L^C} \nu_L \Rightarrow (M_\nu)_{\alpha\beta} = Y_{ab}^2 \frac{v^2}{\Lambda}. \tag{III.4}$$

To obtain a neutrino mass lower than 1 eV, one needs $\Lambda > 10^{13}$ GeV (assuming the coupling Y of the order of one), indicating that new physics at high scale is expected, which is a good motivation for GUTs.

There are only three possible tree-level realizations of the Weinberg operator using only renormalizable interactions. These are the so-called seesaw mechanisms. If we “zoom” in the effective vertex, there are only two possible topologies which will generate the Weinberg operator at low energies according to Lorenz invariance (see Fig. 5,6). Now, by further imposing to all possible renormalizable interactions to be consistent with the gauge symmetry of the SM, we are given by the following possibilities:

In the case of topology (a), we have

- $\ell_L \phi \sim (2, -1/2) \otimes (2, 1/2) = (3, 0) \oplus (1, 0) \Rightarrow$ Fermion triplet (3,0) and fermion singlet (1,0), both hyperchargeless.
- $\ell_L \phi^* \sim (2, -1/2) \otimes (2, -1/2) = (3, -1) \oplus (1, -1) \Rightarrow$ Fermion triplet (3,1) and fermion singlet (1,1), both with hypercharge $Y = 1$. We discard both possibilities since they have hypercharge so that they cannot have Majorana mass.

In the case of topology (b), we have

- $\ell_L \ell_L \sim (2, -1/2) \otimes (2, -1/2) = (3, -1) \oplus (1, -1) \Rightarrow$ Scalar triplet (3,1) and scalar singlet (1,1), both with hypercharge $Y = 1$. The scalar singlet is discarded since it does not have any neutral component and, hence, it cannot give mass at tree level. Moreover, it cannot couple with the scalar doublet ϕ.

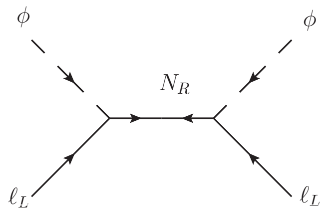


FIG. 7: Seesaw type-I.

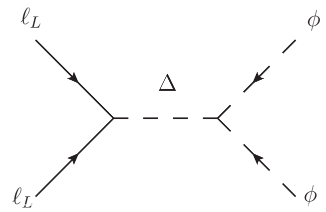


FIG. 8: Seesaw type-II.

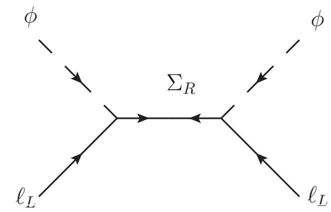


FIG. 9: Seesaw type-III.

Hence, we are left with three possibilities which we could add to the SM in order to generate Majorana masses for the neutrinos at tree level (see Figs. 7,8,9): a fermion singlet N_R (**type-I seesaw**), a scalar triplet Δ (**type-II seesaw**), a fermion triplet Σ_R (**type-III seesaw**) or any combination of those.

A. Type-I seesaw

Let us consider a sterile right handed neutrino $N_R \sim (1,1,0)$, i.e. a singlet under all SM gauge symmetries. The most general Lagrangian that can be written regarding this new fermion is given by (apart from the usual $\mathcal{L}_{SM}$)

$$\mathcal{L} \supset \overline{l_L} Y_e \phi e_R - \overline{l_L} Y_\nu \tilde{\phi} N_R + \frac{1}{2}\overline{N_R^C} N_R + \text{h.c.} \tag{III.5}$$

where $\Psi^C_{L/R} \equiv (\Psi_{L/R})^C$. Here, M_R is a n×n symmetric matrix and Y_ν a 3×n matrix, where n corresponds to the number of right-handed neutrino generations. After spontaneous symmetry breaking, the neutrino mass becomes a combination of a Dirac mass term and a Majorana one:

$$\mathcal{L}_{\text{mass}} = \underbrace{\overline{\nu_L} M_D N_R + \text{h.c}}_{\mathcal{L}_{\text{Dirac}}} + \underbrace{\frac{1}{2}\overline{N_R^C} M_R N_R + \text{h.c.}}_{\mathcal{L}_{\text{Majorana}}} \tag{III.6}$$

Notice that neither N_R nor ν_L are Majorana particles (even though we call M_R "Majorana mass") since they are 2-component spinors. However, one can bridge the gap with the familiar 4-component Dirac case and construct Majorana spinors as follows:

$$\nu \equiv \nu_L + (\nu_L)^C, \tag{III.7}$$
$$N \equiv N_R + (N_R)^C. \tag{III.8}$$

Here, one can see explicitly that $\nu^C = \nu$ and $N^C = N$, confirming their Majorana nature.

The above Lagrangian can be rewritten in terms of the new Majorana spinors by taking into account that $P_L(\nu) = \nu_L, P_R(\nu) = \nu_L^C$ and $P_L(N) = N_R^C, P_R(N) = N_R$,

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}(M_D \bar{\nu} N + M_D^T \bar{N} \nu) + \frac{1}{2} M_R \bar{N} N, \tag{III.9}$$

where the identity $M_D \bar{\nu} N = M_D^T \bar{N^C} \nu^C = M_D^T \bar{N} \nu$ has been used. Here, $M_D \equiv Y_\nu/\sqrt{2}$. Hence, one ends up with the following mass-matrix in the basis (ν, N):

$$\mathcal{L} = \frac{1}{2}\begin{pmatrix}\bar{\nu} & \bar{N}\end{pmatrix}\begin{pmatrix}0 & M_D \\ M_D^T & M_R\end{pmatrix}\begin{pmatrix}\nu \\ N\end{pmatrix}. \tag{III.10}$$

In the limit $M_R \gg M_D$ the above matrix can be diagonalized (block-diagonalized where more than one family of neutrinos is considered) by using the unitary transformation defined as $U = \begin{pmatrix}1 & \rho \\ -\rho^\dagger & 1\end{pmatrix}$, where $\rho^\dagger = M_R^{-1}M_D^T$.

The eigenvalues are given by,

$$M = U\begin{pmatrix}M_1 & 0 \\ 0 & M_2\end{pmatrix}U^\dagger \Rightarrow \begin{cases} M_1^\dagger \simeq -M_D M_R^{-1} M_D^T \\ M_2 \simeq M_R \end{cases} \tag{III.11}$$

and the corresponding eigenstates read as

$$\chi = \begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix} = U^{-1}\begin{pmatrix}\nu \\ N\end{pmatrix} = U^\dagger\begin{pmatrix}\nu \\ N\end{pmatrix} = \begin{pmatrix}\nu - \rho N \\ \rho^\dagger \nu + N\end{pmatrix}. \tag{III.12}$$

From a mass scale point of view, one can see that for a m_D fixed (we follow the notation: m -small letter-refers to the scale of the mass matrix), $M_1 \propto 1/m_R$ and $M_2 \propto m_R$. Therefore, the heavier M_2, the lighter M_1. This effect justifies the name of "seesaw" (see Fig. 10). The matrix ρ can be assumed to be

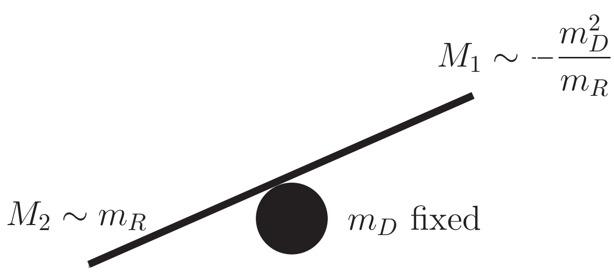


FIG. 10: Seesaw mechanism: the heavier one side, the lighter the other side.

proportional to the scale $\rho \sim m_D^T/m_R$. In the limit $m_R \gg m_D$, ρ is very suppressed so that the eigenstates in Eq. (III.12) can be approximated by $\chi_1 \sim \nu$ and $\chi_2 \sim N$, and thus the physical masses are given by $M_1 \sim M_\nu$ and $M_2 \sim M_N$.

From Eq. (III.11) one could guess the scale of the new RH-neutrino. By assuming $m_\nu \sim 0.1$ eV and $m_D \sim v$, we have that

$$\frac{(10^2)^2}{m_R} \lesssim 10^{-10} \Rightarrow m_R \gtrsim 10^{14}\text{GeV}, \tag{III.13}$$

as already predicted by the Weinberg operator.

Due to the fact that the new particle introduced is sterile and pretty heavy, it is hard to check the viability of the mechanism. There is no interesting phenomenology to look at but the fact that neutrinos are predicted to be Majorana. Therefore, one can hope to test the Majorana nature of the neutrinos through double-beta decay experiments. If neutrinos were found to be Dirac, the seesaw mechanism would be automatically ruled out.

Realization of type-I seesaw in SU(5)

In the context of SU(5), type-I seesaw can be easily realized by adding to the field content three singlets which will play the role of right-handed neutrinos, completely sterile [21]. However, the addition of singlets to the minimal theory does not help to achieve unification since singlets do not contribute in the running of the couplings (a complete representation does not contribute to the beta-functions unless the fields which reside there have different mass scales), so that this minimal extension (SU(5)-GG plus three singlets) is ruled out. Moreover, the addition of these singlets implies three unknown mass scales which are in principle not constrained by any phenomenological bound or unification requirement. The arbitrariness of the new scales makes this alternative not specially attractive. Apart of the addition of these singlets, to make the GUT realistic one needs to further add other representations in order to satisfy the unification constraints (the relation between the charged leptons and the down-type quarks can be corrected either introducing suitable extra representations or by non-renormalizable interactions).

B. Type-II seesaw

The simplest seesaw type-II realization consists on adding a triplet scalar field $\Delta \sim (1,3,1)$ under the gauge symmetry of the SM [22]. The representation of the triplet is given by

$$\Delta = \frac{\sigma^i}{\sqrt{2}}\Delta_i = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \tag{III.14}$$

Notice that the triplet corresponds to the adjoint representation of SU(2) so that it transforms as a tensor with two indices (2×2 matrix) with the restriction of being traceless, i.e. a total of 3 d.o.f ($\Delta_1 = (\delta^{++} + \delta^0)/\sqrt{2}, \Delta_2 = i(\delta^{++} - \delta^0)/\sqrt{2}, \Delta_3 = \delta^+$). The Lagrangian of the SM is modified not only in its Yukawa and kinetic parts (as in the type-I seesaw) but also in the scalar potential since now we are introducing a scalar which also couples to the SM Higgs:

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_Y - V(H,\Delta), \tag{III.15}$$

where

$$\mathcal{L}_{\text{kinetic}} = \mathcal{L}^{SM}_{\text{kinetic}} + \text{Tr}\left[(D_\mu\Delta)^\dagger(D^\mu\Delta)\right], \tag{III.16}$$

$$\text{and } \mathcal{L}_Y = \mathcal{L}^{SM}_Y - Y_\Delta \overline{\ell^C}\epsilon_2\Delta\ell + \text{h.c.} \tag{III.17}$$

The covariant derivative for the adjoint representation is defined as (see appendix),

$$D_\mu\Delta = \partial_\mu\Delta + i\frac{g}{2}[\sigma^a W^a_\mu, \Delta] + i\frac{g^{'}}{2}B_\mu\Delta, \tag{III.18}$$

where $a = 1,2,3$. The most general scalar potential that one can write with a Higgs doublet H and a Higgs triplet Δ reads as,

$$\begin{aligned} V(H,\Delta) = & -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 \text{Tr}\{\Delta^\dagger\Delta\} + \left(\mu H^T i\sigma_2 \Delta^\dagger H + \text{h.c.}\right) + \lambda_1 (H^\dagger H)\text{Tr}\{\Delta^\dagger\Delta\} \\ & + \lambda_2\left(\text{Tr}\{\Delta^\dagger\Delta\right)^2 + \lambda_4 H^\dagger\Delta\Delta^\dagger H. \end{aligned} \tag{III.19}$$

The neutral component of the Higgs doublet breaks the symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ by getting a non-zero vev. But a non-zero vev for the Higgs doublet immediately implies a non-zero vev for the neutral component of the scalar triplet Δ, otherwise a tadpole would be induced. The spontaneous symmetry breaking is guaranteed by imposing $m_H > 0$ in the above potential.

Once the the neutral components get a vev, i.e. $\langle \delta^0 \rangle = v_\Delta/\sqrt{2}$ and $\langle H^0 \rangle = v_H/\sqrt{2}$, neutrinos get a Majorana mass (from the Yukawa Lagrangian):

$$M_\nu = v_\Delta Y_\Delta, \tag{III.20}$$

and, from the kinetic term in the Lagrangian, some of the gauge bosons get mass too:

$$M_W^2 = \frac{g^2}{2}(v_H^2 + 2v_\Delta^2), \quad M_Z^2 = \frac{g^2}{2\cos^2\theta_W}(v_H^2 + 4v_\Delta^2). \tag{III.21}$$

Notice that the explicit form of the covariant derivative for the adjoint representation (commutator involved) is the responsible of the factor of 2 in front of the vev of the triplet. This leads to the experimental constraint $v = \sqrt{v_H^2 + 2v_\Delta^2} = 246.2$ GeV (see reference [18]). Therefore, the ρ-parameter in this model is given by

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \Rightarrow \rho = \frac{1 + 2v_\Delta^2/v_H^2}{1 + 4v_\Delta^2/v_H^2}, \tag{III.22}$$

which is strongly constrained by the electroweak precision data which requires the ρ-parameter to be very close to the unity, i.e. $\rho = 1.00040$ (1.7 σ above the SM expectation $\rho_{SM} = 1$)[18]. This constraints the vevs of the doublet and triplet Higgs and thus requires that $v_H \gg v_\Delta$. Hence, we can assume that $v_H \sim v = 246.2$ GeV.

The minimum conditions of the potential read as,

$$\frac{\partial V}{\partial v} = -m_H^2 v + \frac{\lambda}{4}v^3 + \frac{1}{2}(\lambda_1 + \lambda_4)v v_\Delta^2 + \frac{2}{\sqrt{2}}\mu v v_\Delta \stackrel{!}{=} 0, \tag{III.23}$$

$$\frac{\partial V}{\partial v_\Delta} = M_\Delta^2 v_\Delta + \frac{1}{2}(\lambda_1 + \lambda_4)v^2 v_\Delta + \lambda_2 v_\Delta^3 + \frac{1}{\sqrt{2}}\mu v^2 \stackrel{!}{=} 0. \tag{III.24}$$

which can be rearranged in a more suitable way as

$$m_H^2 = \frac{\lambda}{4}v^2 + \frac{1}{2}(\lambda_1 + \lambda_4)v_\Delta^2 + \sqrt{2}\mu v_\Delta + \lambda_2 v_\Delta^3, \tag{III.25}$$

$$M_\Delta^2 = \frac{1}{\sqrt{2}}\mu\frac{v^2}{v_\Delta} - \frac{1}{2}(\lambda_1 + \lambda_4)v^2 - \lambda_2 v_\Delta^2. \tag{III.26}$$

In the limit where $v_\Delta \ll v$, from Eq. (III.26) the following expression is obtained,

$$v_\Delta = \frac{1}{\sqrt{2}}\frac{\mu v^2}{M_\Delta^2 + \frac{1}{2}(\lambda_1 + \lambda_4)v^2}. \tag{III.27}$$

For $M_\Delta \gg v$, the above equation simplifies in

$$M_\nu \simeq \left(\frac{\mu}{M_\Delta}\right)\frac{v^2}{\sqrt{2}M_\Delta}Y_\Delta. \tag{III.28}$$

Notice that the term in the parenthesis is dimensionless. In this relation one can see the seesaw effect taking place in the inverse proportional relation $M_\nu \propto M_\Delta^{-1}$ since v is fixed experimentally.

Realization of type-II seesaw in SU(5)

In the context of SU(5), the simplest way in which neutrinos can get mass through the seesaw type-II mechanism is by adding a 15_H representation in the scalar sector, since it contains a scalar triplet under SU(2) (Δ_T):

$$15_H = \underbrace{(1,3,1)}_{\Delta_T} \oplus \underbrace{(3,2,1/6)}_{\Delta_{3,2}} \oplus \underbrace{(6,1,-2/3)}_{\Delta_S} \tag{III.29}$$

The corresponding scalar potential is given by,

$$V_{\text{scalar}} = V(5_H) + V(15_H) + V(24_H) + V(5_H, 15_H) + V(5_H, 24_H) + V(15_H, 24_H), \tag{III.30}$$

where

- V(5_H)=$-\mu_5^2 5^\alpha 5^*_\alpha + \lambda_1 (5^\alpha 5^*_\alpha)^2$,
- V(15_H)=$-\mu_{15}^2 15^{\alpha\beta} 15^*_{\alpha\beta} + \lambda_2 (15^{\alpha\beta} 15^*_{\alpha\beta})^2 + \lambda_3 15^{\alpha\beta} 15^*_{\beta\gamma} 15^{\gamma\delta} 15^*_{\delta\alpha}$,
- V(24_H)=$-\mu_{24}^2 24^\alpha{}_\beta 24^\beta{}_\alpha + \lambda_4 (24^\alpha{}_\beta 24^\beta{}_\alpha)^2 + a_1 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\alpha + \lambda_5 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\delta 24^\delta{}_\alpha$,
- V(5_H,24_H)=$a_2 5^*_\alpha 24^\alpha{}_\beta 5^\beta + \lambda_6 5^*_\alpha 5^\alpha 24^\beta{}_\gamma 24^\gamma{}_\beta + \lambda_7 5^*_\alpha 24^\alpha{}_\beta 24^\beta{}_\gamma 5^\gamma$,
- V(24_H,15_H)=$\lambda_8 15^{\alpha\beta} 15^*_{\alpha\beta} 24^\gamma_\delta 24^\delta_\gamma + a_3 15^{\alpha\beta} 24^\gamma_\beta 15^*_{\gamma\alpha} + \lambda_9 15^{\alpha\beta} 24^\gamma{}_\beta 24^\delta_\gamma 15^*_{\delta\alpha} + \lambda_{10} 15^{\alpha\beta} 24^\gamma_\beta 15^*_{\gamma\delta} 24^\delta_\alpha$,
- V(5_H,15_H)=$\lambda_{11} 5^*_\alpha 5^\alpha 15^*_{\beta\gamma} 15^{\beta\gamma} + a_4 5^*_\alpha 5^*_\beta 15^{\alpha\beta} + a_4^* 5^\alpha 5^\beta 15^*_{\alpha\beta} + \lambda_{12} 5^*_\alpha 15^{\alpha\beta} 15^*_{\beta\gamma} 5^\gamma$.

The Yukawa interactions of this SU(5) extension read as,

$$-\mathcal{L}_Y = Y_1\, \bar{5}\; 10\; 15_H + Y_2\; 10\; 10\; 5_H \epsilon_5 + Y_\nu\, \bar{5}\; \bar{5}\; 15_H, \tag{III.31}$$

where ϵ_5 refers to the Levi-Civita tensor in 5 dimensions.

The relevant interactions for the type-II seesaw mechanism are thus given by

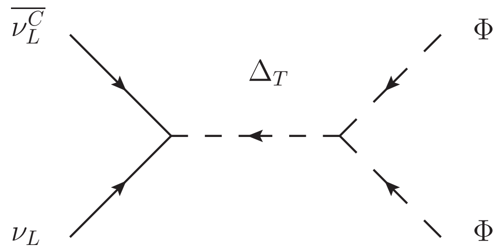
$$\mathcal{L}_{\text{seesaw}} = -M^2_{\Delta_T} \text{Tr}\{\Delta_T^\dagger \Delta_T\} - Y_\nu \ell_L^T C \Delta_T \ell_L + a_4 \Phi^T \Delta_T^\dagger \Phi + \text{h.c.} \tag{III.32}$$

Here,

$$\Phi \sim 5^\alpha_H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \text{ and } \Delta_T \sim 15^{\alpha\beta}_H = \begin{pmatrix} \Delta^{++} & -\Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & \Delta^0 \end{pmatrix} \tag{III.33}$$

where α and β are SU(2) indices, i.e. $\alpha, \beta = 4, 5$ (notice that in SU(5) $\ell_L \sim \begin{pmatrix} e \\ -\nu \end{pmatrix}$ since $\ell_L \supset \bar{5}^\alpha$).

Once the triplet under SU(2) contained in 15_H acquires a vev, the SM symmetry is spontaneously broken so that neutrinos get a Majorana mass through the following interaction:



$$\Rightarrow M_\nu \sim a_4\, Y_\nu \frac{\langle\Phi\rangle^2}{M_{\Delta_T}^2}. \tag{III.34}$$

The addition of the 15_H is an attractive extension of the minimal SU(5) theory since it has enough power to recover unification, as it is studied in detail in Refs. [14, 23, 24]. However, in contrast to the 45_H representation, the relation between the charged lepton and down-type quark masses cannot be corrected only by 15_H. Nevertheless, non-renormalizable interactions can solve this problem.

C. Type-III seesaw

The simplest version of the seesaw type-III [25] consists in adding a hyperchargeless triplet of fermions (ρ) to the SM for each fermion family.

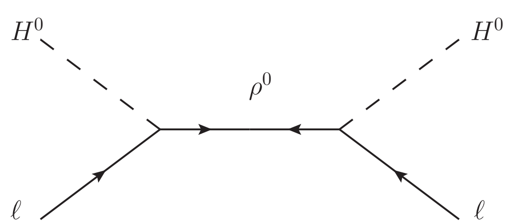
$$\rho = \frac{1}{2}\begin{pmatrix} \rho^0/\sqrt{2} & \rho^+ \\ \rho^- & -\rho^0/\sqrt{2} \end{pmatrix} \sim (3, 1, 0). \tag{III.35}$$

One always have to put special attention to the addition of new fermions since they may spoil the hopeful anomaly cancellation taking place in the SM. In this case, there is nothing to worry about since adjoint representations do not influence into anomalies (see Table II).

The most general renormalizable Lagrangian that can be written by taking into account this new triplet of fermions is given by,

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\text{Beyond}}, \text{ where } \mathcal{L}_{\text{Beyond}} = \text{Tr}\{\bar{\rho} i \not{D} \rho\} - \frac{1}{2} M_\rho \text{Tr}\{\overline{\rho^C}\rho\} - \sqrt{2} Y_\rho \bar{\ell}\rho\tilde{\phi} + \text{h.c.} \tag{III.36}$$

where $\tilde{\phi}$ refers to the charge conjugate of the SM Higgs field. The first term in $\mathcal{L}_{\text{Beyond}}$ is the kinetic term which mixes the new fermions with the gauge bosons. Notice that for the new fermions one has explicitly in the Lagrangian a Majorana mass term, due to the fact that they are hyperchargeless (otherwise it would violate $U(1)_Y$) and are triplets under $SU(2)_L$ which is the adjoint representation, i.e. they transform as $\Sigma \to U^\dagger \Sigma U$. The Yukawa interaction is the responsible to give a Majorana mass to the neutrinos after the spontaneous symmetry breaking (SSB) through the following interaction:



$$\to m_\nu = Y_\rho^T \frac{1}{M_\rho} Y_\rho v^2. \tag{III.37}$$

Hence, here one finds again the same structure than in type-I seesaw, where the mass of the neutrino is inversely proportional to the mass of the new fermion inserted (Here the ρ^0 plays the role of the right-handed neutrino in the seesaw type-I).

Moreover this seesaw is very characteristic for its rich phenomenology. The new fermions will interact with the gauge bosons through the kinetic term in the Lagrangian $\overline{\rho^{\pm}}\rho^{\pm}Z$ and $\overline{\rho^0}\rho^{\pm}W^{\mp}$, and will mix with the ordinary charged leptons of the SM through the new Yukawa interactions. It is remarkable that this mixing is much easier to see than the mixing of neutral neutrinos induced in seesaw type-I.

Realization of type-III seesaw in SU(5)

A simple and realistic extension of the minimal SU(5)-GG model can be obtained by adding to the usual particle content a 24 fermion representation (notice that the addition of new fermion content in the adjoint representation does not spoil chiral anomaly cancellations). The minimal type-III mechanism The triplet contained in the 24 representation can generate neutrino masses through the type-III seesaw. It is remarkable that, since a fermion singlet is also included in 24, type-I seesaw is obtained as a bonus. This model predicts, as the minimal SU(5) does, equal masses for the up type quarks and charged lepton masses. In order to correct this relation, higher dimensional operators must be included, loosing then the renormalizability of the theory.

The new Yukawa interactions read as,

$$\mathcal{L}_Y \supset y_0^i \bar{5}_F^i 24_F 5_H + \frac{1}{\Lambda}\bar{5}_F^i \left[y_1^i 24_F 24_H + y_2^i 24_H 24_F + y_3^i \mathrm{Tr}(24_F 24_H) \right] + O(\Lambda^{-2}) \tag{III.38}$$

where the index i accounts for the number of generations of ordinary fermions (i=1,2,3 according to experimental evidence). After the SU(5) breaking to the SM, i.e $\langle 24_H \rangle \neq 0$, the relevant Yukawa interactions responsible for the neutrino masses are given by,

$$\mathcal{L}^{\text{seesaw-III}} \supset L_i(y_T^i T_F + y_S^i S_F)H - \frac{m_S}{2} S_F S_F - \frac{m_T}{2} T_F T_F + \text{h.c.} \tag{III.39}$$

where y_T and y_S are linear combinations of the y_0^i and $y_a^i v_{24}/\Lambda\,(a = 1, 2, 3)$ above and the mass term of the fermion singlet S_F and the triplet T_F come from the term $m_F \mathrm{Tr}\{24_F^2\}$ in the scalar potential. The mass therefore is given by

$$m_\nu^{ij} = v^2 \left(\frac{y_T^i y_T^j}{m_T} + \frac{y_S^i y_S^j}{m_S} \right). \tag{III.40}$$

Since only one generation of new fermions is introduced, the theory predicts one massless neutrino. This can be easily understood by noticing that the Yukawas in this case are simply vectors (here, only one generation of new fermions is added, so that there is only one family index involved in the Yukawas). The triplet and singlet Yukawas define a plane in the 3-dim family space. One can rotate this plane in Eq. (III.39) such that one of the neutrinos does not couple to both the triplet and singlet new fermions. Therefore, it is straightforward to see that, in Eq. (III.40), one of the eigenvalues of the neutrino mass matrix is zero (the determinant of a matrix is an invariant so that it does not depend on the basis chosen).

About the masses of the extra fermion content, they are highly constrained by the unification constraints. The contribution of the fields from the 24 fermion representation to unification is shown in Table (III). As one can see from the beta functions above, the fermion triplet in 24, T_F, is the only field which helps to increase the unification ratio, i.e. helps to achieve unification. Since, as we have already shown, the particle

TABLE III: Contributions to the B_{ij} coefficients from the 24_F.

	24_F				
B_{ij}	T_F	S_F	O_F	$\Sigma_{(3,2)F}$	$\Sigma_{(\bar{3},2)F}$
B_{12}	$-\frac{4}{3}r_{T_F}$	0	0	$\frac{2}{3}r_{\Sigma_{(3,2)F}}$	$\frac{2}{3}r_{\Sigma_{(\bar{3},2)F}}$
B_{23}	$\frac{4}{3}r_{T_F}$	0	$-2r_{O_F}$	$\frac{1}{3}r_{\Sigma_{(3,2)F}}$	$\frac{1}{3}r_{\Sigma_{(\bar{3},2)F}}$
Does it help?	✓	×	×	×	×
Proton decay	×	×	×	✓	✓

content of SU(5)-GG is not enough to ensure unification of the couplings, a splitting in the 24_F needs to be introduced. The unification constraints set therefore an upper bound to the mass of the fermion triplet (for the Planck scale cutoff):

$$m_T \leq 10^{2.1}\text{TeV}. \tag{III.41}$$

A detailed study on bounds coming from unification constraints and proton decay in the 24_F can be found in reference [27]. The prediction of a triplet of extra fermions below TeV is remarkable in the sense that it is likely to be found at LHC, which shows the predictive power of this SU(5) extension.

D. Zee mechanism

The Zee model is a mechanism proposed by Antony Zee in 1980 [8] that gives mass to neutrinos through first order radiative corrections. In the Zee mechanism for neutrino masses two extra Higgses are needed to generate neutrino masses at one-loop level: a charged scalar singlet under SU(2) and a second Higgs doublet (the standard model one is already counted). Fig. 18 shows the topology of the process in the unbroken phase. We are using the following notation: $H_a \sim (1, 2, -1/2)$ where $a = 1, 2$ and $\delta^+ \sim (1, 1, 1)$. The

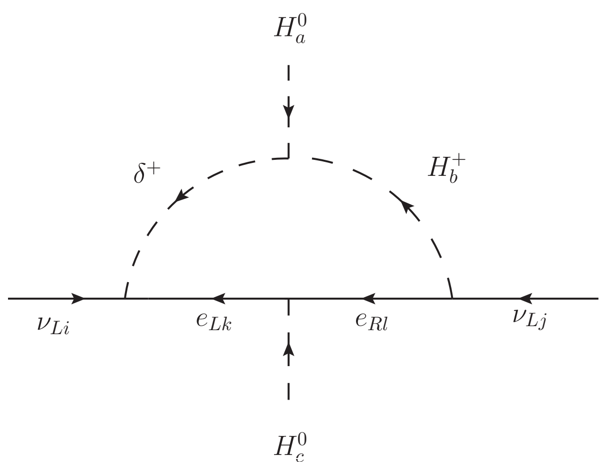


FIG. 11: Zee model generating 1-loop radiative mass to neutrinos in the unbroken phase [8].

relevant interactions are given by (assuming a general effective theory with the usual SM field content plus the extra charged singlet mentioned above),

$$V_{Zee} \supset l_L \lambda l_L \delta^+ + \bar{l}_L Y_a H_a e_R + \mu H_a H_b \delta^- + \text{h.c.} \tag{III.42}$$

where $\ell_L \sim (1,2,-1/2)$, $e_R \sim (1,1,-1)$, λ is an antisymmetric matrix in the flavor space (due to Fermi statistics), and Y_a are the Yukawa matrices for the two Higgses present in the theory. Notice that here the global B-L symmetry is broken due to the simultaneous presence of the Yukawa interaction proportional to λ and the scalar potential term proportional to μ. Non of these terms alone would break the symmetry since the B-L number of the new charged singlet δ could be in principle defined such that the $U(1)_{B-L}$ symmetry holds. Nevertheless the co-presence of the other term forces this symmetry to break.

Explicitly in the SU(2) space, the above interactions read as

$$V_{Zee} \supset \overline{l_L}(Y_1 H_1 + Y_2 H_2)e_R + \lambda\, \overline{l_L^C} i\sigma_2 l_L \delta^+ + \mu\, H_1 i\sigma_2 H_2 \delta^- + \text{h.c.} \tag{III.43}$$

Although one could compute the mass correction coming from the loop shown in Fig. 18 in the unbroken phase by using the dimension five Weinberg operator in an effective theory context, since we do not know how heavy the Higgses running inside the loop are, we are computing the mass assuming the symmetry is already broken (broken phase). Indeed, the heavier are the Higgses, the better is the approximation in the unbroken phase. However the computation in the broken phase gives us the exact expression for the radiative mass.

After the symmetry breaking, the doublets read as

$$H_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(H_1^0 + v_1 + iA_1^0) \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H_2^0 + v_2 + iA_2^0) \end{pmatrix},$$

where H_1^0 and H_2^0 correspond to the CP-even Higgses, and A_1^0 and A_2^0 to the CP-odd ones. Expanding the terms of equation (III.43) as a function of the fields contained in the doublets,

$$\begin{aligned} V_{Zee} \supset\ & \overline{\nu_L}(Y_1 H_1^+ + Y_2 H_2^+)e_R + \frac{1}{\sqrt{2}}\overline{e_L}\left(Y_1[H_1^0 + v_1 + iA_1^0] + Y_2[H_2^0 + v_2 + iA_2^0]\right)e_R + 2\lambda\, \overline{\nu_L^c} e_L \delta^+ + \\ & + \mu\left\{H_1^+(H_2^0 + v_2 + iA_2^0) - (H_1^0 + v_1 + iA_1^0)H_2^+\right\}\delta^- \end{aligned} \tag{III.44}$$

But these Higgses are not physical since they do not have a diagonal mass; their masses are mixed by the scalar potential terms listed below, which is the most general renormalizable potential that can be written as a function of the mentioned fields,

$$V(H_1, H_2, \delta^+) = V(H_1) + V(H_2) + V(\delta^+) + V(H_1, H_2) + V(H_1, H_2, \delta^+), \tag{III.45}$$

where

- $V(H_1)$=$m_1^2 H_1^\dagger H_1 + \lambda_1 (H_1^\dagger H_1)^2$,
- $V(H_2)$=$m_2^2 H_2^\dagger H_2 + \lambda_2 (H_2^\dagger H_1)^2$,
- $V(\delta^+)$=$m_\delta^2 \delta^* \delta + \lambda_\delta (\delta^* \delta)^2$,
- $V(H_1, H_2)$=$m_{12}^2(H_1^\dagger H_2 + \text{h.c.}) + a_1 (H_1^\dagger H_1)(H_2^\dagger H_2) + a_2 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{a_3}{2}\Big((H_1^\dagger H_2)^2 + \text{h.c.}\Big)$,
- $V(H_1, \delta)$=$b_1 H_1^\dagger H_1 \delta^* \delta$,
- $V(H_2, \delta)$=$b_2 H_2^\dagger H_2 \delta^* \delta$,
- $V(H_1, H_2, \delta) = b_3(H_1^\dagger H_2 + \text{h.c.}) + \mu H_1^\dagger \mathrm{i}\sigma_2 H_2^* \delta^* + \text{h.c.}$

which minimum conditions are,

$$\frac{\partial \langle V \rangle}{\partial v_1} = m_1^2 v_1 + \lambda_1 v_1^3 + m_{12}^2 v_2 + \frac{1}{2}(a_1 + a_2 + a_3) v_1 v_2^2 \stackrel{!}{=} 0, \tag{III.46}$$

$$\frac{\partial \langle V \rangle}{\partial v_2} = m_{12}^2 v_1 + m_2^2 v_2 + \frac{1}{2}(a_1 + a_2 + a_3) v_1^2 v_2 + \lambda_2 v_2^3 \stackrel{!}{=} 0. \tag{III.47}$$

The mass matrix for the charged Higgs doublets is given by,

$$\begin{pmatrix} H_1^- \\ \\ H_2^- \end{pmatrix}^T \begin{pmatrix} m_1^2 + \lambda_1 v_1^2 + \frac{a_1}{2} v_2^2 & m_{12}^2 + a_3 v_1 v_2 + \frac{a_2}{2} v_1 v_2 \\ \\ m_{12}^2 + a_3 v_1 v_2 + \frac{a_2}{2} v_1 v_2 & m_2^2 + \lambda_2 v_2^2 + \frac{a_1}{2} v_1^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ \\ H_2^+ \end{pmatrix}, \tag{III.48}$$

which, when applying the minimum conditions (III.46) and (III.47), can be rewritten as

$$\begin{pmatrix} H_1^- \\ \\ H_2^- \end{pmatrix}^T \begin{pmatrix} -\frac{v_2}{v_1} A & A \\ A & -\frac{v_1}{v_2} A \end{pmatrix} \begin{pmatrix} H_1^+ \\ \\ H_2^+ \end{pmatrix}, \tag{III.49}$$

where $A \equiv m_{12}^2 + \left(\frac{a_2}{2} + a_3\right) v_1 v_2$. The eigensystem of the mass matrix is thus given by,

$$m_{G^\pm}^2 = 0, \quad G^\pm = (\sin\beta H_1^\pm + \cos\beta H_2^\pm)^T, \tag{III.50}$$

$$m_{H^\pm} = -\frac{2A}{\sin 2\beta}, \quad H^\pm = (\cos\beta H_1^\pm - \sin\beta H_2^\pm)^T. \tag{III.51}$$

The mixing angle β is defined as $\tan\beta = v_1/v_2$ by the diagonalization of the mass matrix for the H_1^+ and H_2^+. Here v_1 and v_2 are the vacuum expectation values of the neutral components of the Higgses H_1 and H_2 respectively, satisfying the relation $v_1^2 + v_2^2 = v^2$, where $v = 246$ GeV. In the above equation $G^\pm$ are the Goldstone bosons which will be eaten up by the gauge bosons in the unitary gauge.

It turns out that the rotation matrix that rotates the CP-odd Higgses to the physical basis is the same that diagonalizes the charged Higgses above:

$$\begin{pmatrix} A_1^0 \\ A_2^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} A^0 \\ G^0 \end{pmatrix} \tag{III.52}$$

The diagonalization of the neutral Higgses H_1^0 and H_2^0 gives rise to two physical Higgses: one heavy, H, and one light, h:

$$\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \tag{III.53}$$

Hence, we are left with three Goldstones which we can get rid off them by going to the unitary gauge. By applying the above rotations one can express the relevant interactions in (III.43) as a function of the new fields,

$$\begin{aligned}V_{Zee} \supset \quad & \overline{\nu_L}(Y_1\cos\beta - Y_2\sin\beta)H^+ e_R + \frac{1}{\sqrt{2}} i\overline{e_L}(Y_1\cos\beta - Y_2\sin\beta)A^0\, e_R + \bar{e_L}M_e\, e_R + \\ & + \frac{1}{\sqrt{2}}\overline{e_L}(Y_1\cos\alpha - Y_2\sin\alpha)He_R + \frac{1}{\sqrt{2}}\overline{e_L}(Y_1\sin\alpha + Y_2\cos\alpha)h\, e_R + \text{Goldstone terms} + \\ & + 2\,\lambda\,\overline{\nu_L^c}\, e_L\,\delta^+ + \mu\, v\, H^+\delta^- + \mu\cos\beta\left(\frac{v_1}{v_2}v_2 - v_1\right)\delta^- G^+ + \text{higher order int.,} \qquad \text{(III.54)}\end{aligned}$$

where $M_e = \frac{1}{\sqrt{2}}v(Y_1\cos\beta + Y_2\sin\beta)$ is the mass of the electron. In the above equation only the second order interactions are written explicitly since are the ones of our interest. Notice that the new fields written above are not all physical yet. We still expect mixing between the two new charged Higgses, $G^\pm$ and $H^\pm$, with the charged singlet $\delta^\pm$, which we did not consider yet. However, as the above expression shows, the charged Goldston, G^+, decouples from the δ^+ so that only the mixing between δ^+ and H^+ has to be considered. Defining θ_+ as the mixing between the H^+ and the singlet δ^+, these fields can be written as a linear combination of the now physical charged Higgses $h_1^\pm$ and $h_2^\pm$ in the following way:

$$\delta^\pm = \cos\theta_+ h_1^\pm + \sin\theta_+ h_2^\pm, \qquad \text{(III.55)}$$

$$H^\pm = -\sin\theta_+ h_1^\pm + \cos\theta_+ h_2^\pm. \qquad \text{(III.56)}$$

Thus, we can finally write the potential as a function of the physical Higgses, which in the unitary gauge reads as

$$\begin{aligned}\mathcal{L} \supset \quad & \bar{\nu_L}\;(Y_1\cos\beta - Y_2\sin\beta)(\cos\theta_+\; h_2^+ - \sin\theta_+\; h_1^+)\; e_R + \frac{1}{\sqrt{2}}\;\bar{e_L}\;(Y_1\cos\alpha - Y_2\sin\alpha)\; H\; e_R + \\ & + \frac{1}{\sqrt{2}}\;\bar{e_L}\;(Y_1\sin\alpha + Y_2\cos\alpha)\; h\; e_R + \frac{i}{\sqrt{2}}\;\bar{e_L}\;(Y_1\cos\beta - Y_2\sin\beta)\; A^0\; e_R + \\ & + 2\,\lambda\,\bar{\nu_L^c}\;(\cos\theta_+\; h_1^+ + \sin\theta_+\; h_2^+)\; e_L. \qquad \text{(III.57)}\end{aligned}$$

Fig. 12 shows the topology of the Zee mechanism in the unbroken phase. Two different diagrams are contributing to the 1-loop radiative correction to the neutrino masses (h_1 and h_2). The Feynman rules of the

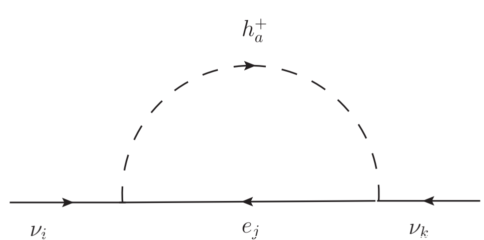
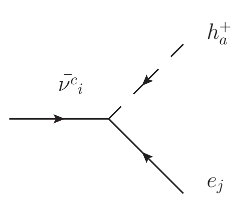
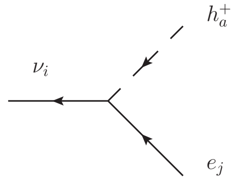


FIG. 12: Zee model generating 1-loop radiative mass to neutrinos in the broken phase, where $a = 1, 2$.

interactions taking place in the Zee-mechanism are:

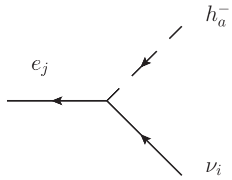


$$: \Gamma_a = \begin{cases} a = 1 : \lambda_{ij}\Gamma_1, \text{ where } \Gamma_1 = 2i\cos\theta_+ P_L, \\ a = 2 : \lambda_{ij}\Gamma_2, \text{ where } \Gamma_2 = 2i\sin\theta_+ P_L, \end{cases} \qquad \text{(III.58)}$$



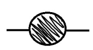
$$: \tilde{\Gamma_a} = \begin{cases} a = 1 : -(Y_{1ij}\cos\beta - Y_{2ij}\sin\beta)\sin\theta_+ P_R, \\ a = 2 : \ \ (Y_{1ij}\cos\beta - Y_{2ij}\sin\beta)\cos\theta_+ P_R, \end{cases} \tag{III.59}$$

$\Rightarrow$



$$: \tilde{\Gamma_a}^{\dagger}. \tag{III.60}$$

Therefore, the 1-loop correction is given by,



$$\begin{aligned} &= \int \frac{d^4k}{(2\pi)^4} \lambda^{ij}\Gamma_a P_L \frac{\not{k} + m_{e_j}}{k^2 - m_{e_j}^2} P_L \tilde{\Gamma}_a^{\dagger jk} \frac{1}{k^2 - m_{h_a}^2} \\ &= \lambda^{ij}\Gamma_a P_L m_{e_j} \tilde{\Gamma}_a^{\dagger jk} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{h_a}^2} \frac{1}{k^2 - m_{e_j}^2}. \end{aligned} \tag{III.61}$$

The above integral can be computed through the dimensional regularization method by introducing the so-called Feynman parameters, so that

$$\begin{aligned} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_a^2} \frac{1}{k^2 - m_b^2} &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \, \frac{1}{[x(k^2 - m_a^2) + (1-x)(k^2 - m_b^2)]^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \, \frac{1}{(k^2 - \Delta)^2}, \end{aligned} \tag{III.62}$$

where $\Delta \equiv x\ m_a^2 + (1-x)m_b^2$. By Wick-rotating the limits of integration and moving to the Euclidean space, momenta can be integrated assuming d dimensions (see appendix for more details) so that,

$$\int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \, \frac{1}{(k^2 - \Delta)^2} = \frac{i}{(4\pi)^2} \int_0^1 dx \, \left(\frac{2}{\epsilon} - \text{Log}(x\ m_a^2 + (1-x)m_b^2) - \gamma + \text{Log}(4\pi) + O(\epsilon) \right) \mu_R^{-\epsilon}, \tag{III.63}$$

where μ_R refers to the renormalization scale. By integrating the feynman parameter,

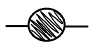
$$\int_0^1 \text{Log}\left(x\ m_a^2 + (1-x)m_b^2\right) dx = 1 + \text{Log}(m_a^2) + \frac{m_b^2 \text{Log}\left(\frac{m_a^2}{m_b^2}\right)}{m_a^2 - m_b^2}, \tag{III.64}$$

the integral in Eq. (III.63) reads,

$$= \frac{i}{(4\pi)^2} \left(\frac{2}{\epsilon} - 1 - \gamma + \text{Log}\left(\frac{4\pi\mu_R^2}{m_a^2}\right) + \frac{m_b^2 \text{Log}\left(\frac{m_a^2}{m_b^2}\right)}{m_b^2 - m_a^2} \right) \mu_R^{-\epsilon}, \tag{III.65}$$

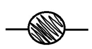
which has been computed by using dimensional regularization. Here, ϵ comes from performing the integral in $d = 4 - \epsilon$ dimensions. Notice that in the limit $d \to 4$, $\epsilon \to 0$ and the above integral diverges. The implementation of the dimensional regularization is nothing else than a mathematical trick to parametrize the divergences. But divergences are unphysical (we do not observe them in the physical measurements) so that one expects they do not show up in the final result. Explicitly, by adding the 1-loop correction of each of the two diagrams contributing to the radiative mass one can see how divergences do cancel, as expected (notice the imaginary unit can be absorbed by the free Majorana phase of the neutrinos).

The total radiative correction reads as



$$= \frac{1}{16\pi^2}\lambda^{ij}\mu_R^{-\epsilon}P_L m_{e_j}\left(\Gamma_1\tilde{\Gamma}_1^{\dagger ij}\left[\dots + \frac{m_{h_1}^2\,\mathrm{Log}\left(\frac{m_{e_j}^2}{m_{h_1}^2}\right)}{m_{h_1}^2 - m_{e_j}^2} + \dots\right] + \Gamma_2\tilde{\Gamma_2}^{\dagger jk}\left[\dots + \frac{m_{h_2}^2\,\mathrm{Log}\left(\frac{m_{e_j}^2}{m_{h_2}^2}\right)}{m_{h_2}^2 - m_{e_j}^2} + \dots\right]\right). \tag{III.66}$$

Notice that the renormalizable scale "corrects" the dimensions of the coupling λ in the sense that $\tilde{\lambda} = \lambda\mu_R^{-\epsilon}$ is dimensionless at any dimension d. Here, we are interested in the limit $d \to 4$, i.e. $\epsilon \to 0$, and thus the renormalization scale μ_R disappear as one would expect since it is completely unphysical. By assuming $m_{e_j}^2 \ll m_{h_a}^2$,



$$= \frac{1}{16\pi^2}\lambda^{ij}m_{e_j}(Y_1^{\dagger jk}\cos\beta - Y_2^{\dagger jk}\sin\beta)\left[\frac{\mathrm{Log}\left(\frac{m_{h_2}^2}{m_{h_1}^2}\right)}{16\pi^2}\sin(2\theta_+)\right]P_L. \tag{III.67}$$

Therefore, the Majorana mass matrix for the neutrinos reads as (notice the factor of two coming from the fact that the mass is Majorana),

$$M_\nu \;=\; \frac{1}{8\pi^2}\left(\lambda M_e(Y_1^\dagger\cos\beta - Y_2^\dagger\sin\beta) + (Y_1^*\cos\beta - Y_2^*\sin\beta)\,M_e^T\lambda^T\right)\sin 2\theta_+\mathrm{Log}\left(\frac{\mathrm{m}_{\mathrm{h}_2^+}^2}{\mathrm{m}_{\mathrm{h}_1^+}^2}\right). \tag{III.68}$$

Notice that the mixing angle θ_+ is proportional to the μ parameter. Hence, the neutrino mass matrix, according to the above expression, is also proportional to μ, which as we have already discussed, is responsible for the breaking of the global B-L symmetry. When $\mu \to 0$, the B-L symmetry is recovered. Then, loop corrections to this parameter must be proportional to μ itself, which means that, if μ is small, loop corrections to μ cannot be large. In this sense, it is said that the μ parameter is protected by the symmetry. Notice that the Zee-Wolfenstein model [8, 28] requires only one Higgs coupled with the fermions, which leads to a resulting mass matrix with zero diagonal entries. This particular model has been ruled out by the experiments [29, 30]. However, in the general scenario for the Zee mechanism where two different Higgs doublets with different couplings to the fermions are involved, one has enough freedom to reproduce the values for neutrino mixings and masses. See for instance [31] for a recent study of the Zee model.

IV. REALISTIC RENORMALIZABLE SU(5) MODEL

From the first section of this report, we can summarize the "weak points" of the simplest $SU(5)$ gauge unified theory in three main problems:

(i) $Y_e = Y_d$ is not satisfied according to experimental data at the GUT scale, i.e. fermion masses predicted by the SU(5) model are wrong.

(ii) The SM-couplings do not unify at any energy scale.

(iii) The model predicts massless neutrinos.

Those are the main aspects for which the SU(5) model has been ruled out. So now one could ask, "*What is the simplest realistic renormalizable model based on SU(5)?*".

The "easiest" way (in the sense of adding the least possible number of particles), to fix problem (i) by keeping the renormalizability of the Lagrangian is by introducing a 45_H representation (see appendix for its field content), i.e. 45 scalars [32]. Problem (ii) can be solved by introducing new representations whose beta functions contribute positively to achieve the unification constraints imposed by Eqs. (II.56,II.57). We will show that the addition of the 45_H is enough to solve problem (i) and (ii) simultaneously. In order to give mass to neutrinos, i.e. tackle problem (iii), SU(5) can be extended by adding an extra singlet representation (seesaw type-I) [33], a symmetric 15_H representation (seesaw type-II) [34] or a 24 matter representation (seesaw type-I and type-III) [35, 36], since the adjoint representation is the only one whose addition does not spoil the beautiful anomaly cancellation of SU(5).

Apart from the seesaws, there are other ways to give mass to the neutrinos, as we have already reviewed in last section. By restricting ourselves to the fermion fields that exist in the standard model, the light neutrino mass m_ν comes from a dimension 5 operator which may be generated at tree-level by the seesaw mechanism, or at the n-loop level with an extra suppression factor of $(1/16\pi^2)^n$, along with the suppression of new coupling constants which appear in the loop diagram. These new Yukawa matrices can be constrained by the structure of neutrino mass matrix which is determined by the neutrino oscillation data.

As it is shown in table IV, the realistic model which requires the least number of extra fields and could solve the presented problems would be the type-I seesaw SU(5) model, where one has at least two singlets [7], right-handed neutrinos, and the extra 45_H Higgses. However, as we have already discussed in the section of neutrino masses, the introduction of a singlet implies a new scale in the theory: one naively expects the fermion singlets to get mass from above the GUT scale since their masses are not protected by the SU(5) gauge symmetry.

Thus, the next candidate would be the Zee-SU(5) model. We bet for this option under the motivation of building the most economic renormalizable unified theory being able to solve, one by one, the main problems of the original SU(5) (without the addition of extra singlets).

[7] one neutrino could be massless since we in principle only know mass differences between the three generations of them.

TABLE IV: Possible renormalizable extensions of SU(5)-GG. By "extra field content" is understood any extra representation apart from the field content of SU(5)-GG + 45_H.

$Y_e \neq Y_d^T$	$m_\nu \neq 0$	Unification	Extra field content	number of new d.o.f.
45_H	type-I seesaw	✓	1_F, 1_F	47
	type-II seesaw	✓	15_H	60
	type-III seesaw	✓	24_F	69
	Zee model	✓	10_F	55
	Colored seesaw	✓	24_F	69

A. Theoretical framework

In the Zee model, in order for the neutrinos to get mass, a charged scalar, which is a singlet under SU(2) and SU(3), and a second Higgs doublet are needed, which are contained in 10_H and 45_H respectively.

The simplest extension of the SU(5) model consists on the basic fields composing SU(5)-GG (see first section of this report) plus a 45_H and 10_H representations. We address the field content of these representations, their quantum numbers and the Lagrangian of the model in detail in the appendix. In the context of this Zee-SU(5) unified model (SU(5)-GG + 45_H + 10_H) the Yukawa Lagrangian reads as,

$$\mathcal{L}_Y = \bar{5}\,10\,(Y_1\,5_H^* + Y_2\,45_H^*) \;+\; 10\,10\,(Y_3\,5_H + Y_4\,45_H)\epsilon_5 + Y_5\,\bar{5}\,\bar{5}\,10_H\epsilon_2 + \text{h.c.} \tag{IV.1}$$

where ϵ_2 and ϵ_5 refer to the Levi-Civita symbol with 2 and 5 Lorentz indices, respectively. Explicitly written in the flavor (a,b) and color (i,j,k) spaces,

$$\begin{aligned}\mathcal{L}_Y =& Y_1^{ab}\bar{5}_{i\,a}10_b^{ij}5^*_{H\,j} + Y_2^{ab}10_a^{ij}\bar{5}_{k\,b}45^{*\,k}_{H\,ij} + Y_3^{ab}10_a^{ij}10_b^{kl}5_H^m\epsilon_{ijkml} + Y_4^{ab}10_a^{ij}10_b^{kn}45_{H\,n}^{\;lm}\epsilon_{ijklm} \\ & Y_5^{ab}\bar{5}_{i\,a}\,\bar{5}_{j\,b}10_H^{ij} + \text{h.c.}\end{aligned} \tag{IV.2}$$

From the above expression it is straight forward to realize that Y_5 is antisymmetric due to Fermi statistics, which constraint the degrees of freedom of the matrices: it only has three d.o.f. (as long as we consider three families of fermions in Nature).

The Yukawa Lagrangian gives mass to the fermions of our theory once the symmetries are spontaneously broken and the Higgses get a vev. Therefore, the mass matrices of the different fermions can be written as a function of the Yukawa couplings. By looking at each term of the Yukawa Lagrangian is straight forward to see that,

$$\begin{aligned} M_d &= M_d(Y_1, Y_2), \\ M_u &= M_u(Y_3, Y_4), \\ M_e &= M_e(Y_1, Y_2). \end{aligned} \tag{IV.3}$$

All Yukawa interactions can be found explicitly written in the appendix, but here we will obtain step by step the fermion masses in order to illustrate the process.

Fermions get mass once H_1^0 and H_2^0 get their corresponding vevs. These fields live in

$$\begin{aligned} &H_1^\alpha \sim {5_H}^\alpha, \\ &H_2^\alpha \sim {45_H}_i^{j\alpha}\delta_j^i - \frac{1}{3}\epsilon_{\beta\gamma}\epsilon^{\delta\alpha}{45_H}_\delta^{\beta\gamma}, \end{aligned} \tag{IV.4}$$

where the roman letters refer to color indices, $i, j, k = 1, ..., 3$, and the greek ones to the $SU(2)$ indices, $\alpha, \beta = 4, 5$. To the down-type quarks mass and the electron mass contributes the first term in Eq. (IV.2), i.e.

$$\mathcal{L}_Y \supset Y_1(\bar{5}_i 10^{i\alpha} + \bar{5}_\beta 10^{\beta\alpha})5^*_{H\alpha} \; + \; 2Y_2\bar{5}_i 10^{j\alpha}(45^*_H)^i_{j\alpha} + Y_2\bar{5}_\alpha 10^{\beta\gamma}(45_H)^\alpha_{\beta\gamma} + \text{h.c.} \tag{IV.5}$$

where the factor of two comes from the contraction of the 10_H and the two lower indices of the 45^*_H, which are both antisymmetric. In terms of the fields,

$$\mathcal{L}_Y \supset Y_1\{d_i^C q^{i\alpha} + \ell_\beta\epsilon^{\beta\alpha}e^C\}H^*_{1\,\alpha} \; + \; 2Y_2 d_i^C q^{j\alpha}H_{2\alpha}\delta^i_j + Y_2\ell_\alpha e^C\epsilon^{\beta\gamma}(-3)\epsilon_{\beta\gamma}\epsilon^{\alpha\delta}H_\delta + \text{h.c.} \tag{IV.6}$$

Taking into account only the neutral component of the Higgs doublets, i.e. $\alpha = 5$ and $\delta = 5$, which is responsible for the fermion masses, we have

$$\mathcal{L}_Y \supset Y_1\{d_i^C\; d^i + e\; e^C\}H_1^0 + 2\;Y_2\; d_i^C\; d^i\; H_2^0 - 6\;Y_2\; e\; e^C\; H_2^0 + \text{h.c.} \tag{IV.7}$$

Therefore, after the spontaneous symmetry breaking,

$$\mathcal{L}_Y \supset d_i^C\; d^i\left(Y_1\frac{v_1^*}{\sqrt{2}} + 2Y_2\frac{v_2^*}{\sqrt{2}}\right) + e^C\; e\left(Y_1^T\frac{v_1^*}{\sqrt{2}} - 6Y_2^T\frac{v_2^*}{\sqrt{2}}\right) + \text{h.c.} \tag{IV.8}$$

where we have written explicitly the transpose of the $e\; e^C$ term since our convention for the mass term definition is $M_f^2\overline{f^C}f$.

On the other hand, for the up-type quark masses, terms leaded by Y_3 and Y_4 are the ones that matter. All possible combinations contributing to the quarks mass can be written as follows,

$$\begin{aligned} \mathcal{L}_Y \supset& Y_3(10^{ij}10^{k\alpha} + 10^{i\alpha}10^{jk})5^\beta\epsilon_{ijk\alpha\beta} \; + \; Y_4(10^{i\alpha}10^{jk}(45_H)_k^{l\beta} \\ &- 10^{ij}10^{\alpha k}(45_H)_k^{l\beta})\epsilon_{ijl\alpha\beta} \; + \; Y_4 10^{ij}10^{k\alpha}(45_H)_\alpha^{\beta\gamma}\epsilon_{ijk\beta\gamma}. \end{aligned} \tag{IV.9}$$

Now, by splitting the Levi-Civita tensor in its SU(2) and SU(3) indices $\epsilon_{ijk\alpha\beta} = \epsilon_{ijk}\epsilon_{\alpha\beta}$ and taking into account the possible miscounting in the mixing between weak isospin and color indices as we have already commented in last section, we have

$$\begin{aligned} \mathcal{L}_Y \supset& \;2\;Y_3(10^{ij}10^{k\alpha} + 10^{i\alpha}10^{jk})5^\beta\epsilon_{ijk}\epsilon_{\alpha\beta} + \; Y_4(4\;10^{i\alpha}10^{jk} - 2\;10^{ij}10^{\alpha k})(45_H)_k^{l\beta}\epsilon_{ijl}\epsilon_{\alpha\beta} \\ &+ \; Y_4 10^{ij}10^{k\alpha}(45_H)_\alpha^{\beta\gamma}\epsilon_{ijk}\epsilon_{\beta\gamma}. \end{aligned} \tag{IV.10}$$

By writing it in terms of the fields,

$$\begin{aligned} \mathcal{L}_Y \supset& 2\;Y_3(u_l^C\epsilon^{ijl}q^{k\alpha} + q^{i\alpha}u_m^C\epsilon^{jkm})H_1^\beta\epsilon_{ijk}\epsilon_{\alpha\beta} + \; Y_4(4\;q^{i\alpha}\epsilon^{jkm}u_m^C - 2\;\epsilon^{ijn}u_n^C q^{\alpha k})\delta_k^l H_2^\beta\epsilon_{ijl}\epsilon_{\alpha\beta} \\ &- \;3\;Y_4\epsilon^{ijm}u_m^C q^{k\alpha}\epsilon^{\beta\gamma}\epsilon_{\alpha\delta}H^\delta\epsilon_{ijk}\epsilon_{\beta\gamma} + \text{h.c.} \end{aligned} \tag{IV.11}$$

and using the following contractions: $\epsilon_{ab}\epsilon^{ab} = 2$ and $\epsilon_{ija}\epsilon^{ijb} = 2\delta_a^b$, we have

$$\begin{aligned} \mathcal{L}_Y \supset& \;4\;Y_3(u_l^C q^{l\alpha} + q^{i\alpha}u_i^C)H_1^\beta\epsilon_{\alpha\beta} + \; Y_4(8\;q^{i\alpha}u_i^C - 4\;u_l^C q^{\alpha l})\epsilon_{\alpha\beta}H_2^\beta - \; 12Y_4 u_k^C q^{k\alpha}\epsilon_{\alpha\delta}H^\delta \\ &= 4(Y_3 + Y_3^T)u_i^C q^{i\alpha}\epsilon_{\alpha\beta}H_1^\beta - 8(Y_4 - Y_4^T)u_i^C q^{i\alpha}\epsilon_{\alpha\beta}H_2^\beta + \text{h.c.} \end{aligned} \tag{IV.12}$$

Focusing on $\beta = \delta = 5$, after the spontaneous symmetry breaking, the above expression reads as

$$\mathcal{L}_Y \supset 4(Y_3 + Y_3^T)u_i^C u^i \frac{v_1}{\sqrt{2}} - 8(Y_4 - Y_4^T)u_i^C u^i \frac{v_2}{\sqrt{2}}. \tag{IV.13}$$

Therefore, altogether, the dependence of the mass matrices with the Yukawa couplings is explicitly given by:

$$\begin{aligned} M_d &= Y_1 \frac{v_5^*}{\sqrt{2}} + 2Y_2 \frac{v_{45}^*}{\sqrt{2}} \\ M_e &= Y_1^T \frac{v_5^*}{\sqrt{2}} - 6Y_2^T \frac{v_{45}^*}{\sqrt{2}} \\ M_u &= 4(Y_3 + Y_3^T)\frac{v_5}{\sqrt{2}} - 8(Y_4 - Y_4^T)\frac{v_{45}}{\sqrt{2}} \end{aligned} \tag{IV.14}$$

In contrast with the prediction $Y_e^T = Y_d$ from SU(5)-GG, here there is enough freedom to reproduce the measured fermion masses at the EW scale, so that problem (i) can be solved within the proposed extension of the SU(5). However, the Yukawa Lagrangian does not allow neutrinos to get mass at tree level. As we have already mentioned, we can give mass to the neutrinos through the Zee model (radiative corrections), by using the "new" 10_H introduced and the second Higgs doublet contained in 45_H. We will refer in detail to the generation of neutrino masses in the upcoming sections but first we will focus in the study of the unification of the Zee-SU(5) model.

B. Unification and proton decay

In this section we study the consistency of the theory by showing that unification can be achieved in the context of the Zee-SU(5) model and we constraint the available parameter space for the mass scales of the new fields by imposing experimental bounds (proton decay lifetimes and collider bounds), which will give rise to interesting predictions.

Unification constraints

As we have shown, in the original SU(5)-GG couplings do not unify at any energy. In order to satisfy the unification constraints new representations must be added. Each of the fields living in the extra representations may or may not help to satisfy the unification constraints (II.56,II.57), and the strength (or relevance) in which they contribute to unification is weighted by the mass scale of each field (see Eq. II.53). In this subsection we analyze which are the contributions of the extra fields considered in the proposed model and we study the possibility of unification according to the mass scale of these fields and the experimental constraints coming from the LHC or from the proton decay lifetimes of different channels.

In the proposed extension of SU(5), we consider an extra 45_H representation, able to correct the relation between charged fermion masses in a renormalizable way, and a 10_H representation, which contains the charged singlet able to give mass to neutrinos through the Zee mechanism. The contribution of these new fields to the beta functions is listed in Table V. As we can see, from the 45_H representation, only the fields Φ_3 and H_2 can help to achieve unification. Even thought Φ_1 strictly does not help to satisfy the unification constraints since $B_{23} > 0$, due to the negative value of its B_{12} (see Eq. (II.57)) it may help to increase the GUT scale and thus suppress proton decay. Apart from the doublet-triplet splitting problem in 5_H, which was needed to satisfy proton decay experimental constraints, here in the 45_H we have the same fine-tuning

TABLE V: Contributions to the B_{ij} coefficients of the extra fields in Zee-SU(5) model.

	45_H							10_H		
B_{ij}	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2	δ^+	$\delta_{(3,2)}$	δ_T
B_{12}	$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$	$-\frac{1}{15}r_{H_2}$	$\frac{1}{5}r_{\delta^+}$	$-\frac{7}{15}r_{\Delta_{(3,2)}}$	$\frac{4}{15}r_{\bar{T}}$
B_{23}	$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$	$\frac{1}{6}r_{H_2}$	0	$\frac{1}{6}r_{\Delta_{(3,2)}}$	$-\frac{1}{6}r_{\bar{T}}$
Does it help?	×	×	✓	×	×	×	✓	×	✓	×
Proton decay	×	×	✓	×	✓	✓	✓	×	✓ [a]	×

[a] Through the term $\lambda_{39} 5^*_i 10^{i\alpha} (45^*)^i_{i\alpha}$, the proton can decay via the tree-level process shown in Fig. 13 after the electroweak symmetry is spontaneously broken.

problem, because H_2 must be light in order to have a large expectation value to correct the fermion masses relations whereas other fields sitting in 45_H need to be heavy since they mediate proton decay (see next subsection).

In the 10_H representation, however, only $\delta_{(3,2)}$ could help to unification, but one has to be careful with this field since $\delta_{(3,2)}$ couples to fermions through the term $\lambda\,\bar{5}\,\bar{5}\,10_H$ in the following way $\lambda \epsilon_{\alpha\beta} d^c \ell^\alpha \delta^\beta_{(3,2)}$, where $\delta_{(3,2)} = (\delta^{2/3}_{(3,2)}, \delta^{-1/3}_{(3,2)})$ in the $SU(2)_L$ space. Hence, $\delta_{(3,2)}$ alone cannot mediate proton decay. However, the term $2\mu T^*_i H_2{}^*_\alpha \delta^{i\alpha}_{(3,2)} \in \mu 5^*_H 45^*_H 10_H$ in the scalar potential together with the above interaction may contribute to proton decay through the process shown in Fig. 13.

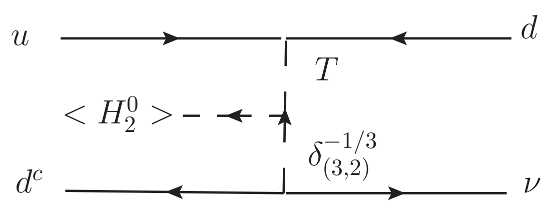


FIG. 13: B-L violating proton decay contribution.

A qualitative study on the bounds of the delta mass scale can be performed by considering the effective coupling of the process shown in Fig. 1, which is given by

$$\mathcal{L}_{eff} \supset Y_3 \frac{\lambda\,\mu\,v_2}{M^2_{\delta_{(3,2)}} M^2_T} u d \bar{\nu} \bar{d}^c. \tag{IV.15}$$

In order to satisfy the bounds on proton decay, $\mu\,\lambda\,v_2 / M^2_{\delta_{(3,2)}} M^2_T \lesssim 1/(10^{12}\mathrm{GeV})^2$ as in the usual Higgs mediated $d = 6$ proton decay contribution. Notice that, due to the presence of the triplet mass squared in the denominator, the mass of $\delta_{(3,2)}$ is not necessarily required to be heavy (the parameters λ and μ are constrained to be small since they appear in the neutrino mass matrix). In this way the B-L violating contribution to proton decay mediated by $\delta_{(3,2)}$ can be understood. Therefore, in principle it could be relatively light and it would still be in agreement with proton decay bounds.

In spit of this, as we will show, there is no need to assume any particular mass scale for the fields sitting in 10_H, i.e. a degenerate mass scale for the representation can be assumed. The unification constraints allow us to keep the mass scale of the 10_H unfixed. It is enough to consider the splitting in the 45_H

to achieve unification. Therefore, for simplicity and proceeding in the most natural way, we will not assume any splitting in the 10_H neither in the 24_H. In Fig. 14 we show the region where unification of

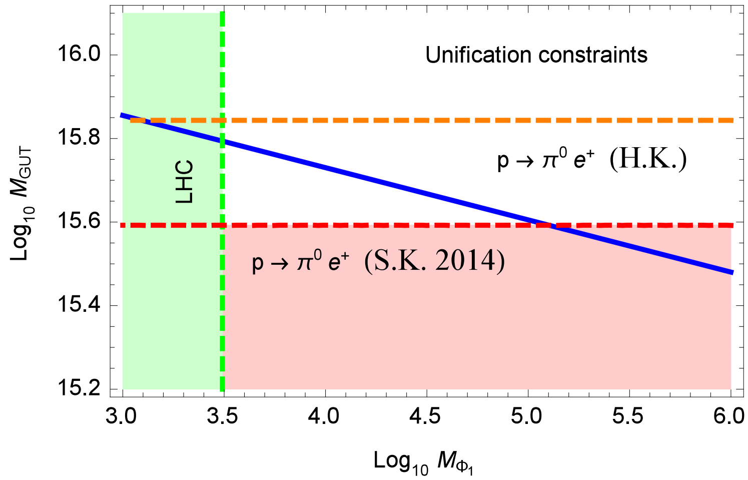


FIG. 14: Dependence of the scale M_{ϕ_1} with M_{GUT} dictated by the unification constrains (blue line) when $M_{H_2} = 1$ TeV. The dashed green line shows the naive LHC bound on the colored octet mass, $M_{\Phi_1} > 3.1$ TeV [37]. The red dashed line shows the limit on the GUT scale from the current experimental value on proton decay lifetime, $\tau_p(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years [20]. The orange dashed line shows the projected limit on the proton decay lifetime from the Hyper-Kamiokande collaboration, $\tau_p(p \to \pi^0 e^+) > 1.3 \times 10^{35}$ years [38]. The mass of the Φ_3 (implicit) is in the range $10^{8.6} - 10^{8.9}$ GeV from left to right.

the gauge couplings at 1-loop is satisfied (see Eqs. (II.56,II.57)). The red shadowed region is ruled out by the current experimental bounds on proton decay, $\tau_p(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years [20], and the green shadowed region is dismissed due to LHC bounds on the colored octet mass, $M_{\Phi_1} \geq 3.1$ TeV [37]. We also show the limit projected (orange line) by the Hyper-Kamiokande collaboration on proton decay bounds, $\tau_p(p \to \pi^0 e^+) > 1.3 \times 10^{35}$ years [38]. The mass of $\Phi_3 \sim (3, 3, -1/3)$ is implicit in the plot, and it changes from $10^{8.6}$ to $10^{8.9}$ GeV. One may argue that M_{Φ_3} cannot be set arbitrarily light since Φ_3 potentially contributes to the proton decay through the Yukawa Lagrangian (see appendix), and indeed the mass scale must be thus far limited, but as we are showing in next section, the constraint can be relaxed due to the freedom of having a product of two Yukawa couplings, one of them (Y_4) not being restricted by the fermion masses, so that the range of M_{Φ_3} used in Fig. 14 is justified. M_{H_2} has been assumed to be light in order to avoid fine tunning in Eq.(IV.14), concretely we took $M_{H_2} = 1$ TeV.

The allowed parameter space fixes the upper and lower bounds for the field Φ_1 (notice that this is only true in the case where the mass scale of the fields in 10_H is either degenerate or very heavy), $\phi_1 \sim [10^{3.5}, 10^{5.1}]$ GeV. Hence, the model predicts a light colored Higgs with large cross sections through QCD interactions, as one can see from the Yukawa Lagrangian (see appendix for more details):

$$\mathcal{L}_Y \supset 2\, d^c Y_2 \Phi_1^\dagger q_L \;+\; 4\, u^c (Y_4 - Y_4^T) q_L \Phi_1 \;+\; \text{h.c.} \tag{IV.16}$$

It is remarkable that, due to the antisymmetry of the second coupling above, decays of ϕ_1 into two top-quarks would not be observed. Therefore one could have exotic signatures such as signals with one top quark and three light jets (gluons can produce a pair of ϕ_1 colored scalars which may decay into a pair of quarks from different families. From all possible decays, the pattern top plus three light quarks -which will hadronize into three light jets- is of special relevance since this signal is not predicted by the SM). Moreover, if M_{Φ_1} is close to the TeV, one might see these signals at the LHC. The phenomenology of colored octets has been investigated in the literatures [39–57]. The existence of this light colored octet is one of the main predictions of the model.

So far we have shown that, in the context of this model, unification can be achieved in agreement with experimental bounds and, moreover, we come out with a prediction of a colored light Higgs whose exotic phenomenology could be found in the LHC.

Proton decay

In the Zee-SU(5) model there are several fields contributing to proton decay. In the first place, one has the leptoquark gauge bosons predicted by the $SU(5)$ theory, $X^\mu \sim (3, 2, -5/6)$ and $Y^\mu \sim (\bar{3}, 2, 5/6)$, which sit in the adjoint representation. These bosons couple to leptons and quarks through the kinetic terms in the Lagrangian, as we have already discussed in last chapter. As we know, in the SU(5)-GG model, one also has the colored triplet T, sitting in 5_H, which couples to both leptons and quarks and, hence, also mediates proton decay. In the 45_H representation, from the Yukawa interactions (see appendix) one can see that Φ_3 and Φ_5 are also proton decay candidates. The field Φ_6 may also mediate proton decay but its two body decay at tree level is killed by the antisymmetry in the flavor space of the effective Yukawa (see Table VI) [58]. However, one cannot dismiss the contribution of the three-body decay although it is higher suppressed. In the 10_H representation at first sight there is no mediator of proton decay, at least at tree level, but there is one field in this representation, the $\delta_{(3,2)}$, which could mediate proton decay through the process shown in Fig. 13. However, as we have shown in last section, this process is quite suppressed by the mass of the triplet squared. In last section we already showed that unification constraints require, from the extra Higgses, only H_2 and Φ_1 to be light, which do not contribute to proton decay, so that the safety of the theory is in principle guaranteed.

In Table VI are shown the mediators of proton decay and the correspondent naive estimation of the decay rate from dim-6 effective operators, which are listed in the appendix. The case of Φ_3 is slightly more

TABLE VI: List of tree level exchange (d=6) operators in Zee-SU(5) which contribute to proton decay. The relevant coefficients are shown in the last column. $\tilde{Y}_4$ represents the effective Yukawa coupling, which is antisymmetric. The effective Lagrangian leading to proton decay is written in detail in the appendix.

field	$\mathcal{L}^{p.d}_{d=6} \sim \frac{1}{m^2} qqql$	decay channel	decay width
X_μ, Y_μ	A	$p \to e^+ (\mu^+)\, \pi^0$	$\Gamma \sim \alpha_s^2 \frac{m_p^5}{M_X^4}$
T	$\mathcal{L}_T \supset \frac{1}{m_T^2} \left\{ (l_\alpha\, Y_1\, q^\alpha)(q_\beta\, Y_3\, q_\gamma)\epsilon^{\beta\gamma}) + (d^c\, Y_1\, u^c)(u^c\, Y_3\, e^c) \right\}$	$p \to \pi^0\, e^+ (\mu^+)$ $p \to \bar{\nu}\, \pi^+$	$\Gamma \sim (Y_1\, Y_3)^2 \frac{m_p^5}{m_T^4}$
Φ_3	$\mathcal{L}_{\Phi_3} \supset \frac{1}{m_{\Phi_3}^2} (l_\alpha\, Y_2\, q_\beta)(q_\alpha\, \tilde{Y}_4\, q_\gamma)\epsilon^{\beta\gamma}$	$p \to \bar{\nu}\, K^+$ $p \to e^+\, \pi^0$	$\Gamma \sim (Y_2\, \tilde{Y}_4)^2 \frac{m_p^5}{m_{\Phi_3}^4}$
Φ_5	$\mathcal{L}_{\Phi_5} \supset \frac{1}{m_{\Phi_5}^2} (d^c\, Y_2\, u^c)(u^c\, \tilde{Y}_4\, e^c)$	$p \to \pi^0 \mu^+ (\tau^+)$	$\Gamma \sim (Y_2 \tilde{Y}_4)^2 \frac{m_p^5}{m_{\Phi_5}^4}$
Φ_6	$\mathcal{L}_{\Phi_6} \supset \frac{1}{m_{\Phi_6}^2} (d^c\, Y_2\, e^c)(u^c\, \tilde{Y}_4\, u^c)$	$p \to \pi^0 e^+ (\mu^+)(\tau^+)$	$\Gamma \sim (Y_2\, \tilde{Y}_4)^2 \frac{m_p^5}{m_{\Phi_6}^4}$

delicate. The unification constraints require $M_{\Phi_3} \sim 10^{8.6} - 10^{8.9}$ GeV but this field is a proton decay mediator. From the above table, we have

$$\Gamma \sim (Y_2 \tilde{Y}_4)^2 \frac{M_p^5}{M_{\Phi_3}^4} \lesssim \frac{\hbar}{10^{34}(\text{yr})}. \tag{IV.17}$$

where $\tau(p \to K^0 e^+) = 5.9 \times 10^{33}$ yr [59].

Entering in this equation the proton mass $M_p = 0.938$ GeV, the lowest possible value of M_{Φ_3} allowed by the unification constraints, i.e $M_{\Phi_3} \sim 10^{8.6}$, and assuming that the coupling $Y_2 \sim 10^{-3}$ (it is constrained by the fermion masses), one gets that

$$\tilde{Y}_4 = Y_4 - Y_4^T \sim 10^{-12}. \tag{IV.18}$$

The smallness of the coupling may startle the reader (or may not), so that we stress a couple of points in order to make clear that this number is perfectly consistent.

(a) In contrast to the Yukawas $Y_1 = f(M_e, M_d)$ and $Y_2 = f(M_e, M_d)$, which are constrained by the charged leptons and down-type quark masses (see Eq. (IV.3) and therefore cannot be arbitrarily small (unless fine-tuning was assumed), the Yukawas Y_3 and Y_4 are only constrained by the up-type quark mass. Hence, the model fixes a combination of both Yukawas (see Eq. (IV.3)) so that one degree of freedom is left, which can be used to set Y_4 arbitrarily to any value (always regarding perturbation constraints). This shows the consistency of the coupling.

(b) Moreover, just as a comment regarding the "aesthetics, the $\tilde{Y}_4$ coupling is a combination $\tilde{Y}_4 = Y_4 - Y_4^T$ which means that, if the Y_4 matrix is almost symmetric, $\tilde{Y}_4$ will be very suppressed in a natural way.

From table VI, one can see that the main contribution to proton decay comes from the new gauge bosons (the rest of the candidates are suppressed by the product of Yukawa couplings).

In order to estimate the proton decay rate in a more rigorous way, we will appeal to an effective operator theory. We assume that the proton and the positron play the role of chiral fermions, whereas the meson is a scalar field. Under this assumptions, the proton decay can be represented roughly through the process shown in Fig. 15. We assume Λ to be the coupling of the feynman rule of this interaction.

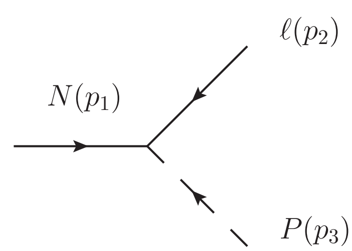


FIG. 15: Decay process $N_L(p_1) \to \bar{\ell}(p_2)\, P(p_3)$

The partial decay width of the process above with an initial nucleon state (N) and a final state containing a pseudo-scalar meson (P) and anti-lepton ($\bar{\ell}$) reads as,

$$\Gamma(N \to P\bar{\ell}) = \frac{m_N}{32\pi}\left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 |\left\langle \pi^0 \middle| O_I^{B-L} \middle| p \right\rangle|^2. \tag{IV.19}$$

where O_I^{B-L} are the effective operators involved in the process. By using the effective operator approach, the possible dimension-six (three quarks and one lepton, obtained by integrating out the heavy gauge fields

X^μ and Y^μ) operators which are $SU(3)_c$ color singlets and $SU(2)_L \times U_Y(1)$ invariant are:

$$O_i^{B-L} = k_1^2 \,\epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u_{i\alpha}^C}\gamma^\mu Q_{j\alpha a}\overline{e_b^C}\gamma_\mu Q_{k\beta b}, \tag{IV.20}$$

$$O_{ii}^{B-L} = k_1^2 \,\epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u_{ia}^C}\gamma^\mu Q_{j\alpha a}\overline{d_{kb}^C}\gamma_\mu L_{\beta b}. \tag{IV.21}$$

where $Q = (u, d),\ L = (\nu, e)$; i, j and k $= 1, 2, 3$, are the color indices, a and b $= 1, 2, 3$ are the family indices, and $\alpha, \beta = 1, 2$ are the $SU(2)$ indices. From the above, one can write down the effective operators for each decay channel in the physical basis [60]:

$$O(e_\alpha^c, d_\beta) = C(e_\alpha^C, d_\beta)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j\overline{e_\alpha^C}\gamma_\mu d_{k\beta}, \tag{IV.22}$$

$$O(e_\alpha, d_\beta^C) = C(e_\alpha, d_\beta^C)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j\overline{d_{k\beta}^C}\gamma_\mu e_\alpha, \tag{IV.23}$$

$$O(\nu_l, d_\alpha, d_\beta^C) = C(v_l, d_\alpha, d_\beta^C)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu d_{j\alpha}\overline{d_{k\beta}^C}\gamma_\mu\nu_l. \tag{IV.24}$$

The C^I are the perturbative estimate of Wilson coefficients in GUT models, and concretely in SU(5) they are given by $C^I = k_1^2 c^I$ where $k_1 = g_{GUT}/\sqrt{2}M_{X,Y}$ and the c^I have the following form [61],

$$c(e_\alpha^c, d_\beta) = V_1^{11}V_2^{\alpha\beta} + (V_1V_{UD})^{1\beta}(V_2V_{UD}^\dagger)^{\alpha 1}, \tag{IV.25}$$

$$c(e_\alpha, d_\beta^c) = V_1^{11}V_3^{\beta\alpha}, \tag{IV.26}$$

$$c(\nu_l, d_\alpha, d_\beta^c) = (V_1V_{UD})^{1\alpha}(V_3V_{EN})^{\beta l}. \tag{IV.27}$$

Here, the V's are mixing matrices defined as

$$\begin{aligned} &V_1 = U_C^\dagger U, V_2 = E_C^\dagger D,\ V_3 = D_C^\dagger E, \\ &V_{UD} = U^\dagger D \text{ and } V_{EN} = E^\dagger N. \end{aligned} \tag{IV.28}$$

The matrices U, E, D and N define the Yukawa couplings diagonalization, so that

$$\begin{aligned} &U_C^T Y_u U = Y_u^{\text{diag}},\ D_C^T Y_d D = Y_d^{\text{diag}}, \\ &E_C^T Y_e E = Y_e^{\text{diag}},\ N^T Y_\nu N = Y_\nu^{\text{diag}}. \end{aligned} \tag{IV.29}$$

By using the identity $\Psi^T C\gamma^\mu\epsilon = (\Psi^T C\gamma^\mu\epsilon)^T = -\epsilon^T C\gamma^\mu\Psi$, the above effective operators can be rewritten in a more suitable way:

$$O(e_\alpha^c, d_\beta) = -2\, C(e_\alpha^C, d_\beta)\, \epsilon_{ijk}\, u_{jL}^T C\gamma^\mu u_{iR}\, e_{\alpha R}^T C\gamma_\mu d_{k\beta L}, \tag{IV.30}$$

$$O(e_\alpha, d_\beta^C) = -2\, C(e_\alpha, d_\beta^C)\, \epsilon_{ijk}\, ujL^T C\gamma^{mu} u_i iR\, d_{k\beta R}^T C\gamma_\mu e_{\alpha L}, \tag{IV.31}$$

$$O(\nu_l, d_\alpha, d_\beta^C) = -2\, C(v_l, d_\alpha, d_\beta^C)\, \epsilon_{ijk}\, d_{j\alpha L}^T C\gamma^\mu u_{iR}\, d_{k\beta R}^T C\gamma_\mu\nu_{lL}, \tag{IV.32}$$

regarding the application of the following Fierz identity [62], where h refers to a certain chirality and $-h$ to the opposite one,

$$(A_h^T C\gamma^\mu B_{-h})(C_{-h}^T C\gamma_\mu D_h) = -2(A_h^T C D_h)(C_{-h}^T C B_{-h}). \tag{IV.33}$$

This identity allows us to eliminate the dependence of the effective operators on the Dirac matrices, so that the rearranged expression gains a global factor of 2:

$$O(e^c_\alpha, d_\beta) \;=\; 2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, (u^T_{jL} C d_{k\beta L})(e^T_{\alpha R} C u_{iR}), \tag{IV.34}$$

$$O(e_\alpha, d^C_\beta) \;=\; 2\, C(e_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (u^T_{jL} C e_{\alpha L})(d^T_{k\beta R} C u_{iR}), \tag{IV.35}$$

$$O(\nu_l, d_\alpha, d^C_\beta) \;=\; 2\, C(v_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (d^T_{j\alpha L} C \nu_{lL})(d^T_{k\beta R} C u_{iR}). \tag{IV.36}$$

By defining the product $(x, y)_{R/L} = x^T C\, P_{R/L}\, y$ as introduced by reference [63] we can rewrite them as follows:

$$O(e^c_\alpha, d_\beta) \;=\; 2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, (u_j d_{k\beta})_L (e_\alpha u_i)_R, \tag{IV.37}$$

$$O(e_\alpha, d^C_\beta) \;=\; 2\, C(e_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (u_j e_\alpha)_L (d_{k\beta} u_i)_R \tag{IV.38}$$

$$O(\nu_l, d_\alpha, d^C_\beta) \;=\; 2\, C(v_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (d_{j\alpha} \nu_l)_L (d_{k\beta} u_i)_R. \tag{IV.39}$$

Hence, the proton decay width in the context of the effective operator approach reads as,

$$\Gamma(N \to P\bar{\ell}) = A\, \frac{m_N}{8\pi} \left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 \left| \sum_I C^I W^I_0(N \to P) \right|^2, \tag{IV.40}$$

where $W^I_0(N \to P)$ corresponds to the matrix element of the three-quarks $\left\langle N \right| (q_1 q_2)_{L/R}\, q_{3\,L/R} \left| P \right\rangle$ and A refers to the running of the operators, which must also be considered, defined as

$$A = A_{QCD} A_{SR} = \left(\frac{\alpha_3(m_b)}{\alpha_3(M_Z)}\right)^{6/23} \left(\frac{\alpha_3(Q)}{\alpha_3(m_b)}\right)^{6/25} \left(\frac{\alpha_3(M_Z)}{\alpha_3(M_{GUT})}\right)^{2/7}. \tag{IV.41}$$

Here, $A_{QCD} \approx 1.2$ corresponds to the running from the M_Z to the $Q \approx 2.3$ GeV scale, while $A_{SR} \approx 1.5$ defines the running from the GUT scale to the electroweak scale [61].

Particularly, for the most relevant channels,

$$\Gamma(p \to \pi^0 e^+_\beta) \;=\; \frac{m_p}{8\pi} A^2 k_1^4 |\left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle|^2 \left(|c(e^c, d)|^2 + |c(e, d^c)|^2 \right), \tag{IV.42}$$

$$\begin{aligned} \Gamma(p \to K^+ \bar{\nu}) \;=\; & \frac{m_p}{8\pi} \left(1 - \frac{m^2_{K^+}}{m^2_p}\right)^2 A^2 k_1^4 \sum_i \left| c(\nu_i, d, s^c) \left\langle K^+ \right| (us)_R d_L \left| p \right\rangle + \right. \\ & \left. + \, c(\nu_i, s, d^c) \left\langle K^+ \right| (ud)_R s_L \left| p \right\rangle \right|^2. \end{aligned} \tag{IV.43}$$

where it has been taken into account that $\left\langle \pi^0 \right| (ud)_L u_R \left| p \right\rangle \overset{P}{=} \left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle$, i.e. the matrix elements are invariant under parity transformations.

In general the Wilson coefficients cannot be predicted since the above matrices are unknown. In our analysis we have assumed the most conservative scenario (in the sense of less optimistic case in which the diagonal entries of the mixing matrices products are equal to the unity) in which $c(e, d^c) = 1$, and $c(e^c, d) = 2$ for $p \to \pi^0 e^+$ and, in the case of $p \to K^+ \bar{\nu}$, we use $c(\nu_l, d, s^c) = (V_3 V_{EN})^{2l}$ and $c(\nu_l, s, d^c) = V^{12}_{CKM} (V_3 V_{EN})^{1l}$.

The quantities $\left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle$, $\langle K^+ | (us)_R d_L | p \rangle$ and $\langle K^+ | (ud)_R s_L | p \rangle$ entering in the decay amplitude are the different matrix elements computed in lattice calculations. Here we use the values reported in

Ref. [64]:

$$\begin{aligned} \left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle &= -0.103, \\ \left\langle K^+ \right| (us)_R d_L \left| p \right\rangle &= -0.054, \\ \left\langle K^+ \right| (ud)_R s_L \left| p \right\rangle &= -0.093. \end{aligned} \tag{IV.44}$$

In Fig. 16 we show the conservative predictions for the proton decay lifetime and the current experimental bounds, $\tau_p(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years [20] and $\tau_p(p \to K^+ \bar{\nu}) > 5.9 \times 10^{33}$ years [59]. As one can see, the proton decay predictions are not far from the reach of the Hyperkamiokande experiment so that proton decay could be found according to this model (however, since we have taken the most conservative bounds, it cannot strictly speaking completely ruled out). We have shown that unification can be achieved in this model in agreement with the experimental bounds on proton decay lifetime.

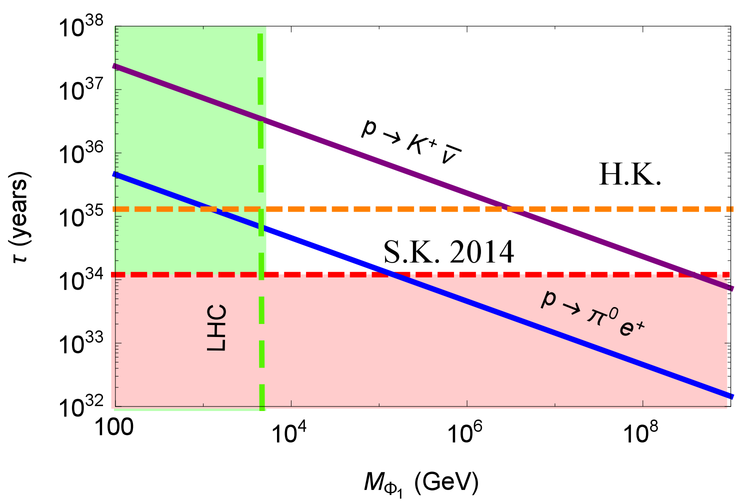


FIG. 16: Predictions for the proton decay lifetimes. The blue line shows the predictions for the decay $p \to \pi^0 e^+$, while the purple line shows the predictions for the decay $p \to K^+ \bar{\nu}$. The horizontal red dashed line shows the current experimental value on proton decay lifetime, $\tau_p(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years [20] from the Super-Kamiokande collaboration. The orange dashed line shows the projected limit on the proton decay lifetime from the Hyper-Kamiokande collaboration, $\tau_p(p \to \pi^0 e^+) > 1.3 \times 10^{35}$ years [38]. The green vertical line represents the LHC bound, $M_{\Phi_1} \geq 3.1$ TeV [37], on the colored octet mass.

C. Neutrino masses

The mass expression (III.68), along with the argumentation followed to compute it, is a completely general result which could be valid in principle for many scenarios as long as they have as an effective field theory the Zee potential we started with. The key point of this work is that we embed this model in a unified theory which allows us to establish certain interesting and powerful relations between the fermions inside the model. The Zee model can be realized in a grand unified theory based on $SU(5)$: the extra charged singlet scalar needed, δ^+ [8], is embedded in the antisymmetric representation 10_H, and the other field required, a second Higgs doubled (as we have discussed above) lives in the 45_H. It is remarkable that this second Higgs doublet is already contained in any realistic and renormalizable SU(5) model, since the 45_H representation is the simplest addition needed to correct the mass relation between the charged leptons and the down-type quarks without loosing the renormalizability of the model. Thus, there is no need to add a new Higgs doublet, which shows the simplicity of the model.

In the $SU(5)$ language the needed interactions for the Zee mechanism read as

$$V_{SU(5)} \supset \lambda \bar{5}\,\bar{5}\,10_H + \bar{5}\,10\left(Y_1^* 5_H^* - \frac{1}{6} Y_2^* 45_H^*\right) - \frac{1}{6}\mu\, 5_H 45_H 10_H^* + \text{ h.c.} \tag{IV.45}$$

where the couplings in the SU(5) theory has been redefined in order to match the V_{Zee} potential defined before in Eq. (III.43) with the SU(5) potential. Therefore, $Y_1 \equiv (Y_1^{SU(5)})^*$, $Y_2 \equiv -6(Y_2^{SU(5)})^*$ and $\mu \equiv -6\,\mu_{SU(5)}$.

Using the relation between the charged fermion masses and the Yukawa couplings, taking into account that the definition of the mass of a fermion f that we have introduced for the fermion masses in SU(5)-Zee was $f^{cT}_L C M_f f_L$ whether in the section of the generation of neutrino masses through the Zee model the mass is defined as $\bar{f}_L M_f f_R$, and writing it in terms of the redefined couplings we have

$$\sqrt{2} M_d = Y_1^T v_5 - \frac{1}{3} Y_2^T v_{45}, \tag{IV.46}$$

$$\sqrt{2} M_e = Y_1 v_5 + Y_2 v_{45}. \tag{IV.47}$$

Here, $v_5 \equiv v_1$ and $v_{45} \equiv v_2$. Thus, the Yukawa couplings read as

$$Y_1 = \frac{1}{2\sqrt{2} v_5}(M_e + 3M_d^T), \tag{IV.48}$$

$$Y_2 = \frac{3}{2\sqrt{2} v_4 5}(M_e - M_d^T). \tag{IV.49}$$

and therefore the neutrino mass matrix can be written as a function of the charged lepton masses and down-type quarks:

$$M_\nu = \lambda M_e \left(c_e M_e^\dagger + 3 c_d M_d^*\right) + \left(c_e M_e^* + 3 c_d M_d^\dagger\right) M_e^T \lambda^T, \tag{IV.50}$$

where the coefficients c_e and c_d are given by

$$c_e = \frac{(1 - 4\sin^2\beta)}{8\pi^2\sqrt{2} v} \frac{\sin 2\theta_+}{\sin 2\beta} \mathrm{Log}\left(\frac{\mathrm{m}^2_{\mathrm{h}_2^+}}{\mathrm{m}^2_{\mathrm{h}_1^+}}\right), \tag{IV.51}$$

$$c_d = \frac{1}{8\pi^2\sqrt{2} v} \frac{\sin 2\theta_+}{\sin 2\beta} \mathrm{Log}\left(\frac{\mathrm{m}^2_{\mathrm{h}_2^+}}{\mathrm{m}^2_{\mathrm{h}_1^+}}\right). \tag{IV.52}$$

according to Eq. (III.68). Clearly one can see that, since Y_1 and Y_2 cannot be simultaneously diagonalized, the diagonal elements of the neutrino mass matrix are not zero even if λ is antisymmetric. Therefore, the model has enough freedom to be consistent with the experimental values for neutrino masses and mixings.

This relation between the masses of the neutrinos and the charged fermions is quite interesting since one would not expect any connection like this in the context of SU(5)-GG. Notice that the antisymmetric matrix λ which enters in Eq. (IV.50) has only three free parameters.

Due to the explicit dependence of the fermion masses on the Yukawa couplings (see Eq. (IV.3)), M_u and M_d cannot be simultaneously diagonalized. Therefore, by working in the basis in which M_e and M_u are diagonal, the down-type quark mass matrix adopts the following form: $M_d = D_c^* M_d^{diag} V_{CKM}^\dagger$, where D_c corresponds to the rotation matrix which diagonalizes the d^C quarks. Notice that it has only three degrees of freedom in the real case (symmetric matrix). In this context one finds

$$M_\nu = \lambda M_e^{diag} \left(c_e M_e^{diag} + 3 c_d D_c M_d^{diag} V_{CKM}^T \right) + \left(c_e M_e^{diag} + 3 c_d V_{CKM} M_d^{diag} D_c^T \right) M_e^{diag} \lambda^T, \tag{IV.53}$$

where all phases have been neglected for simplicity. As it is shown in Eq. (IV.53), the model has enough degrees of freedom to reproduce consistently the experimental values of neutrino masses and mixing angles. We remark that this freedom also refuses the prediction of the ratio between the fermion masses but one can constrain the unknown parameters of Eq. (IV.53) by imposing experimental bounds regarding neutrino masses and mixings. We stress the beauty of this outcome in the sense that the above relation is an intrinsic prediction of the unified model and it comes out in a natural way.

V. LR-SYMMETRIC THEORIES

Left-right (LR) symmetric models are regarded as appealing extensions of the Standard Model, since they present a more symmetric structure in the representations of the fermion sector along with the fact that they give an explanation about the parity violation (left-chiral preferred structure) at the electroweak scale and predict massive neutrinos as a natural outcome.

The simplest LR symmetric theories are based in the following gauge symmetry group [3–6]:

$$G = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}. \tag{V.1}$$

The main difference of LR symmetric models with respect to the SM is the prediction of a right handed neutrino which allows neutrinos to have a Dirac mass. Notice that in this model there is a total of seven generators: $2^2 - 1$ for each SU(2), i.e. $T^a_{L,R}$ where $a = 1, .., 3$ which obey the Lie algebra

$$[T^a_{L,R}, T^b_{L,R}] = i\epsilon_{abc}T^c_{L,R} \tag{V.2}$$

and one for U(1), αI, so that, apart from the SM gauge bosons, three extra massive gauge bosons are expected ($W^{\pm}_R$ and $Z^{'}$) after the LR symmetry is spontaneously broken to $U(1)_{em}$. In this section we summarize the main features of these models by first introducing the basic field content and then discussing the masses of the fields after the gauge symmetry is spontaneously and how this breaking occurs.

A. Field content

LR symmetric models enjoy sixteen Weyl degrees of freedom, so that they can accommodate sixteen chiral fields, i.e. one more than the SM. The matter content is given by fermion multiplets. Fermions are embedded in doublets of $SU(2)_L$ and $SU(2)_R$ and are completely symmetric under $L \leftrightarrow R$,

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (1, 2, 1, 1/3), \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (1, 1, 2, 1/3)$$

for quarks, and for leptons

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, 1, -1), \quad \ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (1, 1, 2, -1),$$

where the third quantum number corresponds to the charge of the abelian group $U(1)_{B-L}$ which is defined by the breaking of the LR-model to the Standard Model in such a way that the electromagnetic charge is recovered after the breaking. The hypercharge operator is defined by the unbroken gauge symmetry in the process $SU(2)_R \otimes U(1)_{B-L} \to U(1)_Y$. The explicit form of the charge operator depends on the breaking pattern to the $U(1)_{em}$, as we have already mentioned in the context of SU(5). By taking an infinitesimal $SU(2)_R \otimes U(1)_{B-L}$ transformation and imposing that it leaves the vacuum state invariant (taking into account that Φ transforms as $\Phi \to \Phi e^{iT^a c_a}$, being T^a the generators and c_a the corresponding phases),

$$\delta\phi = \frac{i}{2}(\xi \cdot \sigma + I\rho)\phi \stackrel{!}{=} 0, \tag{V.3}$$

we find that $\xi_1 = \xi_2 = 0$ and $\xi_3 = \rho$, so that $Y = T_3^R + \frac{B-L}{2}$, where the normalization factor is chosen to recover the already known electric charges in the SM. After the breaking of the SM to $U(1)_{em}$, the charge operator is defined as

$$Q = T_3^L + Y = T_3^L + \underbrace{T_3^R + \frac{B-L}{2}}_{Y} \tag{V.4}$$

It is straightforward to realize that the charge $B-L$ corresponds to the difference between the baryon and lepton number of the field involved, which justifies the label of the charge. Notice that, in the context of LR symmetric theories, the hypercharge has a physical meaning, in contrast with the Standard Model, where it is introduced ad-hoc. Here, the LR symmetry is deeply connected with the baryon-lepton symmetry [13]. Every doublet has a flavor index in addition. The complete set of fermion generations requires three copies of the above multiplets to cover them, as in SU(5) theories. The scalar sector of the theory is discussed in the next section since it depends on the way chosen to break the LR-symmetry to the SM one.

B. LR Symmetry breaking and Dirac neutrinos

Since $M_{W_R} \gg M_{W_L}$ according to experiment, the LR symmetry must be broken at some point. A scalar sector is needed in order to first break the LR gauge symmetry to the SM and then give mass to the fermions in the model by keeping $U(1)_{em}$ unbroken. For the last propose, we are lead to introduce a doublet under both $SU(2)_L$ and $SU(2)_R$, Φ, which we will call "bi-doublet" and it transforms as,

$$\tilde{\Phi} = \begin{array}{c} \xrightarrow{SU(2)_R} \\ \begin{pmatrix} \Phi_1^0 & \Phi_2^+ \\ \Phi_1^- & \Phi_2^0 \end{pmatrix} \end{array} \Big\downarrow_{SU(2)_L} \sim (1,2,2,0)\,,$$

along with the following Yukawa interactions

$$\mathcal{L} \supset \bar{Q}_L \left(Y_1 \Phi + Y_2 \tilde{\Phi} \right) Q_R + \bar{\ell}_L \left(Y_3 \Phi + Y_4 \tilde{\Phi} \right) \ell_R + \text{h.c.}, \tag{V.5}$$

where $\tilde{\Phi}$ is defined as $\sigma_2 \Phi^* \sigma_2$, i.e.

$$\tilde{\Phi} = \begin{pmatrix} (\Phi_2^0)^* & -\Phi_1^+ \\ -\Phi_2^- & (\Phi_1^0)^* \end{pmatrix} \sim (2, \bar{2}, 0) \tag{V.6}$$

This bi-doublet field is required in order to connect left and right fermion multiplets through the Yukawa Lagrangian. However, it turns out that it is not enough to fully break the LR-symmetry group to $U(1)_{em}$ as we show right after.

The most general form of $\langle \Phi \rangle$ such that the electromagnetic gauge-invariance is preserved is the following:

$$\Phi = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix},$$

Once the neutral components of the bi-doublet acquire a vev (as shown above), the symmetry is spontaneously broken and, therefore, fermions become massive. Notice that the condition of Φ being an extremum of the scalar potential requires that

$$\left\langle \frac{\partial^2 V}{\partial \Phi^2} \right\rangle \langle \delta\Phi \rangle = m_\Phi^2 T^a c_a \, \langle \Phi \rangle \overset{!}{=} 0. \tag{V.7}$$

due to the fact that Φ transforms as $\Phi \to \Phi e^{iT^a c_a}$, being T^a the generators of $SU(2)_{L/R}$ ($a = 1, \cdots, 6$) and c_a the corresponding phases. The unbroken linearly independent combinations of generators define the breaking pattern of the process. In this case,

$$(T_L^3 + T_R^3) \, \langle \Phi \rangle = 0, \tag{V.8}$$
$$(B - L)\mathbb{1} = 0. \tag{V.9}$$

Hence, the vev of the bi-doublet breaks the symmetry down to $U(1) \otimes U(1)$ [5]. Therefore, extra Higgs multiplets are required to recover the $U(1)_{em}$. There are many options to perform this breaking. Here we will discuss the two simplest (in the sense of minimality regarding degrees of freedom) ways.

In order to keep the discussion as general as possible, let us introduce first two scalars and specify their quantum numbers later:

$$\varphi_L \overset{P}{\leftrightarrow} \varphi_R \tag{V.10}$$

Actually, as we will shown, only one of them is required to break the LR symmetry group down to $U(1)_{em}$, but the LR symmetry demand the parity-partner to be there. The introduction of the above scalars leads to the following potential,

$$V = -\frac{\mu^2}{2}(\varphi_L^2 + \varphi_R^2) + \frac{\lambda}{4}(\varphi_L^4 + \varphi_R^4) + \frac{\lambda'}{2}\varphi_L^2\varphi_R^2, \tag{V.11}$$

where linear terms do not appear since these scalars should carry quantum numbers under SU(2) [8]. The above potential may be rewritten in the following way,

$$V = -\frac{\mu^2}{2}(\varphi_L^2 + \varphi_R^2) + \frac{\lambda}{4}(\varphi_L^2 + \varphi_R^2)^2 + \frac{\lambda' - \lambda}{2}\varphi_L^2\varphi_R^2. \tag{V.12}$$

Notice that the breaking pattern depends crucially on the sign of $\lambda' - \lambda$ [13]. If $\lambda' - \lambda < 0$, the minimization of the potential requires both that $\langle \phi_R \rangle \neq 0 \neq \langle \phi_L \rangle$. This would be a problem since LR symmetry implies that $\langle \phi_L \rangle = \langle \phi_R \rangle$. On the other hand, if $\lambda' - \lambda > 0$, then $\langle \phi_L \rangle = 0$ and $\langle \phi_R \rangle \neq 0$ or vice versa, which naturally induces a parity breaking. Since we are interested in breaking the LR symmetry, i.e. $SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)} \to SU(2)_L \otimes U(1)_Y$ we opt for the second possibility.

Thus, as summarize, the breaking occurs in two steps:

$$SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)} \overset{\langle \phi_R \rangle}{\to} SU(2)_L \otimes U(1)_Y \overset{\langle \Phi \rangle}{\to} U(1)_{em}. \tag{V.13}$$

[8] which will forbid them due to symmetry invariance.

The minimal way to perform the full breaking is by introducing two Higgs doublets,

$$H_L = \begin{pmatrix} H_L^- \\ H_L^0 \end{pmatrix} \sim (1,2,1), \quad H_R = \begin{pmatrix} H_R^+ \\ H_R^0 \end{pmatrix} \sim (2,1,1). \tag{V.14}$$

There is another alternative for the role of ϕ_R and ϕ_L which leads to seesaw mechanism for the neutrino masses, in which the candidates are triplets under SU(2), i.e. $\Delta_L \sim (1,2,2)$ and $\Delta_R \sim (2,1,2)$. We will discuss the above alternatives in more detail in the following sections.

Independently on the alternative chosen to break the LR symmetry, fermions get mass through the bi-doublet, which is the responsible of breaking the electroweak symmetry. Once Φ gets a vev the masses of the fermions read as,

$$\begin{aligned} M_U &= Y_1 v_1 + Y_2 v_2^*, &\text{(V.15)}\\ M_D &= Y_1 v_2 + Y_2 v_1^*, &\text{(V.16)}\\ M_E &= Y_3 v_2 + Y_4 v_1^*, &\text{(V.17)}\\ M_\nu^D &= Y_3 v_1 + Y_4 v_2^*, &\text{(V.18)} \end{aligned}$$

where v_1 and v_2 are the vacuum expectation values for the fields ϕ_1^0 and ϕ_2^0, respectively. Notice that neutrinos in the context of LR theories get a Dirac mass in a natural way. By taking the limit in which $Y_3 \ll Y_4$ and $v_2 \ll v_1$, one can explain the smallness of neutrino masses without introducing any fine-tuning. In this context, the masses would be given by

$$\begin{aligned} M_E &\approx Y_4 v_1^*, &\text{(V.19)}\\ M_\nu^D &= v_1 \left(Y_3 + M_E \frac{v_2^*}{|v_1|^2} \right). &\text{(V.20)} \end{aligned}$$

It is important to remark that this simple model which predicts Dirac neutrinos does satisfy without any problem any constraint coming from experiment, although it does not provide a ”natural“ explanation about the smallness of the neutrino masses.

C. Majorana neutrinos

The $SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)}$ with two Higgs doublets and one Higgs bi-doublet will be the minimal LR symmetric model on the market till Dirac neutrinos are ruled out. However, physicists tend to think that the smallness of neutrino masses might be an argument for their Majorana nature. We already showed that Dirac masses for neutrinos are obtained by considering the minimal scalar content needed to break the LR symmetry to the SM. As we show in this section, there are alternative ways to break spontaneously the symmetry which imply different scalar contents for the theory.

Type-I seesaw realization in LR

An alternative way to break the LR symmetry down to the SM is by introducing two Higgs triplets, Δ_L and Δ_R [7], defined as,

$$\Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad \Delta_L \sim (3,1,2), \quad \Delta_R \sim (1,3,2). \tag{V.21}$$

The above scalar sector comes along the following terms in the Yukawa Lagrangian:

$$\mathcal{L}_Y \supset \mathcal{L}_\Delta = \frac{1}{2}\left(\ell_L^T C i\tau_2 Y_\Delta \Delta_L \ell_L + \ell_R^T C i\tau_2 Y_\Delta \Delta_R \ell_R\right) + \text{h.c.} \tag{V.22}$$

and the corresponding scalar potential,

$$\begin{aligned} V \; = \; & -\sum_{i,j=1}^{2} \mu_{ij}^2 \mathrm{Tr}\{\Phi_i^\dagger \Phi_j\} + \sum_{i,j,k,l=1}^{2} \lambda_{ijkl} \mathrm{Tr}\{\Phi_i^\dagger \Phi_j\}\mathrm{Tr}\{\Phi_k^\dagger \Phi_l\} + \lambda'_{ijkl}\mathrm{Tr}\{\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l\} - \mu^2 \mathrm{Tr}\{\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R\} \\ & + \rho_1 \left[\mathrm{Tr}\{\Delta_L^\dagger \Delta_L\}^2 + \mathrm{Tr}\{\Delta_R^\dagger \Delta_R\}^2\right] + \rho_2 \left(\mathrm{Tr}\{\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L\} + \mathrm{Tr}\{\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R\}\right) + \rho_3 \mathrm{Tr}\{\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R\} \\ & + \sum_{i,j=1}^{2} \alpha_{ij} \mathrm{Tr}\{\Phi_i^\dagger \Phi_j\}\left(\mathrm{Tr}\{\Delta_L^\dagger \Delta_L\} + \mathrm{Tr}\{\Delta_R^\dagger \Delta_R\}\right) + \sum_{i,j=1}^{2} \beta_{ij}\left(\mathrm{Tr}\{\Delta_L^\dagger \Delta_L \Phi_i \Phi_j^\dagger\} + \mathrm{Tr}\{\Delta_R^\dagger \Delta_R \Phi_i^\dagger \Phi_j\}\right) \\ & + \sum_{i,j=1}^{2} \gamma_{ij} \mathrm{Tr}\{\Delta_L^\dagger \Phi_i \Delta_R \Phi_j^\dagger\}, \end{aligned} \tag{V.23}$$

where $\Phi_1 \equiv \Phi$ and $\Phi_2 \equiv \tilde{\Phi}$. The LR symmetry is spontaneously broken once the neutral components of the scalar sector get a vev according to

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ and } \langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}. \tag{V.24}$$

Notice that only $\langle \Delta_R \rangle$ is needed to break $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \to SU(2)_L \otimes U(1)_Y$. The vevs of the bi-doublet will be responsible, as before, of the SM breaking to $U(1)_{em}$. Let us assume that $v_L = 0$ (this assumption is strictly speaking not correct since the vev of the bi-doublet forces Δ_L to get a non-zero vev [7]. However, as we will shown in next section, this vev is of the order of $O(\langle \Phi \rangle^2 / v_R) \ll \langle \Phi \rangle$). Therefore, once the LR symmetry is broken, the right-handed neutrinos get a Majorana mass through the following Yukawa Lagrangian,

$$\mathcal{L}_\Delta^{SSB} = \frac{1}{2} Y_\Delta v_R \nu_R^T C \nu_R + \text{h.c.} \Rightarrow M_R = Y_\Delta v_R. \tag{V.25}$$

Hence, we are left with an already familiar mixing of Majorana and neutrino masses,

$$\mathcal{L}_\nu = M_R \nu_R^T C \nu_R + m_D \overline{\nu_L} \nu_R + \text{h.c.} \tag{V.26}$$

which in a matrix form reads as,

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2}(Y_3 v_1 + Y_4 v_2^*) \\ \frac{1}{2}(Y_3 v_1 + Y_4 v_2^*) & -Y_\Delta v_R \end{pmatrix}. \tag{V.27}$$

This mixing, after diagonalizing the matrix, leads to the following neutrino masses:

$$M_{\nu_L} \sim -m_D^T M_R^{-1} m_D = \frac{1}{4}\frac{(Y_3 v_1 + Y_4 v_2^*)^2}{Y_\Delta v_R}, \tag{V.28}$$

$$M_{\nu_R} \sim M_R = Y_\Delta v_R. \tag{V.29}$$

From here the smallness of the left-handed neutrinos can be explained through the seesaw type-I mechanism, i.e. the heavier the right-handed neutrino mass, the lighter the left-handed neutrino mass. The process is shown in Fig. 17.

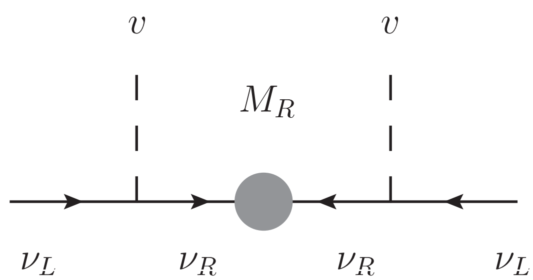


FIG. 17: Type-I seesaw mechanism in the context of LR-symmetric theories.

Type-II seesaw realization in LR

However, it can be shown [7] that, once the bi-doublet gets a vev, the term $\alpha\Delta_L^\dagger\Phi\Delta_R\Phi^\dagger$ in the scalar potential implies that $\langle\Delta_{L/R}\rangle$ cannot vanish. In the case $v_L \neq 0$ one has to put special care since left-handed neutrinos get also a Majorana mass, so that the mass matrix for the neutrinos is given by

$$M_\nu = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \equiv \begin{pmatrix} Y_\Delta v_L & \frac{1}{2}(Y_3 v_1 + Y_4 v_2^*) \\ \frac{1}{2}(Y_3 v_1 + Y_4 v_2^*) & -Y_\Delta v_R \end{pmatrix} \tag{V.30}$$

which leads to the following eigenstates:

$$M_{\nu_L} \sim M_L - m_D^T M_R^{-1} m_D = Y_\Delta v_L + \frac{1}{4}\frac{(Y_3 v_1 + Y_4 v_2^*)^2}{Y_\Delta v_R}, \tag{V.31}$$

$$M_{\nu_R} \sim M_R = Y_\Delta v_R. \tag{V.32}$$

Hence, one has to make sure that $v_L \ll M_R$ in order to apply the seesaw type-I mechanism. But, as we will show here, this vev is actually pretty suppressed by the v_R through the seesaw type-II mechanism (here we follow the approach of Mohapatra and Senjanovíc [7]).

After the spontaneous symmetry breaking, the scalar potential (see Eq. (V.23)) reads as,

$$\begin{aligned} &V(\Delta_L, \Delta_R, v_1, v_2) = \\ &-\mu^2(v_L^2 + v_R^2) + \frac{\rho}{4}(v_L^4 + v_R^4) + \frac{\rho^{'}}{2}v_L^2 v_R^2 + (v_L^2 + v_R^2)(\alpha_{11} + \alpha_{22} + \beta_{11})v_1^2 \\ &+ (\alpha_{11} + \alpha_{22} + \beta_{22})v_2^2 + (4\alpha_{12} + 2\beta_{12})v_1 v_2 + 2v_L v_R\left[(\gamma_{11} + \gamma_{22})v_1 v_2 + \gamma_{12}(v_1^2 + v_2^2)\right] \\ &+ \text{terms which only depend on } v_1 \text{ and } v_2, \end{aligned} \tag{V.33}$$

where $\rho \equiv 4(\rho_1 + \rho_2)$ and $\rho^{'} \equiv 2\rho_3$. By assuming w.l.o.g. that $v_2 \ll v_1$, in order to encourage the suppression of the $W_R - W_L$ mixing [65], the above potential can be rewritten as

$$V(\Delta_L, \Delta_R, v_1) \sim -\mu^2(v_L^2 + v_R^2) + \frac{\rho}{4}(v_L^4 + v_R^4) + \frac{\rho^{'}}{2}v_L^2 v_R^2 + \frac{\alpha}{2}(v_L^2 + v_R^2)v_1^2 + \beta v_L v_R v_1^2, \tag{V.34}$$

where $\alpha \equiv 2(\alpha_{11} + \alpha_{22} + \beta_{11})$ and $\beta \equiv 2\gamma_{12}$. By computing the minimum conditions one gets,

$$\frac{\partial V}{\partial v_L} = -\mu^2 v_L + \rho v_L^3 + \rho^{'} v_L v_R^2 + \alpha v_1^2 v_L + \beta v_1^2 v_R \stackrel{!}{=} 0, \tag{V.35}$$

$$\frac{\partial V}{\partial v_R} = -\mu^2 v_R + \rho v_R^3 + \rho^{'} v_R v_L^2 + \alpha v_1^2 v_R + \beta v_1^2 v_L \stackrel{!}{=} 0. \tag{V.36}$$

After some algebra, i.e. v_R(Eq. (V.35)) - v_L(Eq. (V.36)),

$$\left[(\rho-\rho^{'})v_Lv_R-\beta v_1^2\right](v_L^2-v_R^2)=0. \tag{V.37}$$

Two possible solutions arise from the above expression: (a) $v_L^2=v_R^2$ and (b) $(\rho-\rho^{'})v_Lv_R-\beta v_1^2=0$. Since we are interested in breaking the parity symmetry, we discard solution (a), so that we are left with

$$v_Lv_R=\frac{\beta}{(\rho-\rho^{'})}v_1^2. \tag{V.38}$$

Hence,

$$v_L\propto\frac{v_1^2}{v_R}, \tag{V.39}$$

where the proportionality constant is given by $\beta/(\rho-\rho^{'})$. The above expression reflects the type-II seesaw mechanism and one can see that, as long as $\langle\Delta_L\rangle\neq 0$, one cannot avoid a combination of type-II and type-I seesaws in the process of giving mass to the neutrinos.

Type-III seesaw realization in LR

Let us stick now into the minimal scalar content, i.e. two Higgs doublets as introduced in last section and let us extend, on the other hand, the fermion sector. Neutrinos in LR models can get mass through type-III seesaw mechanism by adding fermion triplets ρ_L (one for each family),

$$\rho_L=\frac{1}{2}\begin{pmatrix}\rho_L^0 & \sqrt{2}\rho_L^{\dagger}\\ \sqrt{2}\rho_L^- & -\rho_L^0\end{pmatrix}\sim(3,1,0)\quad\text{and}\quad\rho_R=\frac{1}{2}\begin{pmatrix}\rho_R^0 & \sqrt{2}\rho_R^{\dagger}\\ \sqrt{2}\rho_R^- & -\rho_R^0\end{pmatrix}\sim(1,3,0), \tag{V.40}$$

and the minimal scalar content required to break the LR symmetry, i.e. two Higgs doublets H_L and H_R. The realization of type-III seesaw in the context of LR symmetric models was first done by Fileviez in [66].

The relevant interactions for the type-III seesaw are given by

$$-\mathcal{L}^{III}\supset Y_\rho(\ell_L^TCi\sigma_2\rho_LH_L+l_R^TCi\sigma_2\rho_RH_R)+M_\rho\mathrm{Tr}\{\rho_L^TC\rho_L+\rho_R^TC\rho_R\}+\text{h.c.} \tag{V.41}$$

Let us build "Majorana" 4-dim spinors as follows,

$$\begin{aligned}\nu &\equiv \nu_L+(\nu_L)^C,\\ N &\equiv N_R+(N_R)^C,\\ \tilde{\rho} &\equiv \rho_R+(\rho_R)^C,\end{aligned} \tag{V.42}$$

so that $\nu^C=\nu$, $N^C=N$ and $\tilde{\rho}=\tilde{\rho}^C$. Once the Higgses get a vev, the LR symmetry is spontaneously broken since $v_R\neq v_L$. Assuming that $v_L=0$ w.l.o.g. (this limit corresponds indeed to a minimum of the scalar potential, see reference [5]), the following mass matrix is obtained in the basis $(\nu,N,\tilde{\rho})$:

$$M_\nu=\begin{pmatrix}0 & M_\nu^D & 0\\ (M_\nu^D)^T & 0 & -\frac{1}{2\sqrt{2}}Y_\rho v_R\\ 0 & -\frac{1}{2\sqrt{2}}Y_\rho v_R & M_\rho\end{pmatrix}. \tag{V.43}$$

Assuming the mass of the triplet is heavy, i.e. $M_\rho\gg Y_\rho v_R/2\sqrt{2}$, the triplet can be integrated out and ν_R

gets mass through the type-III seesaw mechanism (see section "Massive neutrinos"):

$$M_{\nu_R} = \frac{v_R^2}{8} Y_\rho (M_\rho)^{-1} Y_\rho^T . \tag{V.44}$$

Hence, one is left with the following effective mass matrix (in the basis (ν, N)),

$$M_\nu \sim \begin{pmatrix} 0 & M_\nu^D \\ (M_\nu^D)^T & M_{\nu_R} \end{pmatrix} . \tag{V.45}$$

Therefore, a further type-I seesaw occurs in the process of generating neutrino masses. The masses (eigenvalues) of the neutrinos after diagonalizing the above matrix are given by

$$M_{\chi_1} \ \sim \ M_\nu = M_\nu^D M_{\nu_R}^{-1} (M_\nu^D)^T , \tag{V.46}$$
$$M_{\chi_2} \ \sim \ M_N , \tag{V.47}$$

where the new eigenstates χ_1 and χ_2 has been approximated to ν and N, respectively, since $M_{\nu_R} \gg M_\nu^D$. Notice that this model [66] generates neutrino masses in the context of LR theories through a "double seesaw" mechanism, i.e. a combination of type-III and type-I seesaws.

In next section we introduce a simple LR extension which generates neutrino masses through radiative corrections.

VI. SIMPLE LR-SYMMETRIC MODEL WITH MAJORANA NEUTRINOS

We present a simple LR symmetric model with the minimal degrees of freedom that predicts Majorana neutrinos. This model is characterized by the inclusion of a charged scalar singlet which will play an important role in the neutrino mass generation. As opposed to the models introduced before, in this model neutrinos get mass through the Zee mechanism. We therefore will address to this model as "Zee-LR".

This section is organized as follows. First, we present the field content of the model and we show that, after spontaneous symmetry breaking, the limit in which we work is indeed a minimum of the scalar potential and therefore corresponds to a physical scenario. Then, we discuss the Zee-mechanism in the context of our model and we show how neutrinos get Majorana masses. Next, we introduce the new gauge bosons predicted by the Zee-LR model and study their most relevant properties. In the last part of this section, we discuss some phenomenological aspects which are a direct consequence of this particular model.

A. Field content of the Zee-LR model

Apart of the basic field content of any LR theory (i.e. fermion content introduced in last section plus the bi-doublet), the Zee-SU(5) model requires two scalar doublets to break the LR gauge symmetry (H_L and H_R). Moreover, an extra scalar charged singlet δ^+ is included to give mass to the neutrinos through radiative corrections, as we discuss in next section. Hence, the scalar content of our model, excluding the bi-doublet, is composed on:

$$H_L = \begin{pmatrix} H_L^+ \\ H_L^0 \end{pmatrix} \sim (2,1,1), \qquad H_R = \begin{pmatrix} H_R^+ \\ H_R^0 \end{pmatrix} \sim (1,2,1), \qquad \delta^+ \sim (1,1,2).$$

In the context of this model, the most general renormalizable scalar potential that can be written satisfying LR symmetry constraints reads as

$$\begin{aligned} V = &- \mu_H^2 (H_L^\dagger H_L + H_R^\dagger H_R) + \lambda_H ((H_L^\dagger H_L)^2 + (H_R^\dagger H_R)^2) + \lambda_{LR} (H_L^\dagger H_L)(H_R^\dagger H_R) \\ &- (\mu_\Phi^2)_{ij} \mathrm{Tr}(\Phi_i^\dagger \Phi_i) + \lambda_{ijkl}^{(1)} \mathrm{Tr}(\Phi_i^\dagger \Phi_j) \mathrm{Tr}(\Phi_k^\dagger \Phi_l) + \lambda_{ijkl}^{(2)} \mathrm{Tr}(\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l) \\ &+ a_{ij} (H_L^\dagger H_L + H_R^\dagger H_R) \mathrm{Tr}(\Phi_i^\dagger \Phi_j) + b_{ij} (H_L^\dagger \Phi_i \Phi_j^\dagger H_L + H_R^\dagger \Phi_i^\dagger \Phi_j H_R) + c_i (H_L^\dagger \Phi_i H_R + H_R^\dagger \Phi_i^\dagger H_L) \\ &- \mu_\delta^2 \delta^- \delta^+ + \lambda_\delta (\delta^- \delta^+)^2 + d (H_L^\dagger H_L + H_R^\dagger H_R) \delta^- \delta^+ + e_{ij} \mathrm{Tr}(\Phi_i^\dagger \Phi_j) \delta^- \delta^+ \\ &+ \lambda_i (H_L^T i\sigma_2 \Phi_i H_R \delta^- - H_L^* i\sigma_2^T \Phi_i^\dagger H_R^\dagger \delta^+) , \end{aligned}$$

where $(\mu_\Phi^2)_{ij} = (\mu_\Phi^2)_{ji}\,, \quad \lambda_{ijkk}^{(1)} = \lambda_{jikk}^{(1)}\,, \quad \lambda_{ijkl}^{(1)} = \lambda_{klij}^{(1)}\,, \quad \lambda_{ijkl}^{(1)} = \lambda_{jilk}^{(1)}\,,$
$\lambda_{ijkl}^{(2)} = \lambda_{jkli}^{(2)} = \lambda_{klij}^{(2)} = \lambda_{lijk}^{(2)}\,, \quad a_{ij} = a_{ji}\,, \quad b_{ij} = b_{ji}\,, \quad e_{ij} = e_{ji}\,.$

In order to break the LR symmetry and get $M_{W_R} \gg M_{W_L}$, one must assume $v_R \gg v_L, v_1, v_2$. In the appendix we show that this scenario does correspond to a minimum of the above potential and it is therefore realistic. We also derive the masses of the scalar sector as a function of the parameters of the scalar potential for the limit $v_1 = v_2 = 0$.

B. Majorana neutrinos through the Zee mechanism

In the context of the Zee-LR model, neutrinos get radiative mass through the Zee mechanism. Fig. 18 shows the realization of the Zee mechanism in the unbroken phase. Here, neutrinos can get radiative mass at 1-loop, unlike in the usual left-right symmetric models presented in literature, due to the presence of the charged singlet δ^+. Notice that for 1-loop radiative neutrino masses we only need to consider the charged scalar fields Φ_a^+ and δ^+, where $a = 1, 2$.

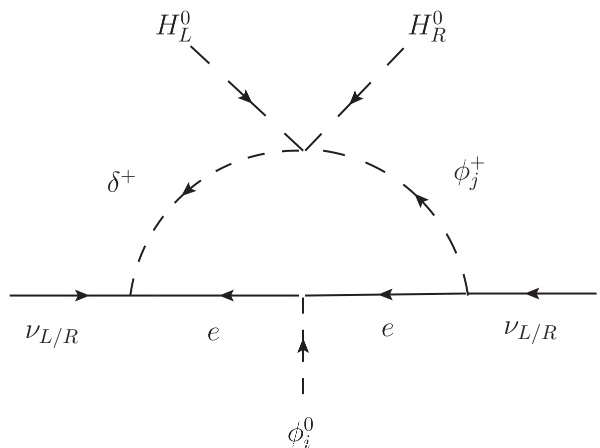


FIG. 18: Zee mechanism generating Majorana left/right-handed neutrino masses.

The relevant interactions in the Lagrangian to generate neutrino masses at the quantum level are given by,

$$-\mathcal{L}^{\text{Zee}}_{\text{unbroken}} \supset \bar{\ell}_L \left(Y_3\Phi + Y_4\tilde{\Phi}\right)\ell_R + \lambda_L \ell_L \ell_L \delta^+ + \lambda_R \ell_R \ell_R \delta^+ + \lambda_1 H_L^T i\sigma_2 \Phi H_R \delta^- + \lambda_2 H_L^T i\sigma_2 \tilde{\Phi} H_R \delta^- + \text{h.c.} \tag{VI.1}$$

Notice that here $\lambda_L \neq \lambda_R$, i.e. we will assume the discrete left-right parity symmetry to hold only in the gauge sector since we are mainly interested in the case where the LR symmetry scale is low and besides, since the discrete symmetry is not spontaneously broken, domain wall problems will be avoided [67].

In the broken phase, the Higgs sector can be written explicitly as,

$$\Phi = \begin{pmatrix} v_1 + \phi_1^0 + iA_1^0 & \phi_2^+ \\ \phi_1^- & v_2 + \phi_2^0 + iA_2^0 \end{pmatrix},\ H_L = \begin{pmatrix} h_L^+ \\ \frac{1}{\sqrt{2}}(v_L + h_L^0 + iA_L^0) \end{pmatrix} \text{ and } \quad H_R = \begin{pmatrix} h_R^+ \\ \frac{1}{\sqrt{2}}(v_R + h_R^0 + iA_R^0) \end{pmatrix}.$$

Once the symmetry is spontaneously broken, one has five charged scalar fields, $\phi_1^\pm$, $\phi_2^\pm$, $h_R^\pm$, $h_L^\pm$, and $\delta^\pm$, four CP-even neutral scalar fields, h_L^0, h_R^0, ϕ_1^0 and ϕ_2^0 and four CP-odd neutral scalar fields A_L^0, A_R^0, A_1^0 and A_2^0. The Higgses get mixed and one needs to define a basis in which fields are physical, i.e. the mass is well-defined. We assume there exists a unitary matrix V which rotates the charged scalar fields from the interaction basis to the physical basis with properly defined mass. This change of basis is defined as

$$\begin{pmatrix} \Phi_1^+ \\ \Phi_2^+ \\ H_L^+ \\ H_R^+ \\ \delta^+ \end{pmatrix} = V \begin{pmatrix} h_1^+ \\ h_2^+ \\ h_3^+ \\ h_4^+ \\ h_5^+ \end{pmatrix} , \tag{VI.2}$$

and thus we find $\Phi_a^+ = V_{ai}\, h_i^+$ and $\delta^+ = V_{5i}\, h_i^+$ where $i = 1, ..., 5$ and $a = 1, 2$. In the broken phase, the Feynman rules for the physical fields are obtained from the following Lagrangian (see appendix for the Feynman rules),

$$\begin{aligned} \mathcal{L}^{Zee}_{broken} =& \bar{e}\left[(Y_3^\dagger P_L - Y_4 P_R)V_{2i}^* + (Y_3 P_R - Y_4^\dagger P_L)V_{1i}^*\right] h_i^- \nu + 2\, \bar{\nu^c}(\lambda_L P_L + \lambda_R P_R)V_{5i} h_i^+ e + \text{h.c.} \\ &+ \text{extra terms}, \end{aligned} \tag{VI.3}$$

where the fields have been rotated to the physical basis. The "extra terms" refers to the rest of interactions which do not participate directly in the process bellow. We are not writing them here explicitly since our interest resides in the amputated amplitude shown in Fig. 19 which, in the broken phase, will give us the radiative left/right-handed neutrino Majorana mass (of course the rest of the terms are participating implicitly through the definition of the components of the mixing matrix of the charged Higgses, which in turn defines the components of the rotation matrix V). Let us first calculate the left-handed neutrino radiative

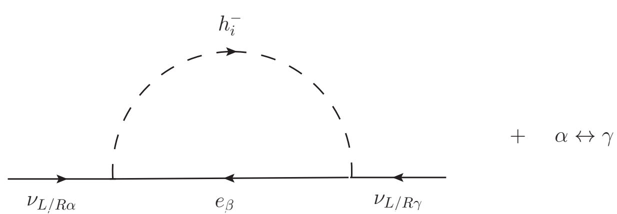


FIG. 19: Zee model generating Majorana left/right-handed neutrino masses.

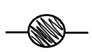
mass. By using the feynman rules in the broken phase, the 1-loop correction reads as,

$$\begin{aligned} &= \sum_i \int \frac{d^4k}{(2\pi)^4} 2\lambda_L^{\alpha\beta} V_{5i} P_L \frac{\not{k} + m_{e_\beta}}{k^2 - m_{e_\beta}^2}\left[(Y_3^\dagger)^{\beta\gamma} V_{2i}^* - (Y_4^\dagger)^{\beta\gamma} V_{1i}^*\right]\frac{1}{k^2 - M_{h_i}^2} = \\ &= 2\lambda_L^{\alpha\beta} P_L m_{e_\beta} \sum_i V_{5i}\left[(Y_3^\dagger)^{\beta\gamma} V_{2i}^* - (Y_4^\dagger)^{\beta\gamma} V_{1i}^*\right] \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_{e_\beta}^2}\frac{1}{k^2 - M_{h_i}^2}. \end{aligned} \tag{VI.4}$$

The above integral can be computed through the dimensional regularization method proceeding as in section "Zee mechanism" (see the mentioned section for the detailed calculation of the integral at issue).

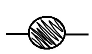
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_e^2}\frac{1}{k^2 - M_{h_i}^2} = \frac{i}{(4\pi)^2}\left(\frac{2}{\epsilon} - 1 - \gamma + \text{Log}\left(\frac{4\pi\mu_R^2}{m_e^2}\right) + \frac{M_{h_i}^2 \text{Log}\left(\frac{m_e^2}{M_{h_i}^2}\right)}{M_{h_i}^2 - m_e^2}\right)\mu_R^{-\epsilon}, \tag{VI.5}$$

and thus one gets a solution which may be split in two terms: one that depends on i (on the charged Higgs that is running over the loop) and the other one which does not. This last term will be killed by the unitarity of the V matrix after we sum over all possible charged Higgses contributing in the 1-loop correction to the mass, i.e. the divergence will cancel along with the unphysical renormalization scale (by taking $\epsilon \to 0$), which shows explicitly the consistency of the theory. Putting everything together [9],



$$= \frac{i}{8\pi^2}\lambda_L^{\alpha\beta} P_L m_{e_\beta} \sum_i V_{5i}\mu_R^{-\epsilon}\left(1+\frac{1}{\epsilon}+\log\left(\frac{\mu_R^2}{m_{e_\beta}^4}\right)+\frac{M_{h_i}^2 \mathrm{Log}\left(\frac{m_{e_\beta}^2}{M_{h_i}^2}\right)}{M_{h_i}^2-m_{e_\beta}^2}\right)\left[(Y_3^\dagger)^{\beta\gamma}V_{2i}^* - (Y_4^\dagger)^{\beta\gamma}V_{1i}^*\right]. \tag{VI.6}$$

Assuming $M_{h_i}^2 \gg m_{e_\beta}^2$,



$$\begin{aligned} &= \frac{i}{8\pi^2}\lambda_L^{\alpha\beta} P_L m_{e_\beta}\Bigg[\left(1+\frac{1}{\epsilon}+\mathrm{Log}\left(\frac{\mu R^2}{m_{e_\beta}^4}\right)\right)\left((Y_3^\dagger)^{\beta\gamma}\delta_{52}-(Y_4^\dagger)^{\beta\gamma}\delta_{51}\right) \\ &\quad - \sum_i \mathrm{Log}\left(\frac{M_{h_i}^2}{m_{e_\beta}^2}\right) V_{5i}\left((Y_3^\dagger)^{\beta\gamma}V_{2i}^* - (Y_4^\dagger)^{\beta\gamma}V_{1i}^*\right)\Bigg]\mu^{-\epsilon}. \end{aligned} \tag{VI.7}$$

Indeed, the unitarity kills the divergence (i.e. $\delta_{51} = 0 = \delta_{52}$) and μ_R disappears when taking $\epsilon \to 0$. Therefore, the Majorana mass matrix of the neutrinos acquired through the Zee mechanism reads as

$$(M_\nu^L)^{\alpha\gamma} = \frac{1}{4\pi^2}\lambda_L^{\alpha\beta} m_{e_\beta}\sum_i \mathrm{Log}\left(\frac{M_{h_i}^2}{m_{e\beta}^2}\right) V_{5i}\left[(Y_3^\dagger)^{\beta\gamma}V_{2i}^* - (Y_4^\dagger)^{\beta\gamma}V_{1i}^*\right] \;+\; \alpha \leftrightarrow \gamma\,, \tag{VI.8}$$

and proceeding analogously for the right-handed neutrino, its Majorana mass matrix reads as

$$(M_\nu^R)^{\alpha\gamma} = \frac{1}{4\pi^2}\lambda_R^{\alpha\beta} m_{e_\beta}\sum_i \mathrm{Log}\left(\frac{M_{h_i}^2}{m_{e_\beta}^2}\right) V_{5i}\left[(Y_3)^{\beta\gamma}V_{1i}^* - (Y_4)^{\beta\gamma}V_{2i}^*\right] \;+\; \alpha \leftrightarrow \gamma\,. \tag{VI.9}$$

We therefore find that the neutrino mass matrix in the basis $(\nu, \nu^C)_L$ is given by

$$M_\nu = \begin{pmatrix} M_\nu^L & m_\nu^D \\ m_\nu^D & M_\nu^R \end{pmatrix}\,. \tag{VI.10}$$

One-loop corrections to the Dirac mass matrix vanish, so that we only need to consider its tree-level contribution which is given by

$$m_\nu^{D\,\alpha\gamma} = (Y_3)^{\alpha\gamma}v_1 + (Y_4)^{\alpha\gamma}v_2^*\,. \tag{VI.11}$$

Notice that, from Eq. (VI.8), M_ν^L vanishes when, for all i, $Y_3 V_{2i}^* = Y_4 V_{1i}^*$ holds, which imply that $Y_3 = Y_4 = 0$ and thus the complete mass matrix M_ν vanishes.

Regarding the above expressions for the Dirac and Majorana neutrino masses, two different scenarios may take place, which are discussed in the following.

[9] Notice that this renormalization scale corrects the dimensions of the coupling λ in such a way that $\tilde{\lambda} \equiv \lambda\mu_R^{-\epsilon}$ is dimensionless for all d.

Pseudo-Dirac neutrinos

In the limit where the Majorana masses are small in comparison with the Dirac mass, i.e. $M_\nu^R, M_\nu^L \gg M_\nu^D$, a pair of almost degenerate eigenstates with a tiny mass difference will appear. The mixing angle is almost maximal,

$$\tan\theta = \frac{2M_\nu^D}{M_\nu^R - M_\nu^L} \Rightarrow \theta \sim \frac{\pi}{2}\text{rad}, \tag{VI.12}$$

and the sum of both Majorana masses is the responsible of the slightly breaking of the mass degeneracy.

Neutrinos in this context are called Pseudo-Dirac or quasi-Dirac neutrinos [68]. This scenario predicts (almost) maximal mixing solution of neutrino oscillations into a sterile state which is encouraged by some experiments and disfavored by others [69]. Although their current status is not encouraged, they are not ruled out (yet).

Low-scale seesaw mechanism

In the limit where $v_2 \ll v_1$, the lepton mass reads as

$$m_e^{\alpha\gamma} \simeq Y_4^{\alpha\gamma} v_1^* \,. \tag{VI.13}$$

Considering this limit in the neutrino mass terms, we have

$$\begin{aligned}
m_\nu^{D\alpha\gamma} &= (Y_3)^{\alpha\gamma} v_1 + m_e^{\alpha\gamma}\frac{v_2^*}{v_1^*}\,, && \text{(VI.14)}\\
M_\nu^{L\alpha\gamma} &= \frac{1}{4\pi^2}\lambda_L^{\alpha\beta} m_{e_\beta}\sum_i \text{Log}\left(\frac{M_{h_i}^2}{m_{e_\beta}^2}\right) V_{5i}\left[(Y_3^\dagger)^{\beta\gamma}V_{2i}^* - \frac{m_e^{\dagger\beta\gamma}}{v_1}V_{1i}^*\right]\,, && \text{(VI.15)}\\
M_\nu^{R\alpha\gamma} &= \frac{1}{4\pi^2}\lambda_R^{\alpha\beta} m_{e_\beta}\sum_i \text{Log}\left(\frac{M_{h_i}^2}{m_{e_\beta}^2}\right) V_{5i}\left[(Y_3)^{\beta\gamma}V_{1i}^* - \frac{m_e^{\beta\gamma}}{v_1^*}V_{2i}^*\right]\,. && \text{(VI.16)}
\end{aligned}$$

Notice that for $Y_3 \ll Y_4$, as we have already discussed, Dirac masses are suppressed and are naturally small. In this context, the neutrino mass matrix in the basis $\begin{pmatrix}\nu_L & (\nu_R)^c\end{pmatrix}$ reads as,

$$\begin{pmatrix}
\frac{1}{4\pi^2}\lambda_L m_e \sum_i \text{Log}\left(\frac{M_{h_i}^2}{m_e^2}\right) V_{5i}\left[Y_3^\dagger V_{2i}^* - \frac{m_e^\dagger}{v_1}V_{1i}^*\right] & Y_3 v_1 + m_e\frac{v_2^*}{v_1^*} \\
Y_3^T v_1 + m_e\frac{v_2^*}{v_1^*} & \frac{1}{4\pi^2}\lambda_R m_e \sum_i \text{Log}\left(\frac{M_{h_i}^2}{m_e^2}\right) V_{5i}\left[Y_3 V_{1i}^* - \frac{m_e}{v_1^*}V_{2i}^*\right]
\end{pmatrix}. \tag{VI.17}$$

Assuming that the parity breaking couplings fulfill $\lambda_L \ll \lambda_R$,

$$M_\nu^L \sim 0, \tag{VI.18}$$

and thus it can be neglected in front of M_ν^R. Therefore, in the limit where $\lambda_L \ll \lambda_R$ and $M_\nu^D \ll M_\nu^R$ a low scale seesaw takes place, i.e.

$$(M_1)^{\alpha\gamma} \simeq -\frac{[(M_\nu^D)^{\alpha\gamma}]^2}{(M_\nu^R)^{\alpha\gamma}} + \alpha \leftrightarrow \gamma\,, \tag{VI.19}$$

$$(M_2)^{\alpha\gamma} \simeq (M_\nu^R)^{\alpha\gamma} + \alpha \leftrightarrow \gamma\,. \tag{VI.20}$$

For this model (Zee-LR) to be consistent one must check that the limit $v_2 \ll v_1$ we are dealing with corresponds to a physical scenario by studying the scalar potential. In the appendix we show that this limit is indeed realistic since it minimizes the potential. The name of ”low-scale” seesaw refers to the lightness of right-handed neutrinos, which we justify in the following.

We may wonder about the mass scale of the right-handed neutrinos. Notice that the sum over the charged Higgs masses weighted by the combination of mixing matrices and Yukawa couplings is strongly constraining the scale of the neutrino masses due to the unitarity of the mixing matrices. It is straightforward to see that there is no lower bound for the right-handed neutrino mass since it turns to be zero when the charged Higgs masses are degenerated. However, we can infer some upper bounds by assuming a conservative scenario where the entries of λ_R and Y_4 are of order ~ 1 and the mass of the lepton appearing in Eq.(VI.16) corresponds to the tau mass, $m_\tau \sim 1.78$ GeV. Due to the unitarity of the mixing matrices, the highest value that the logarithm could reach is roughly twice the number corresponding to the highest difference between the order of magnitude of the charged Higgs masses. Assuming the extreme case in which one charged Higgs is sitting at 100 GeV and another one at the Plank scale (assuming optimistically that our QFT is valid until 10^{19} GeV), the upper theoretical bound for the right-handed neutrino mass would be

$$M_\nu^R < 150 \text{ GeV} \tag{VI.21}$$

However, more realistically one would expect the mass of the charged Higgses to be around the TeV scale. Assuming the lightest scalar field to have a mass of the order of 100 GeV, we find

$$m_\nu^R \sim 0.4 \text{ GeV}, \tag{VI.22}$$

which give us an idea about how light right handed neutrinos are expected to be in the context of this model.

C. New gauge bosons

Some of the gauge bosons of the theory get mass once they “eat” the Goldstone modes (by going to the unitary gauge, so that the gauge gets fixed) generated after the following breaking pattern:

$$SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \to U(1)_{em}. \tag{VI.23}$$

After the spontaneous symmetry breaking, they get mass through the coupling with the Higgses whose neutral components get a vev in the kinetic term of the Lagrangian,

$$\mathcal{L}_{kinetic} = \text{Tr}\{(D_\mu\phi)^\dagger(D^\mu\phi)\} + (D_\mu H_L)^\dagger(D^\mu H_L) + (D_\mu H_R)^\dagger(D^\mu H_R), \tag{VI.24}$$

where the covariant derivatives are defined as,

$$D_\mu\phi \;=\; \partial_\mu\phi + ig_L W^a_{\mu L} T_{La}\phi - ig_R\phi W^a_{\mu R} T_{Ra}, \tag{VI.25}$$

$$D_\mu H_{L,R} \;=\; \{\partial_\mu + ig_{L,R} W^a_{\mu L,R} T_{L,Ra} + ig_{(B-L)}\frac{(B-L)}{2} Z^0_{BL\mu}\mathbb{1}\}H_{L,R}. \tag{VI.26}$$

For the calculation of the gauge boson mass matrices, the following normalizations have been assumed:

$$T^a_{L,R} = \frac{\sigma^a_{L,R}}{2},\;\; W^a_\mu T_{L,Ra} = \begin{pmatrix} \frac{1}{2}W^3_\mu & \frac{1}{\sqrt{2}}W^+_\mu \\ \frac{1}{\sqrt{2}}W^-_\mu & -\frac{1}{2}W^3_\mu \end{pmatrix} \equiv \begin{pmatrix} \frac{1}{2}W^0_\mu & \frac{1}{\sqrt{2}}W^+_\mu \\ \frac{1}{\sqrt{2}}W^-_\mu & -\frac{1}{2}W^0_\mu \end{pmatrix} \tag{VI.27}$$

where in the last step we have renamed the gauge bosons ${W_\mu}^3_{L/R} \to {W^0_\mu}_{L/R}$ due to their neutral charge nature. In the above, the mass matrices are assumed to be in the following basis,

$$\mathcal{L}_M = \begin{pmatrix} W_L^- & W_R^- \end{pmatrix} M_{\text{charged}} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} + \frac{1}{2} \begin{pmatrix} W_L^0 & W_R^0 & Z_{BL}^0 \end{pmatrix} M_{\text{neutral}} \begin{pmatrix} W_L^0 \\ W_R^0 \\ Z_{BL}^0 \end{pmatrix} . \tag{VI.28}$$

The mass matrix for the charged gauge bosons reads as

$$M_{\text{charged}} = \begin{pmatrix} \frac{g_L^2}{2}(\frac{1}{2}v_L^2 + v^2) & -g_L g_R v_1 v_2 \\ -g_L g_R v_1 v_2 & \frac{g_R^2}{2}(\frac{1}{2}v_R^2 + v^2) \end{pmatrix} , \tag{VI.29}$$

Here, the mixing angle between W^+ and W^- is given by $\tan 2\theta_{LR} \approx 8\frac{g_L}{g_R}\epsilon_{12}$, where $\epsilon_{12} = v_1 v_2 / v_R^2$ [10]. In the limit where $v_R \gg v_1, v_2, v_L$, the charged gauge bosons basically do not mix with each other, i.e. $\theta_{LR} \sim 0$, so we can ignore it and assume W_R^+ and W_L^+ to be (approximately) physical states. Therefore, the W_R mass is given by

$$M_{W_R}^2 \simeq \frac{g_R^2}{4} v_R^2. \tag{VI.30}$$

For the neutral gauge bosons, the mass matrix reads as

$$M_{\text{neutral}} = \begin{pmatrix} \frac{g_L^2}{2}(\frac{1}{2}v_L^2 + v^2) & -\frac{g_L g_R}{2} v^2 & -\frac{g_L g_{BL}}{4} v_L^2 \\ -\frac{g_L g_R}{2} v^2 & \frac{g_R^2}{2}(\frac{1}{2}v_R^2 + v^2) & -\frac{g_R g_{BL}}{4} v_R^2 \\ -\frac{g_L g_{BL}}{4} v_L^2 & -\frac{g_R g_{BL}}{4} v_R^2 & \frac{g_{BL}^2}{4}(v_R^2 + v_L^2) \end{pmatrix} . \tag{VI.31}$$

In general, due to the breaking pattern $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \to U(1)_{em}$ we expect on one hand a total of six massive gauge bosons and, on the other hand, one massless gauge boson, which will correspond to the photon. Hence, the above matrix should contain a zero eigenvalue. In general, the neutral gauge boson mass matrix may be diagonalized by a general rotation matrix V with three Euler angles ($V \in O(3)$), but it turns out that performing the following rotation:

$$\begin{aligned} W_L^0 &= \cos\theta_W Z_L + \sin\theta_W A \,, \\ W_R^0 &= \cos\theta_R Z_R - \sin\theta_W \sin\theta_R Z_L + \cos\theta_W \sin\theta_R A \,, \\ Z_{BL}^0 &= -\sin\theta_R Z_R - \sin\theta_W \cos\theta_R Z_L + \cos\theta_W \cos\theta_R A \,, \end{aligned} \tag{VI.32}$$

the photon decouples automatically and one only needs two angles to rotate the gauge bosons to the physical basis. Here, θ_W corresponds to the Weinberg angle, i.e. $\tan\theta_W = g_Y/g_L$ where $g_Y = g_{BL} g_R / \sqrt{g_{BL}^2 + g_R^2}$, and θ_R is defined as $\tan\theta_R = g_{BL}/g_R$.

The rotation (VI.32), called R, decomposes in two: $R = R_2(\theta_W) R_1(\theta_R)$. The g_Y is defined by the breaking of the LR model to the SM, i.e. $SU(2)_R \otimes U(1)_{B-L} \to U(1)_Y$, which corresponds to the first rotation, which rotates W_R^0 and Z_{BL} to a heavy gauge boson plus the hypercharge operator in the SM, B,

[10] for a 2x2 matrix, the angle which describes the rotation to the physical basis satisfies that $\tan(2\theta) = \frac{A_{12}+A_{21}}{A_{11}-A_{22}}$, where A_{ij} are the entries of the given 2x2 matrix.

leaving the W_L^0 invariant, i.e.

$$\begin{pmatrix} Z' \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} W_R^0 \\ Z_{BL} \end{pmatrix}. \tag{VI.33}$$

Hence,

$$W_R^0 = \cos\theta_R Z^{'} + \sin\theta_R B, \tag{VI.34}$$
$$Z_{BL} = -\sin\theta_R Z^{'} + \cos\theta_R B. \tag{VI.35}$$

On the other hand, from the kinetic terms on the form $\bar{\Psi}\not{D}\Psi$ we know that the terms proportional to the hypercharge operator B (after rotation $R(\theta_R)$) are

$$\bar{\Psi}\not{D}\Psi \supset \left(\frac{(B-L)}{2} g_{BL}\cos\theta_R + g_R T_3^R \sin\theta_R\right)\not{B}. \tag{VI.36}$$

Taking into account the definition of the hypercharge, $\frac{B-L}{2} = Y - T_3^R$, and substituting it in the above equation we have,

$$\bar{\Psi}\not{D}\Psi \supset Y g_{BL}\cos\theta_R \not{B} - T_3(-g_{BL}\cos\theta_R + g_R\sin\theta_R)\not{B}. \tag{VI.37}$$

Hence, since $\cos\theta_R = \frac{g_R}{\sqrt{g_R^2+g_{BL}^2}}$, the hypercharge coupling is identified as

$$g_Y = \frac{g_{BL} g_R}{\sqrt{g_R^2 + g_{BL}^2}}. \tag{VI.38}$$

The second rotation is related with the breaking of the SM down to $U(1)_{em}$. It rotates W_L^0 and B to A and Z, leaving Z' invariant.

The kinetic part of the Lagrangian involving fermion-gauge boson interactions reads as

$$\begin{aligned}\mathcal{L} \in\; & \frac{g_{R,L}}{2\sqrt{2}}\left(\bar{u}\gamma^\mu(1\pm\gamma_5)W^+_{R,L\mu}d + \bar{\nu}\gamma^\mu(1\pm\gamma_5)W^+_{R,L\mu}e + \text{h.c.}\right) \\ & + \frac{g_{R,L}}{4}\left(\bar{u}\gamma^\mu(1\pm\gamma_5)W^3_{R,L\mu}u - \bar{d}\gamma^\mu(1\pm\gamma_5)W^3_{R,L\mu}d + \bar{\nu}\gamma^\mu(1\pm\gamma_5)W^3_{R,L\mu}\nu - \bar{e}\gamma^\mu(1\pm\gamma_5)W^3_{R,L\mu}e\right) \\ & + \frac{g_{BL}}{2}\left(\frac{1}{3}\bar{u}\gamma^\mu Z_{BL\mu}u + \frac{1}{3}\bar{d}\gamma^\mu Z_{BL\mu}d - \bar{\nu}\gamma^\mu Z_{BL\mu}\nu - \bar{e}\gamma^\mu Z_{BL\mu}e\right).\end{aligned} \tag{VI.39}$$

And after rotating the fields to the physical basis (applying rotation (VI.32)) we can easily compute from there the Feynman rules, which are listed in the appendix. We show here an explicit example of how this rotation has enough freedom to fully reproduce the SM in a consistent way:

We know that the interaction of the photon with leptons is proportional to their electric charge and that, since neutrinos are chargeless, they must not couple to the photon. We are showing that this is indeed accomplished in the context of LR symmetric theories. From the kinetic term $\bar{\ell}_L D\ell_L + \bar{\ell}_R D\ell_R$, taking into account only the neutral gauge boson interactions,

$$\begin{aligned}
\bar{\ell}_L \not{D} \ell_L + \bar{\ell}_R \not{D} \ell_R \quad & \supset \frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{e_L} \end{pmatrix} \begin{pmatrix} g_L \not{W}_L^0 - g_{BL} \not{Z}_{BL} & 0 \\ 0 & -\not{W}_L^0 - g_{BL} \not{Z}_{BL} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \\
& + \begin{pmatrix} \overline{\nu_R} & \overline{e_R} \end{pmatrix} \begin{pmatrix} g_R \not{W}_R^0 - g_{BL} \not{Z}_{BL} & 0 \\ 0 & -g_R \not{W}_R^0 - g_{BL} \not{Z}_{BL} \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \\
& = \frac{1}{4} \bar{\nu} \left[g_L \not{W}_L^0 (1 - \gamma_5) + g_R \not{W}_R^0 (1 + \gamma_5) - 2 g_{BL} \not{Z}_{BL} \right] \nu \\
& + \frac{1}{4} \bar{e} \left[-g_L \not{W}_L^0 (1 - \gamma_5) - g_R \not{W}_R^0 (1 + \gamma_5) - 2 g_{BL} \not{Z}_{BL} \right] e.
\end{aligned}$$

Rotating the gauge bosons according to (VI.32) and considering only the interactions with the photon, for neutrinos we have (v_γ for the vector coupling and a_γ for the axial one)

$$\begin{aligned}
v_\gamma &= \frac{1}{4} \left(g_L \sin\theta_W + g_R \cos\theta_W \sin\theta_R - 2 g_{BL} \cos\theta_W \cos\theta_R \right) \\
&= \frac{1}{4} \cos\theta_W \cos\theta_R \left(g_{BL} + g_R \tan\theta_R - 2 g_{BL} \right) = 0.
\end{aligned} \tag{VI.40}$$

$$\begin{aligned}
a_\gamma &= \frac{1}{4} \left(g_L \sin\theta_W + g_R \cos\theta_W \sin\theta_R - 2 g_{BL} \cos\theta_W \cos\theta_R \right) \\
&= \frac{1}{4} \cos\theta_W \cos\theta_R \left(g_{BL} + g_R \tan\theta_R - 2 g_{BL} \right) = 0.
\end{aligned} \tag{VI.41}$$

from where we effectively see that there is no coupling with the photon, as expected. On the other hand, for the charged leptons,

$$\begin{aligned}
v_\gamma &= -\frac{1}{4} \left(g_L \sin\theta_W + g_R \cos\theta_W \sin\theta_R + 2 g_{BL} \cos\theta_W \cos\theta_R \right) \\
&= -\frac{1}{4} \cos\theta_W \cos\theta_R \left(g_{BL} + 2 g_{BL} + g_R \tan\theta_R \right) = -g_{BL} \cos\theta_R \cos\theta_W.
\end{aligned} \tag{VI.42}$$

$$a_\gamma = -\frac{1}{4} \left(g_L \sin\theta_W - g_R \cos\theta_W \sin\theta_R \right) = -\frac{1}{4} \cos\theta_W \cos\theta_R \left(g_{BL} - g_R \tan\theta_W \right) = 0. \tag{VI.43}$$

and thus, one has the expected vector coupling with the photon proportional to the electric charge of the lepton, defined as $e = g_L \sin\theta_W = g_{BL} \cos\theta_R \cos\theta_W$ and no axial coupling, as predicted by the SM. The rest of the Feynman rules can be found in the appendix.

Once the photon decouples, we are left with the still mixed massive Z-Z' gauge bosons,

$$\mathcal{M}^2_{Z-Z'} = \begin{pmatrix} M^2_{RR} & M^2_{LR} \\ M^2_{LR} & M^2_{LL} \end{pmatrix}, \tag{VI.44}$$

where the entries of the mass matrix are

$$M^2_{RR} = \frac{v_R^2}{4}\left(g^2_{BL} + g^2_R\right) + \epsilon\,\frac{g_R^4 v_R^2}{2(g^2_{BL} + g^2_R)}\,, \tag{VI.45}$$

$$M^2_{LR} = -\epsilon\,\frac{g^2_R v^2_R \sqrt{g^2_L g^2_R + g^2_{BL}(g^2_L + g^2_R)}}{2(g^2_{BL} + g^2_R)}\,, \tag{VI.46}$$

$$M^2_{LL} = \epsilon\, v^2_R\,\frac{g^2_L g^2_R + g^2_{BL}(g^2_L + g^2_R)}{2(g^2_{BL} + g^2_R)}\,, \tag{VI.47}$$

with $\epsilon = (v_1^2 + v_2^2)/v_R^2$. Hence, the mixing angle between $Z - Z'$ is given by

$$\tan 2\xi \approx \epsilon\,\frac{-4g^2_R\sqrt{g^2_L g^2_R + g^2_{BL}(g^2_L + g^2_R)}}{(g^2_{BL} + g^2_R)^2}\,, \tag{VI.48}$$

Notice that this angle is of the order of ϵ so that it is highly suppressed in the limit $v_R \gg v_1, v_2$, as one would expect from the electroweak precision constraints. Furthermore, for $v_L \to 0$, the mass term of the Z and $W_L^{\pm}$ gauge bosons, $M_Z \sim v^2 \frac{g_L^2 g_R^2 + g_{BL}^2(g_L^2+g_R^2)}{2(g_{BL}^2+g_R^2)}$ and $M_{W_L} \sim \frac{1}{2} v^2 g_L^2$ respectively, are related as

$$M^2_{W_L} = \cos^2\theta_W M_Z^2 \tag{VI.49}$$

where $\cos\theta_W = \frac{g_L}{\sqrt{g_Y^2 + g_L^2}}$ according to the definition of θ_W above. This is a beautiful result that shows the consistency of this model with the standard theory when one takes the limit $M_{W_R} \to \infty$.

In the limit $v_R >> v_1, v_2, v_L$, the masses of the new gauge bosons are given by

$$M_{W_R} \simeq \frac{1}{2} g_R v_R\,, \tag{VI.50}$$

$$M_{Z'} \simeq \frac{1}{4} v_R^2 (g^2_{BL} + g^2_R) = \frac{\sqrt{g^2_{BL} + g^2_R}}{g_R} M_{W_R}. \tag{VI.51}$$

By assuming that the LR symmetry is respected in the gauge sector (as we have already mentioned it, we assume the parity symmetry is not broken in the gauge sector in order to enjoy a low scale LR symmetry and avoid domain wall problems [67]) , i.e. $g_L = g_R$, then

$$M_{Z'} \overset{g_L = g_R}{\simeq} 1.2\, M_{W_R}\,. \tag{VI.52}$$

Through the relation of the masses (VI.52) we can establish a lower bound on the Z' mass from collider bounds $M_W > 3$ TeV [70]:

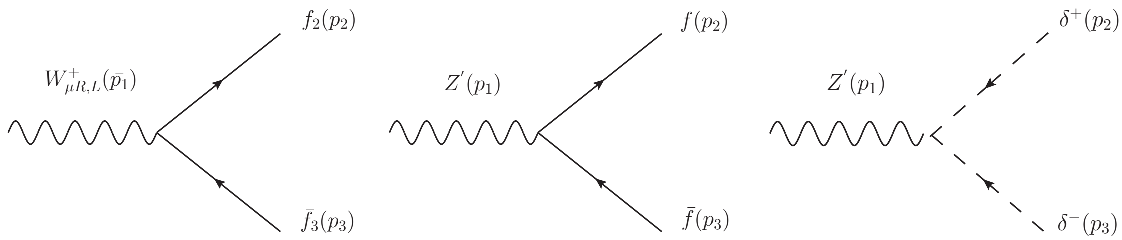
$$M_{Z'} > 3.6\,\text{TeV}. \tag{VI.53}$$

D. New Phenomenological Aspects

In this section we discuss the most relevant phenomenology characterizing the Zee-LR model, i.e. the new predictions of this particular model w.r.t. other already studied LR theories. These new features are the decay widths of the gauge bosons and the lepton family flavor number violation processes due to the presence of the charged singlet δ^+.

Gauge Boson Decays

In this model the new heavy gauge bosons can decay into all the SM fermions plus, since the model predicts light sterile neutrinos, one has extra decay channels involving those.



The W_R decays are

$$W_R^+ \to \bar{d}_i u_i,\ \bar{e}_i \nu_i,\ \bar{e}_i N_i. \tag{VI.54}$$

Here N refers to the heavy Majorana neutrinos present in the theory. Therefore, the branching ratio for the lepton decays of the new gauge bosons will be larger than in the LR models with Higgs triplets where the right-handed neutrinos are very heavy.

The partial widths of $W_R^\pm$ in the center of mass frame are given by (see appendix for more details):

- Massless fermions: $\Gamma(W_R^+ \to e\bar{\nu}) = \Gamma(W_R^+ \to e\bar{N}) = \Gamma_{m_q=0}(W_R^+ \to u\bar{d}) = \dfrac{1}{6\pi} M_W \left(\dfrac{1}{8} g_R^2\right)$,
- Top quark involved: $\Gamma(W_R^+ \to t\bar{b}) = \dfrac{1}{12\pi M_W^3}\left(2M_W^2 - m_t^2 - \dfrac{m_t^4}{m_W^2}\right)\left(\dfrac{1}{8} g_R^2\right)(M_W^2 - m_t^2)$,

where all fermion masses have been neglected except the top quark mass. The total decay width is thus defined as

$$\Gamma_{total} = 3\Gamma(W_R^+ \to e\bar{\nu}) + 3\Gamma(W_R^+ \to e\bar{N}) + 3\big(2\Gamma_{m_q=0}(W_R^+ \to u\bar{d}) + \Gamma(W_R^+ \to t\bar{b})\big), \tag{VI.55}$$

where the color of the quarks has been considered. Thus, the branching ratios of the above channels read as

$$\mathrm{BR}(W_R^+ \to \bar{q}_d q_u) \simeq 60\%\,, \quad \mathrm{BR}(W_R^+ \to \bar{e}\nu) \simeq 20\%\,, \quad \mathrm{BR}(W_R^+ \to \bar{e}N) \simeq 20\%\,.$$

On the other hand, one has the decay modes of the Z' boson.

$$Z' \to \bar{q}q,\ \bar{l}l,\ \delta^-\delta^+. \tag{VI.56}$$

In the broken phase, both the Z_{BL} and W_R^0 gauge bosons contribute to this decay since they can be written as a linear combination of the gauge bosons A, Z_R and Z_L, according to rotation (VI.32). The reader may notice that Z_R and Z_L are still not the physical fields due to some mixing in the mass matrix, but since the mixing angle is small (suppressed by the limit we are considering, as discussed above), we can assume that they do correspond to the physical massive gauge bosons, so that $Z_R \sim Z'$ and $Z_L \sim Z$. The Feynman rules of the interactions $Z'\bar{f}f$ can be found in the appendix. The partial decay widths of Z' are thus given by (see appendix for more details):

- $\Gamma^{m_q=0}(Z' \to \bar{u}u) = \frac{1}{48\pi}\cos^2\theta_R\left(\frac{1}{2}g_R^2 + \frac{1}{9}\frac{g_{BL}^4}{g_R^2} - \frac{1}{3}g_{BL}^2\right)M_{Z'}\,,$
- $\Gamma^{m_q=0}(Z' \to \bar{d}d) = \frac{1}{48\pi}\cos^2\theta_R\left(\frac{1}{2}g_R^2 + \frac{1}{9}\frac{g_{BL}^4}{g_R^2} + \frac{1}{3}g_{BL}^2\right)M_{Z'}\,,$
- $\Gamma(Z' \to \bar{t}t) = \frac{1}{48\pi M_{Z'}^2}\cos^2\theta_R\left[(M_{Z'}^2 + 3m_t^2)\left(\frac{1}{2}g_R - \frac{1}{3}\frac{g_{Bl}^2}{g_R}\right) + (M_{Z'}^2 - 5m_t^2)\frac{1}{4}g_R^2\right]\sqrt{M_{Z'}^2 - 4m_t^2}\,,$
- $\Gamma^{m_e=0}(Z' \to \bar{e}e) = \frac{1}{48\pi}\cos^2\theta_R\left(\frac{1}{2}g_R^2 + \frac{g_{BL}^4}{g_R^2} - g_{BL}\right),$
- $\Gamma(Z' \to \bar{\nu}\nu) = \Gamma(Z' \to \bar{N}N) = \frac{1}{48\pi}\frac{g_R^2}{\cos}\theta_R\left(\frac{g_R}{2} - \frac{g_{BL}^2}{g_R}\right)^2 M_{Z'}\,,$
- $\Gamma(Z' \to \delta^+\delta^-) = \frac{1}{48\pi M_{Z'}^2}g_{Bl}^2\sin^2\theta_R(4m_\delta^2 - M_{Z'}^2)^{3/2}\,,$

in the limit where the final state masses are neglected but the top quark one. We thus find for the branching

$$\begin{aligned}&\mathrm{BR}(Z' \to \bar{u}u) \simeq 32.89\%\,,\quad \mathrm{BR}(Z' \to \bar{d}d) \simeq 58.02\%\,,\quad \mathrm{BR}(Z' \to \bar{e}e) \simeq 7.42\%\,,\\ &\mathrm{BR}(Z' \to \text{inv.}) \simeq 0.28\%\,,\quad \mathrm{BR}(Z' \to \delta^+\delta^-) \simeq 1.38\%\,.\end{aligned}$$

where we have assumed that $g_L = g_R$, i.e. the left-right symmetric gauge symmetry holds in the gauge sector, so that

$$g_{BL} = \left(\frac{\tan^2\theta_W}{1-\tan^2\theta_W}\right)^{1/2} g_R. \tag{VI.57}$$

The branching ratio of the invisible width as a function of $M_{Z_{BL}}$ is shown in Fig 20. For the plots shown in Fig 20 a mass of $M_\delta \sim 1$ TeV has been assumed, since the model allows it to be relatively light. On the other hand, the heavier the charged singlet is, the closer are the branching ratios to the usual L-R symmetric models, as one would expect.

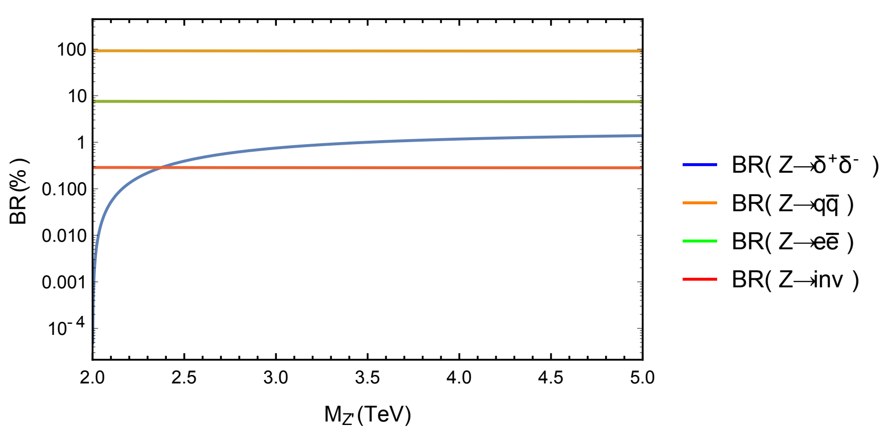


FIG. 20: Branching ratios of the different Z' decays. A $M_\delta \sim 1$ TeV has been assumed for the plots. As it is shown, the branching ratios are characterize by their increasing independence on the Z' mass as long as it grows.

Lepton number violation at the LHC

Another special feature of the introduced model is the family lepton-number violating processes that it may induce at the LHC. The extra singlet charged Higgs, δ^+, couples to fermions through the Majorana terms $\lambda_L \overline{\ell_L^C} \ell \delta^+$ and $\lambda_R \overline{\ell_R^C} \ell \delta^+$, which violate lepton number and therefore may generate lepton-number violating signals at the LHC, as we are showing in the following.

The $\delta^+\delta^-$ at the colliders is produced through the following Drell-Yan process, which afterwards can decay into leptons according to

$$pp \to \gamma, Z, Z^{'} \to \delta^+\delta^- \to e_i^+ e_j^- E_T^{miss}, \tag{VI.58}$$

which in general induces signatures with two leptons of different flavors and missing energy, E_T^{miss}. The number of events of these channels can be estimated by convoluting the combinatorics of the different channels participating in the process such that,

$$N(e_i^+ e_j^- E_T^{miss}) = \mathcal{L} \times \sigma(pp \to \delta^+\delta^-) \times \mathrm{BR}(\delta^+ \to e_i^+ \nu) \times \mathrm{BR}(\delta^- \to \bar{\nu} e_j^-)\,, \tag{VI.59}$$

where $\mathcal{L}$ is the luminosity in fb and $\sigma(pp \to \delta^+\delta^-)$ is the partonic production cross-section of the charged singlet, which is given by

$$\sigma(pp \to \delta^+\delta^-)(s) = \int_{\tau_0}^{1} \mathrm{d}\tau \frac{\mathrm{d}\mathcal{L}_{q\bar{q}}^{pp}}{\mathrm{d}\tau} \sigma(q\bar{q} \to \delta^+\delta^-)(\hat{s})\,, \tag{VI.60}$$

where $\hat{s} = s\tau$ is the partonic center of mass energy, being τ the fraction of the proton center of mass energy carried by the quarks and $\tau_0 = 4M_\delta^2/s$ its threshold. The parton luminosity, $\frac{d\mathcal{L}_{q\bar{q}}}{d\tau}$ is given by the parton distribution functions which are empirical distribution functions that model the topology of the proton. They are shaped from experiment. Here we will use the last update of the PDFs [71]. The parton luminosity is defined as,

$$\frac{\mathrm{d}\mathcal{L}_{q\bar{q}}^{pp}}{\mathrm{d}\tau} = \int_{\tau_0}^{1} \frac{\mathrm{d}x}{x} \left[f_{q/p}(x,\mu)\, f_{\bar{q}/p}\left(\frac{\tau}{x},\mu\right) + f_{q/p}\left(\frac{\tau}{x},\mu\right) f_{\bar{q}/p}(x,\mu) \right]\,, \tag{VI.61}$$

where μ refers to the normalization scale and f estimates the probability of finding the quark q inside the proton with energy x.

The differential cross-section of the quarks is given by,

$$\sigma(q\bar{q} \to \delta^+\delta^-) = \frac{1}{16\pi^2 \hat{s}} \frac{|\overline{p_f}|^{cm}}{|\overline{p_i}|^{cm}} \int \overline{\sum} |\mathcal{M}(q\bar{q} \to \delta^+\delta^-)|^2 d\Sigma_{cm}, \tag{VI.62}$$

where $\mathcal{M}(q\bar{q} \to \delta^+\delta^-)$ refers to the amplitude of the scattering process. The collision $q\bar{q} \to \delta^+\delta^-$ can be mediated only by the gauge boson Z_{BL} in the unbroken phase through the kinetic term $(D_\mu\delta^-)(D^\mu\delta^+)$. Once the symmetry is broken, the Z_{BL} gauge boson becomes a linear combination of the three physical gauge bosons Z, $Z^{'}$ and γ, whose feynman rules with respect to quarks and δ are listed in the appendix. The process before and after symmetry breaking is shown in Fig. 21. The matrix element of this process can be generally expressed as

$$i\mathcal{M} = \bar{v}(\bar{p_2})[V_{\mu\nu} + A_{\mu\nu}\gamma^5]\gamma^\mu (p_3 - p_4)^\nu u(\bar{p_1}), \tag{VI.63}$$

where the coefficients $V_{\mu\nu}$ and $A_{\mu\nu}$ refer to the vector and axial coupling of the interaction, respectively,

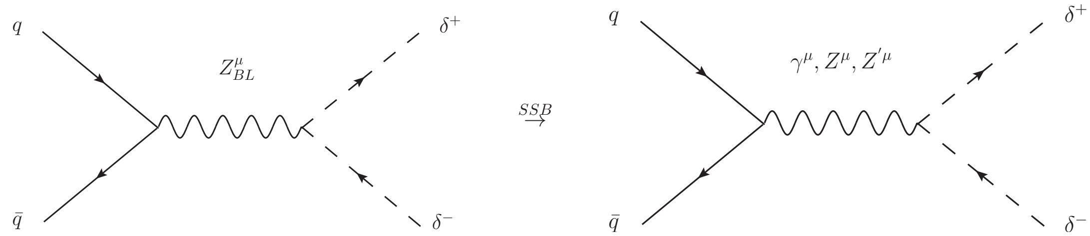


FIG. 21: Drel-Yan process in the unbroken (left) and broken (right) phase.

which are symmetric and depend on each physical neutral gauge boson in the following way

- Boson γ:

$$V^{\gamma}_{\mu\nu} = \frac{e^2 Q_q}{\hat{s}} g_{\mu\nu}, \tag{VI.64}$$

$$A^{\gamma}_{\mu\nu} = 0. \tag{VI.65}$$

- Boson Z:

$$V^{Z}_{\mu\nu} = \frac{V^q_Z a_Z}{\hat{s} - M_{Z'} + iM_{Z'}\Gamma}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2_{Z'}}\right), \tag{VI.66}$$

$$A^{Z}_{\mu\nu} = \frac{A^q_Z a_Z}{\hat{s} - M_{Z'} + iM_{Z'}\Gamma}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2_{Z'}}\right). \tag{VI.67}$$

- Boson Z':

$$V^{Z'}_{\mu\nu} = \frac{V^q_{Z'} a_{Z'}}{\hat{s} - M_{Z'} + iM_{Z'}\Gamma}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2_{Z'}}\right), \tag{VI.68}$$

$$A^{Z'}_{\mu\nu} = \frac{A^q_{Z'} a_{Z'}}{\hat{s} - M_{Z'} + iM_{Z'}\Gamma}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2_{Z'}}\right). \tag{VI.69}$$

The probability amplitude for this process in the center of mass frame reads as

$$\overline{\sum_{spins}}|\mathcal{M}|^2 = (V^2 + A^2)\frac{1}{2}\left[-(t-u)^2 + (s - 2m_q^2)(s - 4M_\delta^2)\right] + (V^2 - A^2)m_q^2(s - 4M_\delta^2), \tag{VI.70}$$

where it has been used that $V_{\mu\nu} = V g_{\mu\nu}$ and $A_{\mu\nu} = A g_{\mu\nu}$, being V and A the scalar part of the expressions defined above. Notice that, since the process takes place in the s-channel, the longitudinal component of the propagator does not contribute to the amplitude. By neglecting the mass of the quarks, the amplitude simplifies to the following expression,

$$\overline{\sum_{spins}}|\mathcal{M}|^2 = 2(t\,u - M_\delta^4)\left((V^q)^2 + (A^q)^2\right), \tag{VI.71}$$

which is a good limit, since all the quarks participating in the process are light. The scattering we are interested in involves protons and therefore the above process with top quarks is quite suppressed by the parton distribution functions [71], which justifies our assumption.

Thus, applying Eq. (VI.60), the partonic production cross-section of the charged scalar singlet is given by

$$\sigma(q\bar{q} \to \delta^+\delta^-)(\hat{s}) = \frac{N_C(\hat{s} - 4M_\delta^2)^{\frac{3}{2}}}{12\pi\sqrt{\hat{s}}} \left(|V^q|^2 + |A^q|^2\right), \tag{VI.72}$$

with

$$V^q = \frac{e^2 Q^q}{\hat{s}} + \frac{V_Z^q a_Z}{\hat{s} - M_Z^2 + i\Gamma_Z M_Z} + \frac{V_{Z'}^q a_{Z'}}{\hat{s} - M_{Z'}^2 + i\Gamma_{Z'} M_{Z'}}, \tag{VI.73}$$

$$\text{and } A^q = \frac{A_Z^q a_Z}{\hat{s} - M_Z^2 + i\Gamma_Z M_Z} + \frac{A_{Z'}^q a_{Z'}}{\hat{s} - M_{Z'}^2 + i\Gamma_{Z'} M_{Z'}}. \tag{VI.74}$$

being N_C the number of colors. In the above formula, since the gauge bosons are unstable particles, one needs to consider that the full propagator will in general have an imaginary part, which will reflect into a Breit-Wigner distribution. Recalling that by the optical theorem

$$\Gamma_{\text{tot}} = \frac{1}{m_P}\text{Im}\Sigma(m_P^2) + \cdots \tag{VI.75}$$

where m_P is the pole mass, $i\Sigma(p^2)$ is defined as the sum of 1PI self-energy diagrams and the $\cdots$ refer to non-1PI diagrams, one can see that, assuming $\Gamma_{\text{tot}} \ll m_P$ (weakly coupled theory), $\text{Im}\Sigma(m_P^2) = m_P\Gamma_{\text{tot}}$. Notice that the imaginary part of the 1PI resummation is non-zero for unstable particles. Therefore, in order to keep the mass real, one needs to modify the definition of the pole mass [72], so that

$$m_P^2 - m_R^2 + \text{Re}\Sigma(m_P^2) = 0, \tag{VI.76}$$

where m_R refers to the renormalized mass, and hence the propagator reads as

$$\frac{i}{p^2 - m_P^2 + i m_P \Gamma_{\text{tot}}}. \tag{VI.77}$$

From experiment, $\Gamma_Z^{total} \sim 2.5$ [18] whereas the total decay width of $Z^{'}$ is given by,

$$\begin{aligned}\Gamma_{Z'}^{total} &= 3\cdot 2\Gamma(Z^{'} \to \bar{u}u) + 3\Gamma(Z^{'} \to \bar{t}t) + 3\cdot 3\Gamma(Z^{'} \to \bar{d}d) + \\ &3\Gamma(Z^{'} \to \bar{e}e) + 3\Gamma(Z^{'} \to \bar{\nu}\nu) + 3\Gamma(Z^{'} \to \bar{N}N) + \Gamma(Z^{'} \to \delta^+\delta^-).\end{aligned} \tag{VI.78}$$

Notice the extra factor of three for the quarks due to color (which to detectors are blind). We considered all quarks and leptons massless except for the top quark.

In Fig. 22 we show the production cross-section of $\delta^+\delta^-$ as a function of M_δ for different values of the $Z^{'}$ mass for a center of mass energy of 13 TeV [10]. For the calculations, the PDFs from the reference [71] has been used. As it can be seen, the effect of the resonance is of relevant importance since it increases considerably the cross-section of such event w.r.t the SM predictions. For instance, notice that the production cross section reaches above 1 fb when the δ mass is below 650 GeV.

For computing the cross-section, the mixing angle in the charged scalar sector has been neglected [11].

[11] the mixing among the charged scalars can modify the rates computed. However, due to the huge amount of freedom in the mixing matrix, one could not compute any rate without making this assumption.

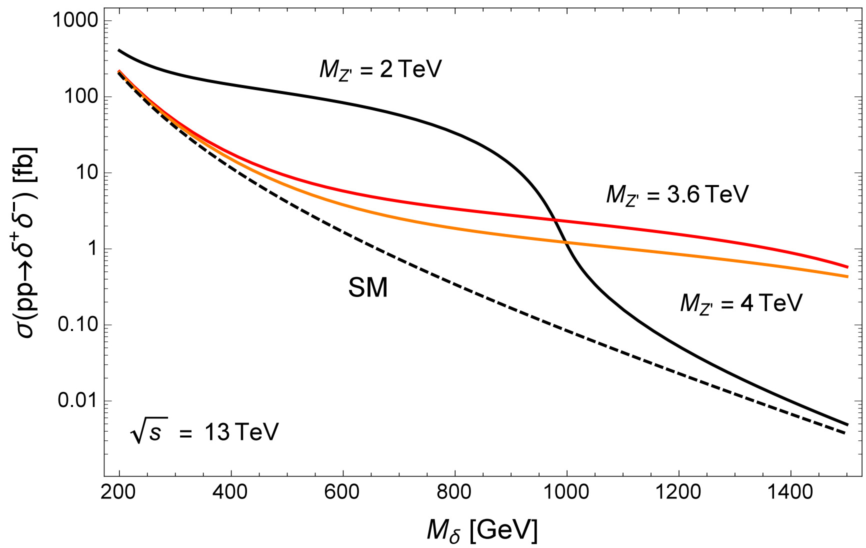


FIG. 22: Drell-Yan production cross-section for charged scalar singlets $\delta^+\delta^-$ as a function of M_δ in the present model. The dashed line corresponds to the SM prediction, whereas the solid lines show the results for different values of Z_{BL}. For the calculations, the $g_L = g_R$ scenario and a center of mass energy $s = \sqrt{13}$ TeV has been assumed.

Notice that a small mixing angle between the charged scalar singlet and the charged scalars living in the bi-doublet would indicate that δ^+ decays mainly into leptons. Since the term which induces the coupling of δ^+ to quarks is also the responsible of generating neutrino masses (see Fig. 18), the decay of the charged singlet to quarks is generally suppressed, which encourages our assumption that δ^+ decays mainly into leptons.

In Fig. 22 (top) we show curves of constant number of events, where the cross sections for Z_{BL} at 13 TeV shown in Fig. 22 has been used, and a luminosity of $\mathcal{L} = 3\ \mathrm{fb}^{-1}$ and a center of mass energy of $s = \sqrt{13}$ TeV has been assumed. In the plot, two different values of the Z' mass are shown. Notice that, for instance, one would expect more than 10 events for a values of $M_{Z_{delta}}$ below 500 GeV and $\mathrm{BR}(\delta^+ \to e_i^+ \nu_j)$ above 0.6.

In Fig. 22 (bottom), the prediction for the number of events assuming $\mathcal{L} = 25$ fb and $s = 14$ TeV is shown. As it would be expected, the number of events increases considerably with the luminosity.

One has to be careful with the noise of other signals. The dominant SM backgrounds of these processes at the LHC are the W and Z pair production. The ZZ channel will never produce two charged leptons with different flavor. However, the WW channel may fake the signatures. To discriminate between this background and the signal processes, one may look into the charged lepton transverse mass, which is defined as (for one isolated lepton and several jets):

$$m_T = \sqrt{2p_T^\ell E_T^{\mathrm{miss}}(1 - \cos\Delta\phi(\vec{\ell}), \vec{p}_T^{\mathrm{miss}}}. \tag{VI.79}$$

Here p_T^ℓ is the lepton transverse momentum and $\Delta\phi(\vec{\ell}, \vec{p}_T^{\mathrm{miss}})$ is the azimuthal angle between the lepton and the missing transverse momentum directions [73]. It is straightforward to see that a distribution of M_T has an end-point at the true mother mass. Therefore, the background due to the W pair production can be reduced by requiring the transverse mass to be above the boson mass. This will make things easier when regarding the search of these signatures at the LHC, so that the present model could be tested in the near future.

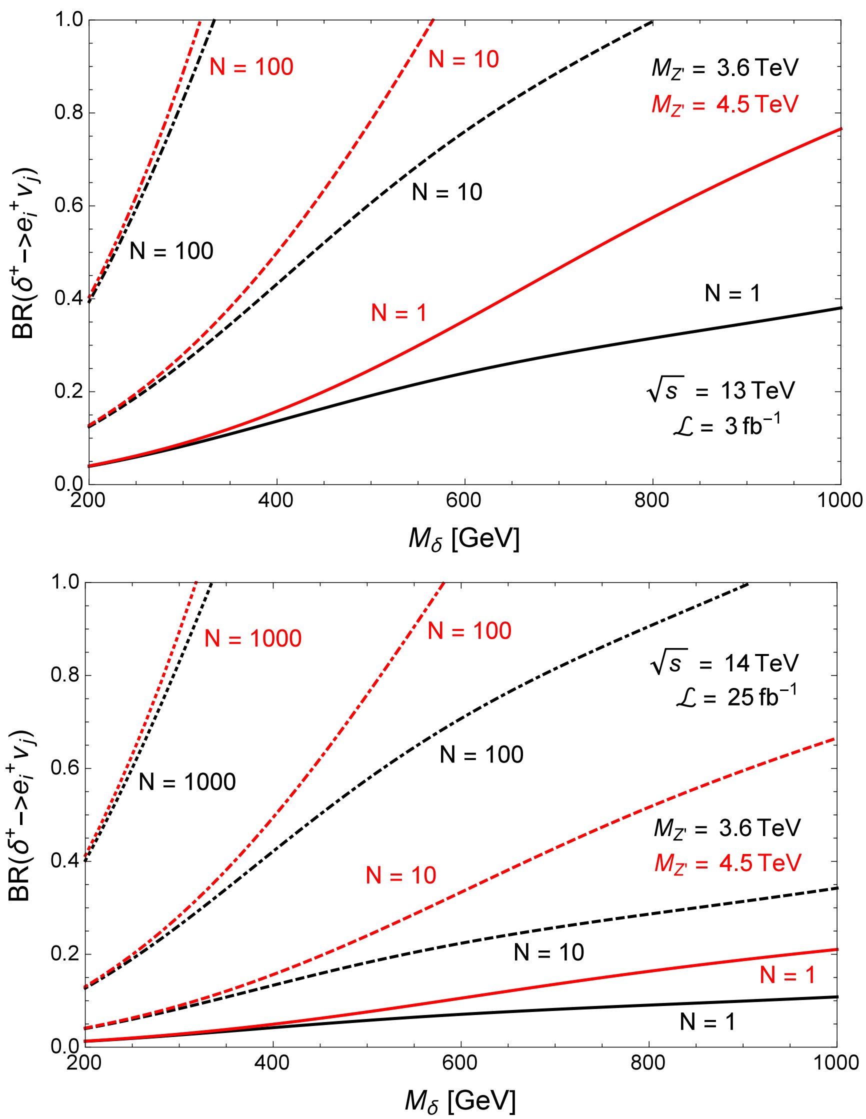


FIG. 23: Contours of expected number of events for 3 fb^{-1} at $\sqrt{s} = 13$ TeV (top) and for 25 fb^{-1} at $\sqrt{s} = 14$ TeV (bottom) in M_δ-BR($\delta^+ \to e_i^+ \nu_j$) plane.

VII. SUMMARY

As we have already discussed, experimental evidence encourages us to interpret the SM as an effective field theory which describes pretty well the low energy experimental observables but does not provide an explanation to other phenomena such as neutrino oscillations, parity violation on the electroweak sector and so on, not even mentioning gravity. Under the powerful effective field theory perspective, one may wonder about new theories (UV-completions of the Standard Model) which are able to explain some features not contemplated by its low energy (electroweak scale) realization.

In this thesis we have proposed two simple and realistic extensions of the Standard Model which improve it at a some extent. Both theories have in common that neutrino masses are generated at the quantum level through the Zee mechanism, so that both approaches include a scalar charged singlet δ^+ and a second higgs doublet in their field content. This mechanism is an economic way to give mass to the neutrinos which naturally explains the smallness of their mass compared with the other leptons.

In the first scenario, a renormalizable and realistic grand unified theory based on SU(5) is introduced. The field content of the theory consists in the usual 5 and 10 fermion representations and the 5_H, 24_H, 45_H and 10_H conforming the scalar sector, being the charged scalar singlet δ^+ embedded in the antisymmetric representation and a complex scalar doublet living in 45_H the crucial fields for the implementation of the Zee mechanism to give mass to the neutrinos. We study the testability of the theory by showing that constraints coming from unification and proton decay bounds can be satisfied, which in turn establishes the range of the parameter space of its fields in which the theory is realistic: for the parameter space shown in Fig. 14 unification is reached and proton decay bounds are satisfied. It is remarkable that, in despite of the fact that Φ_1 does not help to unification, it does help to increase the GUT scale and thus suppress proton decay. One of the main predictions coming from imposing unification and proton decay constraints is the existence of a light colored octet, Φ_1, which mass scale ranges from $\Phi_1 \sim [10^{3.5}, 10^{5.1}]$ GeV. This light field could give rise to exotic signals at the LHC such as one top and three light jets, due to the antisymmetry of its Yukawa coupling with quarks in the flavor space. The Zee mechanism is implemented in order to give mass to the neutrinos through radiative corrections at 1-loop level. As an outcome of the theory, a beautiful relation between the neutrino masses and the charged fermion masses, given by Eq. (IV.53), appears in a natural way, which shows the power of having the Zee model embedded in a grand unified theory.

In the second scenario, the left-right symmetric model able to predict Majorana neutrinos with the least degrees of freedom is introduced: one has the minimal Higgs sector to break the LR symmetry, i.e. two complex doublet scalars H_R and H_L, and a charged singlet Higgs δ^+. This simple model predicts light sterile Majorana neutrinos which play a crucial role in the low energy phenomenology. We study two phenomenological aspects which are characteristic of this particular theory: the branching ratios of the new gauge bosons decaying to fermions and the lepton number violation signature in which two charged leptons of different family plus missing energy are predicted to be produced. The decay of the heavy gauge bosons predicted by the Zee-LR is interesting in the sense that it might be different of the predictions coming from other LR-models due to the fact that this theory predicts a light sterile neutrino in which the heavy bosons can decay, whereas in other LR-theories which predict heavy sterile neutrinos this decay is kinematically forbidden. The branching ratios of the new gauge bosons W_R and Z' are studied. We hope that the induced modifications by the light sterile neutrino can be tested at the LHC. On the other hand, the presence of the charged singlet predicts lepton number violation signatures, $q\bar{q} \to \delta^+\delta^- \to e_i e_j E_T^{miss}$, in which two charged leptons of different family lepton number and missing energy may be produced at energies in the available range of the LHC. The upcoming improvement on the lepton number violation experiments such as the upgrade of MEG, called MEG2, or the experiment Mu2e, which will be available in 2017-2018, motivates the testability of the Zee-LR model.

We stress the beauty of these two BSM-theories: the Zee-SU(5), although it has still too much freedom in their parameters, it has all the advantages of a grand unified theory, such as the quantization of the electric charge, fermion mass relations, baryon and lepton number violation, strong and weak forces merged together, etc. This theory also predicts some interesting outcomes, named above, which are exclusive for this theory, i.e. not contemplated by other GUTs. The Zee-LR model, on the other hand, has a strongly predictive power due to its proximity regarding energy scales. We hope to study in more detail the phenomenology of these models, particularly lepton number violation processes, which could be tested at the LHC in a really near future. Both theories are so far the minimal renormalizable theories which predict majorana massive neutrinos (without adding extra fermion singlets) that can be found on the market. They have been introduced and studied in the following publications:

- P. Fileviez Perez and C. Murgui, ‘Renormalizable SU(5) Unification,” Phys. Rev. D **94** (2016) no.7, 075014 doi:10.1103/PhysRevD.94.075014 [arXiv:1604.03377 [hep-ph]].

- P. Fileviez Perez, C. Murgui and S. Ohmer, “Simple Left-Right Theory: Lepton Number Violation at the LHC,” Phys. Rev. D **94** (2016) no.5, 051701 doi:10.1103/PhysRevD.94.051701 [arXiv:1607.00246 [hep-ph]].

which are the pillars of this master thesis.

VIII. APPENDIX

A. SU(5) generators

The following form for the SU(5) generators is chosen [75], $g_i = \frac{1}{2}\lambda_i, \;\; i = 1, \cdots, 24$, where

$$
\begin{aligned}
\lambda_i \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & \lambda_i & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad i = 1, \cdots, 8, \\
\lambda_9 \;&=\; \begin{pmatrix} & & & 1 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 1 & 0 & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix}, \quad \lambda_{10} = \begin{pmatrix} & & & -i & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ i & 0 & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix}, \\
\lambda_{11} \;&=\; \begin{pmatrix} & & & 0 & 1 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 1 & 0 & 0 & & \end{pmatrix}, \quad \lambda_{12} = \begin{pmatrix} & & & 0 & -i \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ i & 0 & 0 & & \end{pmatrix}, \\
\lambda_{13} \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & & & 1 & 0 \\ & & & 0 & 0 \\ 0 & 1 & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix}, \quad \lambda_{14} = \begin{pmatrix} & & & 0 & 0 \\ & & & -i & 0 \\ & & & 0 & 0 \\ 0 & i & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix}, \\
\lambda_{15} \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 1 \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & 1 & 0 & & \end{pmatrix}, \quad \lambda_{16} = \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & -i \\ & & & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & i & 0 & & \end{pmatrix}, \\
\lambda_{17} \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 1 & 0 \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & \end{pmatrix}, \quad \lambda_{18} = \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & -i & 0 \\ 0 & 0 & i & & \\ 0 & 0 & 0 & & \end{pmatrix}, \\
\lambda_{19} \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 1 \\ 0 & 0 & 0 & & \\ 0 & 0 & 1 & & \end{pmatrix}, \quad \lambda_{20} = \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & -i \\ 0 & 0 & 0 & & \\ 0 & 0 & i & & \end{pmatrix}, \\
\lambda_{20+j} \;&=\; \begin{pmatrix} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{σ_i}} \\ 0 & 0 & 0 & & \end{pmatrix}, \qquad j = 1, 2, 3, \quad \lambda_{24} = \frac{2}{15} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix}.
\end{aligned}
$$

B. Field content and interactions of Zee-SU(5)

Representations if the Zee-SU(5) from the SM point of view:

$$
\begin{aligned}
\bar{5} &\sim \underbrace{(1,\bar{2},-1/2)}_{l_L} \oplus \underbrace{(\bar{3},1,1/3)}_{(d^c)_L} \\
10 &\sim \underbrace{(\bar{3},1,-2/3)}_{(u^c)_L} \oplus \underbrace{(3,2,1/6)}_{q_L} \oplus \underbrace{(1,1,1)}_{(e^c)_L} \\
V_{24} &\sim \underbrace{(8,1,0)}_{G_\mu} \oplus \underbrace{(1,3,0)}_{W_\mu} \oplus \underbrace{(3,2,-5/6)}_{X_\mu} \oplus \underbrace{(\bar{3},2,5/6)}_{Y_\mu} \oplus \underbrace{(1,1,0)}_{\gamma_\mu} \\
5_H &\sim \underbrace{(1,2,1/2)}_{H_1} \oplus \underbrace{(3,1,-1/3)}_{T} \\
24_H &\sim \underbrace{(8,1,0)}_{\Sigma_8} \oplus \underbrace{(1,3,0)}_{\Sigma_3} \oplus \underbrace{(3,2,-5/6)}_{\Sigma_{(3,2)}} \oplus \underbrace{(\bar{3},2,5/6)}_{\Sigma_{(\bar{3},2)}} \oplus \underbrace{(1,1,0)}_{\Sigma_{24}} \\
45_H &\sim \underbrace{(8,2,1/2)}_{\Phi_1} \oplus \underbrace{(\bar{6},1,-1/3)}_{\Phi_2} \oplus \underbrace{(3,3,-1/3)}_{\Phi_3} \oplus \underbrace{(\bar{3},2,-7/6)}_{\Phi_4} \oplus \underbrace{(3,1,-1/3)}_{\Phi_5} \oplus \underbrace{(\bar{3},1,4/3)}_{\Phi_6} \oplus \underbrace{(1,2,1/2)}_{H_2} \\
10_H &= \underbrace{(1,1,1)}_{\delta^+} \oplus \underbrace{(3,2,1/6)}_{\delta_{(3,2)}} \oplus \underbrace{(\bar{3},1,-2/3)}_{\delta_T}
\end{aligned}
$$

Location of the fields inside the representations (from now on $i,j,k = 1..3$ will refer to color indices and $\alpha,\beta,\gamma = 4,5$ to weak isospin indices):

Matter representations

$$
\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}. \tag{VIII.1}
$$

5_H representation

$$
\bar{5} = \begin{pmatrix} T^1 \\ T^2 \\ T^3 \\ H_1^+ \\ H_1^0 \end{pmatrix}.
$$

10_H representation

$(10_H)^{ij}$	$\sim \delta_T^{ij}$	3 d.o.f.
$(10_H)^{\alpha i}$	$\sim \delta_{(3,2)}^{\alpha i}$	6 d.o.f.
$(10_H)^{\alpha\beta}$	$\sim \epsilon^{\alpha\beta}\delta^+$	1 d.o.f.

45_H representation

$(45_H)_i^{jk}$	$\sim f\{\Phi_2, \Phi_5\} = \epsilon^{jkl}\Phi_{2li} + \epsilon^{jkl}\epsilon_{lim}\Phi_5{}^m$ $\Phi_{2il} = \Phi_{2li}$ and $\Phi_{5il} = -\Phi_{5li}$	9 - 3 (trace) = 6 d.o.f 6 + 3
$(45_H)_i^{j\alpha}$	$\sim f\{\Phi_1, H_2\} = [\lambda^a]_i^j\, \Phi_1{}_a^\alpha + \delta_i^j H_2^\alpha$	16 + 2 = 18 d.o.f
$(45_H)_\alpha^{ij}$	$\sim \epsilon^{ijk}\Phi_{4\alpha k}$	6 d.o.f.
$(45_H)_\alpha^{i\beta}$	$\sim f\{\Phi_3 \equiv (\Delta_1, \Delta_2, \Delta_3), \Phi_5\} = \frac{1}{\sqrt{2}}\Phi_3{}^a[\sigma^a]_\alpha^\beta + \delta_\alpha^\beta \Phi_5{}^i$ $45_5^{i4} \sim (\Delta_1{}^i + i\Delta_2{}^i)/\sqrt{2} \equiv (\phi_3{}^{+\frac{2}{3}})^i$ $45_4^{i5} \sim (\Delta_1{}^i - i\Delta_2{}^i)/\sqrt{2} \equiv (\phi_3{}^{-\frac{4}{3}})^i$ $45_4^{i4} \sim \Delta_3{}^i/\sqrt{2} + \Phi_5{}^i \equiv (\phi_3{}^{-\frac{1}{3}})^i/\sqrt{2} + \Phi_5{}^i$ $45_5^{i5} \sim -\Delta_3{}^i/\sqrt{2} + \Phi_5{}^i \equiv -(\phi_3{}^{-\frac{1}{3}})^i/\sqrt{2} + \Phi_5{}^i$	12 d.o.f. 3 3 3 3
$(45_H)_i^{\alpha\beta}$	$\sim \epsilon^{\alpha\beta}\Phi_{6i}$	3 d.o.f.
$(45_H)_\alpha^{\beta\gamma}$	$\sim -3\epsilon^{\beta\gamma}H_2{}^\delta\epsilon_{\alpha\delta}$	2 d.o.f.

where the following explicit form for the triplet in SU(2) ($T_{SU(2)}$) and in SU(3) ($O_{SU(3)}$) is used:

$$T_{SU(2)} : \sigma^a\Delta_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^0 & \Delta^+ \\ \Delta^- & -\frac{1}{\sqrt{2}}\Delta^0 \end{pmatrix},$$

$$O_{SU(3)} : \lambda^a\Omega_a = \begin{pmatrix} \Omega_3 + Y/2 & T^- & V^- \\ T^+ & -\Omega_3 + Y/2 & U^- \\ V^+ & U^+ & -Y \end{pmatrix}.$$

C. Yukawa interactions

$$\mathcal{L}_{Yukawa} = \bar{5}\,10\{Y_1 5^*_H + Y_2(45^*_H)\} + 10\,10(Y_3 5_H + Y_4 45_H) + Y_5 \bar{5}\,\bar{5}\,10_H$$

- 45_H representation -

- Interactions of $\Phi_1 \sim (8, 2, 1/2) \equiv ({\phi_1}^+_a, {\phi_1}^0_a)$, where $a = 1...8$:

$\mathcal{L}_Y \supset Y_2\, q_L\, \Phi_1^\dagger\, (d^C)_L + Y_4\, q_L\, \Phi_1\, (u^C)_L$

$\mathcal{L}_Y \supset 2Y_2\, \bar{5}_i 10^{j\alpha}(45^*_H)^i_{j\alpha} + 4Y_4\, 10^{i\alpha}10^{jk}(45_H)^{m\beta}_k \epsilon_{ijm}\epsilon_{\alpha\beta} - 2Y_4^{ab} 10^{ij} 10^{\alpha k}(45_H)^{l\beta}_k \epsilon_{ijl}\epsilon_{\alpha\beta}$

$\mathcal{L}_Y \supset 2Y_2^{ab}\{(d^c_a)_i {d_b}^j (\phi_1^0)^\dagger_e + (d^C_a)_i {u_b}^j (\phi_1^+)^\dagger_e\}[\lambda^e]^i_j + 4(Y_4^{ab} - Y_4^{ba})\{(u^C_a)_i {u_b}^j \phi^0_{1e} - (u^C_a)_i {d_b}^j {\phi_1^+}_e\}[\lambda^e]^i_j$

$\mathcal{L}^{\Phi_1}_{d=6} \sim \dfrac{1}{m^2_{\Phi_1}} Y_2^{ab}(Y_4^{cd} - Y_4^{dc})\left\{(d^C_a)_i {d_b}^j (u^C_c)_j {u_d}^i - (d^C_a)_i {u_b}^j (u^C_c)_j {d_d}^i\right\}$

- Interactions of $\Phi_2 \sim (\bar{6}, 1, -1/3) \equiv (\Phi_2)_{ij}$, where $(\Phi_2)_{ij} = (\Phi_2)_{ji}$:

$\mathcal{L}_Y \supset Y_2\, (d^C)_L\, \Phi_2^\dagger\, (u^C)_L + Y_4\, q_L\, \Phi_2\, q_L$

$\mathcal{L}_Y \supset Y_2\, \bar{5}_i 10^{jk}(45^*_H)^i_{jk} + Y_4\{4\, 10^{i\alpha}10^{\beta j} + 10^{\alpha\beta}10^{ij}\}(45_H)^{kl}_j \epsilon_{\alpha\beta}\epsilon_{ikl}$

$\mathcal{L}_Y \supset 2\, Y_2^{ab}(d^C_a)_i (u^C_b)_j (\Phi_2^\dagger)^{ij} + 8\,(Y_4^{ab} - Y_4^{ba}) {d_a}^i {u_b}^j \Phi_{2ij}$

$\mathcal{L}^{\Phi_2}_{d=6} \sim \dfrac{1}{m^2_{\Phi_2}} Y_2^{ab}\,(Y_4^{cd} - Y_4^{dc})\,(d^C_a)_i (u^C_b)_j d^i_c u^j_d$

- Interactions of $\Phi_3 \sim (3, 3, -1/3) \equiv \{(\phi_3^{+\frac{2}{3}})^i, (\phi_3^{-\frac{1}{3}})^i, (\phi_3^{-\frac{4}{3}})^i\}$:

$\mathcal{L}_Y \supset Y_2\, q_L\, \Phi_3^\dagger\, l_L\; + Y_4\, q_L\, \Phi_3\, q_L$

$\mathcal{L}_Y \supset 2Y_2\, \bar{5}_\alpha 10^{i\beta}(45^*_H)^\alpha_{i\beta} + 8Y_4\, 10^{i\alpha}10^{j\beta}(45_H)^{k\gamma}_\beta \epsilon_{ijk}\epsilon_{\alpha\gamma}$

$\mathcal{L}_Y \supset 2Y_2^{ab}\{e_a {d_b}^i (\phi_3^{-\frac{4}{3}})^\dagger_i - \nu_a {u_b}^i (\phi_3^{+\frac{2}{3}})^\dagger_i\} + \sqrt{2}Y_2^{ab}\{e_a {u_b}^i + \nu_a {d_b}^i\}(\phi_3^{-\frac{1}{3}})^\dagger_i +$
$+4(Y_4^{ab} - Y_4^{ba})\{{u_a}^i {u_b}^j (\phi_3^{-\frac{4}{3}})^k \epsilon_{ijk} - {d_a}^i {d_b}^j (\phi_3^{+\frac{2}{3}})^k \epsilon_{ijk}\} + 4\sqrt{2}(Y_4^{ab} - Y_4^{ba}) {d_a}^i {u_b}^j (\phi_3^{-\frac{1}{3}})^k \epsilon_{ijk}$

$\mathcal{L}^{\Phi_3}_{d=6} \sim \dfrac{1}{m^2_{\Phi_3}} Y_2^{ab}(Y_4^{cd} - Y_4^{dc})\left(e_a {d_b}^i {u_c}^j {u_d}^k + \nu_{\mathbf{a}} \mathbf{u_b}^{\mathbf{i}} \mathbf{d_c}^{\mathbf{j}} \mathbf{d_d}^{\mathbf{k}} + \mathbf{e_a} \mathbf{u_b}^{\mathbf{i}} \mathbf{d_c}^{\mathbf{j}} \mathbf{u_d}^{\mathbf{k}} + \nu_{\mathbf{a}} \mathbf{d_b}^{\mathbf{i}} \mathbf{d_c}^{\mathbf{j}} \mathbf{u_d}^{\mathbf{k}}\right) \epsilon_{ijk}$

- Interactions of $\Phi_4 \sim (\bar{3}, 2, -7/6) \equiv (\phi_4^{-\frac{5}{3}}{}_i, \phi_4^{-\frac{2}{3}}{}_i)$:

$\mathcal{L}_Y \supset Y_2\,(u^C)_L\,\Phi_4^\dagger\,l_L + Y_4\,(q_L)\,\Phi_4\,(e^C)_L$

$\mathcal{L}_Y \supset Y_2\,\bar{5}_\alpha 10^{ij}(45_H^*)^\alpha_{ij} + Y_4\,\{10^{\alpha\beta}10^{i\gamma}45^{jk}_\gamma\epsilon_{\alpha\beta}\epsilon_{ijk} + 2\,10^{i\alpha}10^{\beta\gamma}45^{jk}_\gamma\epsilon_{ijk}\epsilon_{\alpha\beta}\}$

$\mathcal{L}_Y \supset 2Y_2^{ab}\{e_a(u_b^C)_i(\phi_4^{-\frac{5}{3}})^{i\dagger} - \nu_a(u_b^C)_i(\phi_4^{-\frac{2}{3}})^{i\dagger}\} + 4(Y_4^{ab} - Y_4^{ba})\{e_a^C u_b{}^i(\phi_4^{-\frac{5}{3}})_i + e_a^C d_b{}^i(\phi_4^{-\frac{2}{3}})_i\}$

$\mathcal{L}^{\Phi_4}_{d=6} \sim \frac{1}{m^2_{\Phi_4}} Y_2^{ab}\,(Y_4^{cd} - Y_4^{dc})\,\left\{e_a\,(u_b^C)_i\,e_c^C\,u_d{}^i - \nu_a\,(u_b^C)_i\,e_c^C\,d_d{}^i\right\}$

- Interactions of $\Phi_5 = (3, 1, -1/3) \equiv \Phi_5{}^i$:

$\mathcal{L}_Y \supset Y_2\,\{q_L\,\Phi_5^\dagger l_L + (d^C)_L\,\Phi_5^\dagger\,(u^C)_L\} + Y_4\,\{q_L\,\Phi_5\,q_L + (u^C)_L\,\Phi_5\,(e^C)_L\}$

$\mathcal{L}_Y \supset Y_2\{\bar{5}_i 10^{jk}(45_H^*)^i_{jk} + 2\,\bar{5}_\alpha 10^{i\beta}(45_H^*)^\alpha_{i\beta}\} + Y_4\{4\,10^{i\alpha}10^{\beta j} + \ 10^{\alpha\beta}10^{ij}\}(45_H)^{kl}_j\epsilon_{ikl}\epsilon_{\alpha\beta}$
$-Y_4\{2\,10^{ij}10^{\alpha\beta} + 8\,10^{i\alpha}10^{j\beta}\}(45_H)^{k\gamma}_\beta\epsilon_{ijk}\epsilon_{\alpha\gamma}$

$\mathcal{L}_Y \supset -2Y_2^{ab}(d_a^C)_i(u_b^C)_j\Phi^\dagger_{5k}\epsilon^{ijk} + 2Y_2^{ab}\left(e_a u_b{}^i\Phi^\dagger_{5i} - \nu_a d_b{}^i\Phi^\dagger_{5i}\right) + 8(Y_4^{ab} - Y_4^{ba})e_a^C(u_b^C)_i\Phi_5^i$

$\mathcal{L}^{\Phi_5}_{d=6} \sim \frac{1}{m^2_{\Phi_5}} Y_2^{ab}(Y_4^{cd} - Y_4^{dc})\left(\{e_a u_b{}^i - \nu_a d_b{}^i\}e_c^C(u_d^C)_i - \mathbf{(d_a^C)_i(u_b^C)_j e_c^C(u_d^C)_k}\epsilon^{\mathbf{ijk}}\right)$

- Interactions of $\Phi_6 \sim (\bar{3}, 1, 4/3) \equiv \Phi_{6i}$:

$\mathcal{L}_Y \supset Y_2\,(d^C)_L\,\Phi_6^\dagger\,(e^C)_L + Y_4\,(u^C)_L\,\Phi_6\,(u^C)_L$

$\mathcal{L}_Y \supset Y_2\,\bar{5}_i 10^{\alpha\beta}(45_H^*)^i_{\alpha\beta} + 2Y_4\,10^{ij}10^{kl}(45_H)^{\alpha\beta}_l\epsilon_{ijk}\epsilon_{\alpha\beta}$

$\mathcal{L}_Y \supset 2Y_2^{ab}(d_a^C)_i e_b^C \Phi_6^{\dagger\,i} - 4(Y_4^{ab} - Y_4^{ba})(u_a^C)_i(u_b^C)_j\Phi_{6k}\epsilon^{ijk}$

$\mathcal{L}^{\Phi_6}_{d=6} \sim \frac{1}{m^2_{\Phi_6}} Y_2^{ab}\,(Y_4^{cd} - Y_4^{dc})\,\mathbf{(d_a^C)_i\,e_b^C\,(u_c^C)_j\,(u_d^C)_k}\epsilon^{\mathbf{ijk}}$

- Interactions of $H_2 \sim (1, 2, 1/2) \equiv (H_2^+, H_2^0)$:

$\mathcal{L}_Y \supset Y_1\{(d^C)\,H_2^\dagger\,q + l\,H_2^\dagger\,e^C\} + Y_3\,(u^C)\,H_2\,q$

$\mathcal{L}_Y \supset 2Y_2\,\bar{5}_i 10^{j\alpha}(45_H^*)^i_{j\alpha} + Y_2\,\bar{5}_\alpha 10^{\beta\gamma}(45_H^*)^\alpha_{\beta\gamma} + 4Y_4 10^{i\alpha}10^{kj}(45_H)^{m\beta}_i\epsilon_{ikm}\epsilon_{\alpha\beta} -$
$2Y_4\,10^{ij}10^{\alpha k}(45_H)^{l\beta}_k\epsilon_{ijl}\epsilon_{\alpha\beta} + Y_4\,10^{ij}10^{k\alpha}(45_H)^{\beta\gamma}_\alpha\epsilon_{ijk}\epsilon_{\beta\gamma}$

$\mathcal{L}_Y \supset 2Y_2^{ab}\{(d_a^C)_i u_b{}^i(H_2^+)^\dagger + (d_a^C)d_b{}^i(H_2^0)^\dagger\} - 6Y_2^{ab}\{e_a e_b^C(H_2^0)^\dagger + \nu_a e_b^C(H_2^+)^\dagger\}$
$-8(Y_4^{ab} - Y_4^{ba})\{(u_a^C)_i u_b{}^i H_2^0 - (u_a^C)_i d_b{}^i H_2^+\}$

- 10_H representation -

- Interactions of $\delta^+ \sim (1,1,1)$:

$\mathcal{L}_Y \supset Y_5\, l_L\, \delta^+\, l_L$

$\mathcal{L}_Y \supset 2Y_5\, \bar{5}_\alpha \bar{5}_\beta 10^{\alpha\beta}$

$\mathcal{L}_Y \supset 2(Y_5^{ab} - Y_5^{ba})\nu_a e_b \delta^+$

- Interactions of $\delta_{(3,2)} \sim (3,2,1/6) \equiv (\Delta^i_{+\frac{2}{3}}, \Delta^i_{-\frac{1}{3}})$:

$\mathcal{L}_Y \supset Y_5\, (d^C)_L\, \delta_{(3,2)}\, l_L$

$\mathcal{L}_Y \supset Y_5\, \bar{5}_\alpha \bar{5}_i 10^{\alpha i} + Y_5\, \bar{5}_i \bar{5}_\alpha 10^{i\alpha}$

$\mathcal{L}_Y \supset (Y_5^{ab} - Y_5^{ba}) \left\{ (d_a^C)_i e_b \Delta^i_{+\frac{2}{3}} - (d_a^C)_i \nu_b \Delta^i_{-\frac{1}{3}} \right\}$

- Interactions of $\delta_T \sim (\bar{3}, 1, -2/3)$:

$\mathcal{L}_Y \supset Y_5\, (d^C)_L\, \delta_T\, (d^C)_L$

$\mathcal{L}_Y \supset 2Y_5\, \bar{5}_i \bar{5}_j 10^{ij}$

$\mathcal{L}_Y \supset (Y_5^{ab} - Y_5^{ba})(d_a^C)_i (d_b^C)_j \delta_{Tk} \epsilon^{ijk}$

- 5_H representation -

- Interactions of $T \sim (3,1,-1/3) \equiv T^i$:

$\mathcal{L}_Y \supset Y_1\{q_L\, T^\dagger\, l_L + (d^C)\, T^\dagger\, (u^C)\} + Y_3\{q_L\, T\, q_L + (u^C)\, T\, (e^C)\}$

$\mathcal{L}_Y \supset Y_1\{\bar{5}_\alpha 10^{\alpha i} 5_i^* + 5_i 10^{ij} 5_j^*\} + Y_3 \left\{ -4\, 10^{i\alpha} 10^{j\beta} T^k + 10^{\alpha\beta} 10^{ij} T^k + 10^{ij} 10^{\alpha\beta} T^k \right\} \epsilon_{ijk} \epsilon_{\alpha\beta}$

$\mathcal{L}_Y \supset Y_1^{ab}\{(\nu_a d_b^i - e_a u_b^i) T_i^\dagger - (d_a^C)_i (u_b^C)_j T_k^\dagger \epsilon^{ijk}\} - 4(Y_3^{ab} + Y_3^{ba})\{2\, u_a^i d_b^j T^k \epsilon_{ijk} - e_a^C (u_b^C)_k T^k$

$$\mathcal{L}^T_{d=6} \sim \frac{1}{m_T^2}\, Y_1^{ab}\, (Y_3^{cd} + Y_3^{dc}) \Big\{ \nu_a d_b^i e_c^C (u_d^C)_i - e_a u_b^i e_c^C (u_d^C)_i - (\mathbf{d_a^C})_\mathbf{i} (\mathbf{u_b^C})_\mathbf{j} \mathbf{e_c^C} (\mathbf{u_d^C})_\mathbf{k} \epsilon^{\mathbf{ijk}} + \mathbf{2 e_a u_b^i u_c^j d_d^k} \epsilon_{\mathbf{ijk}}$$
$$- \mathbf{2}\boldsymbol{\nu}_\mathbf{a} \mathbf{d_b^i u_c^j d_d^k} \epsilon_{\mathbf{ijk}} + 2(d_a^C)_i (u_b^C)_j u_c^k d_d^l (\delta_k^i \delta_l^j - \delta_l^i \delta_k^j) \epsilon^{ijk} \Big\}$$

- Interactions of $H_1 \sim (1,2,1/2) \equiv (H_1^+, H_1^0)$:

$$\mathcal{L}_Y \supset Y_1\{(d^C)\, H_1^\dagger\, q + l\, H_1^\dagger\, e^C\} + Y_3\,(u^C)\, H_1\, q$$

$$\mathcal{L}_Y \supset Y_1\{\bar{5}_i 10^{i\alpha} 5_{H\,\alpha}^* + \bar{5}_\alpha 10^{\alpha\beta} 5_{H\,\beta}^*\} + Y_3\{2\,10^{ij}10^{k\alpha}5^\beta \epsilon_{ijk}\epsilon_{\alpha\beta} + 2\,10^{i\alpha}10^{jk}5^\beta \epsilon_{ijk}\epsilon_{\alpha\beta}\}$$

$$\mathcal{L}_Y \supset Y_1^{ab}\left(\left\{(d_a^C)_i u_b^i + \nu_a e_b^C\right\}(H_1^+)^\dagger + \left\{e_a e_b^C + (d_a^C)_i d_b^i\right\}(H_1^0)^\dagger\right)$$
$$+4(Y_3^{ab} + Y_3^{ba})\left\{(u_a^C)_i u_b^i H_1^0 - (u_a^C)_i d_b^i H_1^+\right\}$$

D. Lagrangian of Zee-SU(5)

Definition of the covariant derivatives:

$$\begin{aligned}
&D_\mu 5 = \partial_\mu 5 + i g_{GUT} A_\mu 5\\
&D_\mu \bar{5} = \partial_\mu \bar{5} - i g_{GUT} A_\mu \bar{5}\\
&D_\mu 10 = \partial_\mu 10 + i g_{GUT}(A_\mu 10 + 10 A_\mu^T)\\
&D_\mu 24_H = \partial_\mu 24_H + i g_{GUT}[A_\mu, 24_H]\\
&D_\mu (45_H)_\gamma^{\alpha\beta} = \partial_\mu (45_H)_\gamma^{\alpha\beta} + i g_{GUT}\left((A_\mu)_m^\alpha (45_H)_\gamma^{\alpha m}(A_\mu^T)_m^\beta - (A_\mu^T)_{\gamma\delta}(45_H)_\delta^{\alpha\beta}\right)
\end{aligned}$$

Transformations of the representations involved in Zee-SU(5):

$$\begin{aligned}
&5^i \to U^i{}_j 5^j,\\
&\bar{5}_i \to 5_j (U^\dagger)^j{}_i,\\
&10^{ij} \to U^i{}_k 10^{kl} (U^T)_l{}^j,\\
&\overline{10}_{ij} \to (U^*)_i{}^a 10_{ab} (U^\dagger)^b{}_j,\\
&24^i{}_j \to U^i{}_k 24^k_l (U^\dagger)^l{}_j,\\
&45^{ij}_k \to U^i{}_a 45^{ab}_c (U^T)_b{}^j (U^\dagger)^c{}_k.
\end{aligned}$$

In general,

$$\Psi^{i_1 i_2\cdots}_{j_1 j_2\cdots} \to U^{i_1}_{a_1} U^{i_2}_{a_2}\cdots \Psi^{a_1 a_2\cdots}_{b_1 b_2\cdots}(U^\dagger)^{b_1}_{j_1}(U^\dagger)^{b_2} j_2 \cdots .$$

where the superscript (subscript) refers to the fundamental (antifundamental) representation.

The Lagrangian of the minimal renormalizable extension of SU(5) is composed of the following terms:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermions} + \mathcal{L}_{scalars} + \mathcal{L}_{Yukawa}, \tag{VIII.2}$$

where

- $\mathcal{L}_{\text{gauge}} = -\dfrac{1}{4} Tr\{F^{\mu\nu}F_{\mu\nu}\}$ being $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_{GUT}[A_\mu, A_\nu]$,
- $\mathcal{L}_{\text{fermions}} = \bar{5}^\dagger \gamma^0 i \gamma^\mu D_\mu \bar{5} + \frac{1}{2}\text{Tr}\{10^\dagger \gamma^0 i\gamma^\mu D_\mu 10\}$,
- $\mathcal{L}_{\text{Yukawa}} = \bar{5}\,10\{Y_1 5_H^* + Y_2(45_H^*)\} + 10\,10(Y_3 5_H + Y_4 45_H) + Y_5 \bar{5}\,\bar{5}\,10_H$,

- $\mathcal{L}_{\text{scalars}} = \frac{1}{2}\text{Tr}\{(D^\mu 24_H^\dagger)(D_\mu 24_H)\} + \text{Tr}\{(D^\mu 45_H^\dagger)(D_\mu 45_H)\} + \text{Tr}\{(D^\mu 10_H^\dagger)(D_\mu 10_H)\} + V(5_H, 24_H, 45_H, 10_H)$.

(Remark: the bar in $\bar{5}$ means that the fermions are in the anti-fundamental representation. The adjoint here is written explicitly, i.e. $\psi^\dagger\gamma^0$, just a question of notation), where V is the scalar potential which explicitly takes the following form:

$$\begin{aligned}V(5_H,24_H,45_H,10_H) =& V(5_H)+V(10_H)+V(24_H)+V(45_H)+V(5_H,10_H)+V(5_H,24_H)+\\&+V(5_H,45_H)+V(10_H,24_H)+V(10_H,45_H)+V(24_H,45_H)+\\&+V(24_H,45_H,5_H)+V(45_H,10_H,5_H)+V(5_H,24_H,45_H,10_H)\end{aligned}$$

where

$$V(5_H) = -\mu_5^2 5^\alpha 5^*_\alpha + \lambda_1(5^\alpha 5^*_\alpha)^2,$$

$$V(10_H) = -\mu_{10}^2 10^{\alpha\beta}10^*_{\alpha\beta} + \lambda_2(10^{\alpha\beta}10^*_{\alpha\beta})^2 + \lambda_3 10^{\alpha\beta}10^*_{\beta\gamma}10^{\gamma\delta}10^*_{\delta\alpha},$$

$$V(24_H) = -\mu_{24}^2 24^\alpha{}_\beta 24^\beta{}_\alpha + \lambda_4(24^\alpha{}_\beta 24^\beta{}_\alpha)^2 + a_1 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\alpha + \lambda_5 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\delta 24^\delta{}_\alpha,$$

$$\begin{aligned}V(45_H) = &-\mu_{45}^2 45^{\alpha\beta}_\gamma 45^{*\gamma}_{\alpha\beta} + \lambda_6(45^{\alpha\beta}_\gamma 45^\gamma_{\alpha\beta})^2 + \lambda_7 45^{\alpha\beta}_\gamma 45^{*\delta}_{\alpha\beta}45^{\kappa\lambda}_\delta 45^{*\gamma}_{\kappa\lambda} + \lambda_8 45^{\alpha\beta}_\gamma 45^{*\delta}_{\alpha\beta}45^{\kappa\gamma}_\lambda 45^{*\lambda}_{\kappa\delta}\\&+\lambda_9 45^{\alpha\delta}_\beta 45^{*\beta}_{\alpha\gamma}45^{\kappa\gamma}_\lambda 45^{*\lambda}_{\kappa\delta} + \lambda_{10}45^{\alpha\gamma}_\delta 45^{*\beta}_{\gamma\lambda}45^{\kappa\delta}_\alpha 45^{*\lambda}_{\kappa\beta} + \lambda_{11}45^{\alpha\gamma-}_\delta 45^{*\beta}_{\gamma\lambda}45^{\kappa\lambda}_\alpha 45^{*\delta}_{\kappa\beta}+\\&+\lambda_{12}45^{\alpha\gamma}_\delta 45^{*\beta}_{\gamma\lambda}45^{\kappa\delta}_\beta 45^{*\lambda}_{\kappa\alpha} + \lambda_{13}45^{\alpha\gamma}_\delta 45^{*\beta}_{\gamma\lambda}45^{\kappa\lambda}_\beta 45^{*\delta}_{\kappa\alpha},\end{aligned}$$

$$V(5_H,10_H) = \lambda_{14}5^*_\alpha 5^\alpha 10^*_{\beta\gamma}10^{\beta\gamma} + a_5 5^*_\alpha 10^{\alpha\beta}5^*_\beta + a_5^* 5^\alpha 10^*_{\alpha\beta}5^\beta + \lambda_{15}5^*_\alpha 10^{\alpha\beta}10^*_{\beta\gamma}5^\gamma,$$

$$V(5_H,24_H) = a_3 5^*_\alpha 24^\alpha{}_\beta 5^\beta + \lambda_{16}5^*_\alpha 5^\alpha 24^\beta{}_\gamma 24^\gamma{}_\beta + \lambda_{17}5^*_\alpha 24^\alpha{}_\beta 24^\beta{}_\gamma 5^\gamma,$$

$$V(5_H,45_H) = \lambda_{18}45^{\alpha\beta}_\gamma \bar{45}^\gamma_{\alpha\beta}5^*_\delta 5^\delta + \lambda_{19}45^{\alpha\beta}_\delta 5^*_\gamma 45^{*\gamma}_{\alpha\beta}5^\delta + \lambda_{20}45^{\alpha\beta}_\gamma 45^{*\gamma}_{\alpha\delta}5^*_\beta 5^\delta,$$

$$V(10_H,24_H) = \lambda_{21}10^{\alpha\beta}10^*_{\alpha\beta}24^\gamma{}_\delta 24^\delta{}_\gamma + a_4 10^{\alpha\beta}24_\beta^{\ \gamma}10^*_{\gamma\alpha} + \lambda_{22}10^{\alpha\beta}24_\beta^{\ \gamma}24_\gamma^{\ \delta}10^*_{\delta\alpha} + \lambda_{23}10^{\alpha\beta}24_\beta^{\ \gamma}10^*_{\gamma\delta}24^\delta{}_\alpha,$$

$$\begin{aligned}V(10_H,45_H) = &\lambda_{25}45^{\alpha\beta}_\gamma 45^{*\gamma}_{\alpha\beta}10^*_{\gamma\delta}10^{\gamma\delta} + \lambda_{26}45^{\alpha\beta}_\gamma 10^*_{\alpha\beta}45^{*\gamma}_{\delta\epsilon}10^{\delta\epsilon}+\\&\lambda_{27}45^{\alpha\beta}_\gamma 10^*_{\alpha\delta}10^{\delta\epsilon}45^{*\gamma}_{\alpha\epsilon} + \lambda_{28}45^{\alpha\beta}_\gamma 45^{*\delta}_{\alpha\beta}10^{\gamma\epsilon}10^*_{\delta\epsilon} + \lambda_{29}45^{\alpha\beta}_\gamma 10^*_{\alpha\delta}45^{*\delta}_{\beta\epsilon}10^{\gamma\epsilon}\end{aligned}$$

$$\begin{aligned}V(24_H,45_H) = &a_2 45^{\alpha\beta}_\gamma 24^\gamma{}_\delta \bar{45}^{*\delta}_{\alpha\beta} + \lambda_{30}(45^{\alpha\beta}_\gamma 45^{*\gamma}_{\alpha\beta})24^\delta{}_\epsilon 24^\epsilon{}_\delta + \lambda_{31}45^{\alpha\beta}_\gamma 24^\delta{}_\alpha 24^\epsilon{}_\beta 45^{*\gamma}_{\delta\epsilon} + \lambda_{32}45^{\alpha\beta}_\gamma 24^\gamma{}_\beta 24^\delta{}_\epsilon 45^{*\epsilon}_{\alpha\delta}+\\&\lambda_{33}45^{\alpha\beta}_\gamma 24^\gamma{}_\epsilon 24^\delta{}_\beta 45^{*\epsilon}_{\alpha\delta} + \lambda_{34}45^{\alpha\beta}_\gamma 24^\kappa{}_\alpha 24^\lambda{}_\kappa 45^{*\gamma}_{\lambda\beta} + \lambda_{35}45^{\alpha\beta}_\gamma 24^\gamma{}_\kappa 24^\kappa{}_\lambda 45^{*\lambda}_{\alpha\beta},\end{aligned}$$

$$V(24_H,45_H,5_H) = \lambda_{36}5^*_\alpha 24^\gamma{}_\beta 45^{\alpha\beta}_\gamma + \lambda_{37}5^*_\alpha 24^\gamma{}_\delta 24^\delta{}_\beta 45^{\alpha\beta}_\gamma + \lambda_{38}5^*_\alpha 24^\alpha{}_\beta 24^\gamma{}_\delta 45^{\beta\delta}_\gamma + \text{h.c.}$$

$$V(45_H,10_H,5_H) = \mu 45^{\alpha\beta}_\gamma 10^*_{\alpha\beta}5^\gamma + \text{h.c.}$$

$$V(5_H,24_H,45_H,10_H) = \lambda_{39}45^{\alpha\beta}_\gamma 24^\gamma{}_\delta 10^*_{\alpha\beta}5^\delta + \lambda_{40}45^{\alpha\beta}_\gamma 5^\gamma 24^\delta{}_\alpha 10^*_{\beta\delta} + \text{h.c.}$$

E. Scalar potential of the Zee-LR model

The most general renormalizable potential that can be constructed with the fields $\Phi_1 \equiv \Phi$, $\Phi_2 \equiv \tilde{\Phi}$, H_L, H_R and δ^+ which is invariant under left-right parity symmetry, which implies

$$H_L \leftrightarrow H_R \text{ and } \Phi_i \leftrightarrow \Phi_i^\dagger, \tag{VIII.3}$$

reads as

$$\begin{aligned}
V = &-\mu_H^2(H_L^\dagger H_L + H_R^\dagger H_R) + \lambda_H((H_L^\dagger H_L)^2 + (H_R^\dagger H_R)^2) + \lambda_{LR}(H_L^\dagger H_L)(H_R^\dagger H_R) \\
&- (\mu_\Phi^2)_{ij}\mathrm{Tr}(\Phi_i^\dagger \Phi_i) + \lambda_{ijkl}^{(1)}\mathrm{Tr}(\Phi_i^\dagger \Phi_j)\mathrm{Tr}(\Phi_k^\dagger \Phi_l) + \lambda_{ijkl}^{(2)}\mathrm{Tr}(\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l) \\
&+ a_{ij}(H_L^\dagger H_L + H_R^\dagger H_R)\mathrm{Tr}(\Phi_i^\dagger \Phi_j) + b_{ij}(H_L^\dagger \Phi_i \Phi_j^\dagger H_L + H_R^\dagger \Phi_i^\dagger \Phi_j H_R) + c_i(H_L^\dagger \Phi_i H_R + H_R^\dagger \Phi_i^\dagger H_L) \\
&- \mu_\delta^2 \delta^-\delta^+ + \lambda_\delta(\delta^-\delta^+)^2 + d(H_L^\dagger H_L + H_R^\dagger H_R)\delta^-\delta^+ + e_{ij}\mathrm{Tr}(\Phi_i^\dagger \Phi_j)\delta^-\delta^+ \\
&+ \lambda_i(H_L^T i\sigma_2 \Phi_i H_R \delta^- - H_L^* i\sigma_2^T \Phi_i^\dagger H_R^\dagger \delta^+)\,,
\end{aligned}$$

where

$$\begin{aligned}
&(\mu_\Phi^2)_{ij} = (\mu_\Phi^2)_{ji}\,, \quad \lambda_{ijkk}^{(1)} = \lambda_{jikk}^{(1)}\,, \quad \lambda_{ijkl}^{(1)} = \lambda_{klij}^{(1)}\,, \quad \lambda_{ijkl}^{(1)} = \lambda_{jilk}^{(1)}\,, \\
&\lambda_{ijkl}^{(2)} = \lambda_{jkli}^{(2)} = \lambda_{klij}^{(2)} = \lambda_{lijk}^{(2)}\,, \quad a_{ij} = a_{ji}\,, \quad b_{ij} = b_{ji}\,, \quad e_{ij} = e_{ji}\,.
\end{aligned}$$

In this section we show the consistency of the theory by first showing that the limit $v_R \gg v_L, v_1, v_2$ (since we know that $M_{W_R} \gg M_{W_L}$ experimentally) is physical, i.e. minimizes the potential, and then by defocussing in the limit $v_2 \ll v_1$ since it corresponds to the scenario of the low-scale seesaw we are interested in.

For simplicity and w.l.o.g. we assume that both v_1 and v_2 are negligible compared with v_L and v_R. In this context a soft breaking of the symmetry in the doublets mass term should be required in order avoid the trivial extremum $v_1 = v_2 = v_R = v_L = 0$. The extremum conditions lead to

$$\begin{aligned}
\frac{\partial V}{\partial v_L} &= v_L(v_L^2\lambda_H + \frac{v_R^2}{2}\lambda_{LR} - \mu_L^2) \stackrel{!}{=} 0\,, \\
\frac{\partial V}{\partial v_R} &= v_R(v_R^2\lambda_H + \frac{v_L^2}{2}\lambda_{LR} - \mu_R^2) \stackrel{!}{=} 0.
\end{aligned}$$

We assume that the vacuum expectation values are given by the unique pair of positive solutions:

$$\begin{aligned}
v_L &= \frac{\sqrt{4\lambda_H\mu_L^2 - 2\lambda_{LR}\mu_R^2}}{\sqrt{4\lambda_H^2 - \lambda_{LR}^2}}\,, \\
v_R &= \frac{\sqrt{4\lambda_H\mu_R^2 - 2\lambda_{LR}\mu_L^2}}{\sqrt{4\lambda_H^2 - \lambda_{LR}^2}}\,.
\end{aligned}$$

Thus, the above vevs for the Higgs doublets plus $v_1 = v_2 = 0$ extremize the potential. Now we show that, in this context, a positive definite mass matrix for the Higgs are obtained, which in turn justifies that this solution corresponds to a minimum of the potential.

The CP-even Higgs masses are then given by

$$\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} h_L^0 & h_R^0 & \phi_1^0 & \phi_2^0 \end{pmatrix} \begin{pmatrix} M_{11}^2 & M_{12}^2 & 0 & 0 \\ M_{12}^2 & 0 & 0 & 0 \\ 0 & 0 & M_{33}^2 & M_{34}^2 \\ 0 & 0 & M_{34}^2 & M_{44}^2 \end{pmatrix} \begin{pmatrix} h_L^0 \\ h_R^0 \\ \phi_1^0 \\ \phi_2^0 \end{pmatrix} ,$$

with

$$\begin{aligned} M_{11}^2 &= \frac{4\lambda_H (2\lambda_H \mu_L^2 - \lambda_{LR}\mu_R^2)}{4\lambda_H^2 - \lambda_{LR}^2} , \\ M_{12}^2 &= \frac{2\lambda_{LR}\sqrt{2\lambda_H \mu_R^2 - \lambda_{LR}\mu_L^2}\sqrt{2\lambda_H \mu_L^2 - \lambda_{LR}\mu_R^2}}{4\lambda_H^2 - \lambda_{LR}^2} , \\ M_{33}^2 &= \frac{2\left(a_{11} + a_{22} + b_{22}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 2\left(\left(\mu_\Phi^2\right)_{11} + \left(\mu_\Phi^2\right)_{22}\right) , \\ M_{34}^2 &= \frac{2\left(2a_{12} + b_{12}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 4\left(\mu_\Phi^2\right)_{12} , \\ M_{44}^2 &= \frac{2\left(a_{11} + a_{22} + b_{11}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 2\left(\left(\mu_\Phi^2\right)_{11} + \left(\mu_\Phi^2\right)_{22}\right) . \end{aligned}$$

The CP-odd Higgs masses are given by

$$\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} A_1^0 & A_2^0 \end{pmatrix} \begin{pmatrix} \mathsf{M}_{33}^2 & \mathsf{M}_{34}^2 \\ \mathsf{M}_{34}^2 & \mathsf{M}_{44}^2 \end{pmatrix} \begin{pmatrix} A_1^0 \\ A_2^0 \end{pmatrix} ,$$

with

$$\begin{aligned} \mathsf{M}_{33}^2 &= \frac{2\left(a_{11} + a_{22} + b_{22}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 2\left(\left(\mu_\Phi^2\right)_{11} + \left(\mu_\Phi^2\right)_{22}\right) , \\ \mathsf{M}_{34}^2 &= \frac{2\left(2a_{12} + b_{12}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 4\left(\mu_\Phi^2\right)_{12} , \\ \mathsf{M}_{44}^2 &= \frac{2\left(a_{11} + a_{22} + b_{11}\right)\left(\mu_L^2 + \mu_R^2\right)}{2\lambda_H + \lambda_{LR}} - 2\left(\left(\mu_\Phi^2\right)_{11} + \left(\mu_\Phi^2\right)_{22}\right) , \end{aligned}$$

where A_L^0 and A_R^0 are Nambu-Goldstone bosons eaten by the gauge bosons Z and Z_{BL}.

The charged scalar masses are given by

$$\mathcal{L} \supset \begin{pmatrix} \delta^+ & \phi_1^+ & \phi_2^+ \end{pmatrix} \begin{pmatrix} \mathtt{M}_{33}^2 & \mathtt{M}_{34}^2 & \mathtt{M}_{35}^2 \\ \mathtt{M}_{34}^2 & \mathtt{M}_{44}^2 & \mathtt{M}_{45}^2 \\ \mathtt{M}_{35}^2 & \mathtt{M}_{45}^2 & \mathtt{M}_{55}^2 \end{pmatrix} \begin{pmatrix} \delta^- \\ \phi_1^- \\ \phi_2^- \end{pmatrix} ,$$

with

$$\mathtt{M}_{33}^2 = +d\frac{(\mu_L^2+\mu_R^2)}{2\lambda_H+\lambda_{LR}} - \mu_\delta^2\,,$$
$$\mathtt{M}_{34}^2 = \frac{2f_2\sqrt{2\lambda_H\mu_R^2-\lambda_{LR}\mu_L^2}\sqrt{2\lambda_H\mu_L^2-\lambda_{LR}\mu_R^2}}{4\lambda_H^2-\lambda_{LR}^2}\,,$$
$$\mathtt{M}_{35}^2 = \frac{-2f_1\sqrt{2\lambda_H\mu_R^2-\lambda_{LR}\mu_L^2}\sqrt{2\lambda_H\mu_L^2-\lambda_{LR}\mu_R^2}}{4\lambda_H^2-\lambda_{LR}^2}\,,$$
$$\mathtt{M}_{44}^2 = -\frac{\lambda_{LR}\left((a_{11}+a_{22}+b_{22})\mu_L^2+(a_{11}+a_{22}+b_{11})\mu_R^2\right)+2\lambda_H\left((a_{11}+a_{22}+b_{11})\mu_L^2+(a_{11}+a_{22}+b_{22})\mu_R^2\right)}{4\lambda_H^2-\lambda_{LR}^2}$$
$$-\left(\left(\mu_\Phi^2\right)_{11}+\left(\mu_\Phi^2\right)_{22}\right)\,,$$
$$\mathtt{M}_{45}^2 = 2\left(\mu_\Phi^2\right)_{12} - \frac{(2a_{12}+b_{12})\left(\mu_L^2+\mu_R^2\right)}{2\lambda_H+\lambda_{LR}}\,,$$
$$\mathtt{M}_{55}^2 = -\frac{\lambda_{LR}\left((a_{11}+a_{22}+b_{11})\mu_L^2+(a_{11}+a_{22}+b_{22})\mu_R^2\right)+2\lambda_H\left((a_{11}+a_{22}+b_{22})\mu_L^2+(a_{11}+a_{22}+b_{11})\mu_R^2\right)}{\lambda_H^2-\lambda_{LR}^2}$$
$$-\left(\left(\mu_\Phi^2\right)_{11}+\left(\mu_\Phi^2\right)_{22}\right)\,,$$

where $h_L^\pm$ and $h_R^\pm$ are the Nambu-Goldstone bosons eaten by the gauge bosons $W_L^\pm$ and $W_R^\pm$.

Although the limit $v_1, v_2 \ll v_L, v_R$ works as we can see from above, we are interested in the separation of v_1 and v_2 and the parameter space where $v_2 \to 0$ in order to enjoy small Dirac neutrino masses.

$$V \supset -(\mu_\Phi^2)_{ij}\mathrm{Tr}(\Phi_i^\dagger\Phi_i) + b_{ij}(H_L^\dagger\Phi_i\Phi_j^\dagger H_L + H_R^\dagger\Phi_i^\dagger\Phi_j H_R) + c_i(H_L^\dagger\Phi_i H_R + H_R^\dagger\Phi_i^\dagger H_L)\,,$$

and assume for simplicity $(\mu_\Phi^2)_{11} = (\mu_\Phi^2)_{22} = (\mu_\Phi^2)_{12} =: \mu_\Phi^2$ and $b_{12} = 0$ we find

$$v_1 = -\frac{v_L v_R\left(2c_1\mu_\Phi^2 + c_2\left(-2\mu_\Phi^2 + b_{11}(v_L^2+v_R^2)\right)\right)}{(v_L^2+v_R^2)\left(-2b_{22}\mu_\Phi^2 + b_{11}(-2\mu_\Phi^2 + b_{22}(v_L^2+v_R^2))\right)}\,,$$
$$v_2 = -\frac{v_L v_R\left(2c_2\mu_\Phi^2 + c_1\left(-2\mu_\Phi^2 + b_{22}(v_L^2+v_R^2)\right)\right)}{(v_L^2+v_R^2)\left(-2b_{22}\mu_\Phi^2 + b_{11}(-2\mu_\Phi^2 + b_{22}(v_L^2+v_R^2))\right)}\,.$$

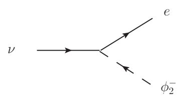
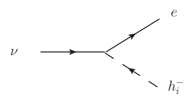
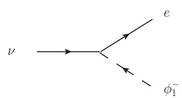
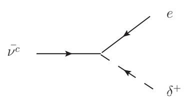
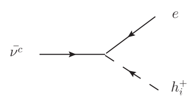
We can infer that to separate v_1 and v_2 with $v_2 \to 0$ the scalar potential has to be in the regime $\mu_\Phi^2, c_1 \to 0$ and $c_2 \neq 0$. This can be seen more easily in the limit $\mu_\Phi \to 0$ where we find

$$v_1 \simeq -\frac{c_2 v_L v_R}{b_{22}(v_L^2+v_R^2)}\,,$$
$$v_2 \simeq -\frac{c_1 v_L v_R}{b_{11}(v_L^2+v_R^2)}\,,$$

and hence if $|c_2/b_{22}| \gg |c_1/b_{11}|$ then $v_1 \gg v_2$ and for $c_1 \to 0$ also $v_2 \to 0$.

F. Feynman rules of the Zee-LR model

Feynman rules involved in the Zee mechanism:

Unbroken Phase		Broken Phase	
	$= Y_3^\dagger P_L - Y_4 P_R$		$= (Y_3^\dagger P_L - Y_4 P_R)V_{2i}^* + (Y_3 P_R - Y_4^\dagger P_L)V_{1i}^*$
	$= Y_3 P_R - Y_4^\dagger P_L$		
	$= 2(\lambda_L P_L + \lambda_R P_R)$		$= 2(\lambda_L P_L + \lambda_R P_R)V_{5i}$

Feynman rules involved in the scattering $qq \to \delta^+\delta^-$:

- $\bar{q}q\gamma : -ieQ^q\gamma^\mu$,
- $\bar{q}qZ : -i(V_Z^q - A_Z^q\gamma^5)\gamma^\mu$,
- $\bar{q}qZ' : -i(V_{Z'}^q - A_{Z'}^q\gamma^5)\gamma^\mu$,
- $\bar{e}eZ' : -i(V_{Z'}^e - A_{Z'}^e\gamma^5)\gamma^\mu$,
- $\bar{\nu}\nu Z' : iA_{Z'}^\nu\gamma^5\gamma^\mu$,
- $\bar{N}NZ' : iA_{Z'}^N\gamma^5\gamma^\mu$,
- $\delta^+\delta^-\gamma : -ie(p_3 - p_4)^\mu$,
- $\delta^+\delta^- Z : -ia_Z(p_3 - p_4)^\mu$,
- $\delta^+\delta^- Z' : -ia_{Z'}(p_3 - p_4)^\mu$.

where

- $V_Z^q = \frac{1}{2}\cos\theta_W\left(g_L T_3 - \frac{g_{BL}^2}{g_L}\cos^2\theta_R\left(\frac{1}{3} + T_3\right)\right)$,
- $A_Z^q = -\frac{T_3}{2}\cos\theta_W\left(\frac{g_R^2}{g_L}\sin^2\theta_R + g_L\right)$,
- $V_{Z'}^q = \frac{1}{2}\cos\theta_R\left(g_R T_3 - \frac{1}{3}\frac{g_{BL}^2}{g_R}\right)$,
- $A_{Z'}^q = \frac{g_R}{2}T_3\cos\theta_R$,
- $V_{Z'}^e = \frac{1}{2}\cos\theta_R\left(\frac{g_{BL}^2}{g_R} - \frac{g_R}{2}\right)$,
- $A_{Z'}^e = -\frac{g_R}{4}\cos\theta_R$,

- $A^{\nu}_{Z'} = A^{N}_{Z'} = \frac{1}{2}\cos\theta_R\left(\frac{g_R}{2} - \frac{g^2_{BL}}{g_R}\right)$,
- $a_Z = -g_{BL}\sin\theta_W\cos\theta_R$,
- $a_{Z'} = -g_{BL}\sin\theta_R$.

where $e = g_L\sin\theta_W = \frac{g_L g_R g_{BL}}{\sqrt{g_L^2 g_R^2 + g_{BL}^2(g_L^2+g_R^2)}}$ and T_3 is the $SU(2)_{L/R}$ isospin of the quark.

G. Dimensional regularization

Performing a Wick rotation , i.e. $x^0 \to ix^0$, the integral can be performed in the Euclidean space,

$$\int \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta)^n} = (-)^n i\int \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2+\Delta)^n}, \tag{VIII.4}$$

and using d-dimensional generalized spherical coordinates, i.e. $d^d l = l^{d-1} dl d\Omega_d$,

$$= \int \frac{d\Omega_d}{(2\pi)^d}\int_0^\infty dl \frac{l^{d-1}}{(l^2+\Delta)^2}, \tag{VIII.5}$$

Taking into account that $\int dx e^{-x^2} = \sqrt{\pi}$,

$$(\sqrt{\pi})^d = \left(\int dx e^{-x^2}\right)^d = \int d^d x e^{-\sum^d x_i^2} = \int d\Omega_d \int_0^\infty dx x^{d-1}e^{-x^2} = \left(\int d\Omega_d\right)\frac{1}{2}\int_0^\infty d(x^2)(x^2)^{\frac{d}{2}-1}e^{-x^2} \tag{VIII.6}$$

where the chain's rule $d(x^2) = 2xdx$ was used. By identifying the gamma function, which is defined as $\Gamma(x) = \int_0^\infty d(y^2)(y^2)^{x-1}e^{-y^2}$,

$$(\sqrt{\pi})^d = \left(\int d\Omega_d\right)\frac{1}{2}\Gamma(d/2), \tag{VIII.7}$$

so that $\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$. Applying this result to the above integral,

$$\int_0^\infty \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2+\Delta)^n} = \frac{2}{\Gamma(d/2)}\frac{1}{\sqrt{4\pi^d}}\int_0^\infty dl \frac{l^{d-1}}{(l^2+\Delta)^n}, \tag{VIII.8}$$

focusing in the remaining integral, we can do a couple of changes of variables in order to write it in an adequate way. First, by using again the chain's rule in variable l, i.e. $dl^2 = 2ldl$ we get

$$\int_0^\infty \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2+\Delta)^n} = \frac{1}{2}\int_0^\infty dl^2 \frac{(l^2)^{\frac{d}{2}-1}}{(l^2+\Delta)^n}, \tag{VIII.9}$$

and then performing the following change of variables $\xi = \frac{\Delta}{(l^2+\Delta)}$,

$$= \frac{1}{2}\Delta^{d/2-n}\int d\xi\, \xi^{n-d/2-1}(1-\xi)^{d/2-1} \tag{VIII.10}$$

by identifying the following definition of the beta function,

$$\int_0^1 d\xi\, \xi^{\alpha-1}(1-\xi)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{VIII.11}$$

we are done with the integral over ξ. Putting altogether, the final result for the d-dimensional integral reads as

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}}\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}},. \tag{VIII.12}$$

Proceeding analogously, the following d-dimensional integrals in Minkowski space [74] can be obtained:

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2-\Delta)^n} \;=\; \frac{(-1)^{n-1} i}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}, \tag{VIII.13}$$

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^\mu\ell^\nu}{(\ell^2-\Delta)^n} \;=\; \frac{(-1)^{n-1} i}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}. \tag{VIII.14}$$

Concretely, for Eq. (VIII.12) and $n=2$, i.e.

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-\Delta)^2} = \frac{i}{(4\pi)^{d/2}}\Gamma\left(n-\frac{d}{2}\right)\Delta^{d/2-2}, \tag{VIII.15}$$

which, by redefining $d=4-\epsilon$, reads as,

$$= \frac{i}{(4\pi)^2}\frac{\Gamma\left(\frac{\epsilon}{2}\right)}{(4\pi)^{\frac{\epsilon}{2}}}\left(\frac{1}{\Delta}\right)^{\frac{\epsilon}{2}}. \tag{VIII.16}$$

Since we are interested in the limit $\epsilon\to 0$, i.e. $d\to 4$, we can expand the powers on ϵ by taking into account that $\chi^\epsilon = e^{\epsilon \mathrm{Log}(\chi)} = 1+\epsilon\mathrm{Log}(\chi)+O(\epsilon^2)$ in the following way,

- $\Gamma\left(\frac{\epsilon}{2}\right) = \frac{2}{\epsilon} - \gamma + O(\epsilon)$,
- $(4\pi)^{\frac{\epsilon}{2}} = 1 + \frac{\epsilon}{2}\mathrm{Log}(4\pi) + O(\epsilon^2)$,
- $\Delta^{-\frac{\epsilon}{2}} = 1 - \frac{\epsilon}{2}\mathrm{Log}(\Delta) + O(\epsilon^2)$,

where γ is the Euler-Mascheroni constant, $\gamma\sim 0.5772$. Applying the above expansions, the integral (VIII.16) reads as,

$$= \frac{i}{(4\pi)^2}\left(\frac{2}{\epsilon} - \mathrm{Log}\left(\frac{\Delta}{4\pi}\right) - \gamma + O(\epsilon)\right), \tag{VIII.17}$$

where we truncated the result at order ϵ since $\epsilon\to 0$. Notice that we have a dimensionful logarithm. We may solve that by performing the following trick: let us add an arbitrary scale, μ_R, called the renormalization scale, in the following way,

$$= \frac{i}{(4\pi)^2}\mu^{-\epsilon}\left(\frac{2}{\epsilon} - \mathrm{Log}\left(\frac{\Delta}{4\pi\mu_R^2}\right) - \gamma + O(\epsilon)\right), \tag{VIII.18}$$

since $\mu^\epsilon = 1+\epsilon/2\mathrm{Log}(\mu^2)+O(\epsilon^2)$, so that now our integral looks much nicer.

H. Amplitudes and decay widths of the gauge boson decays in the c.o.m frame

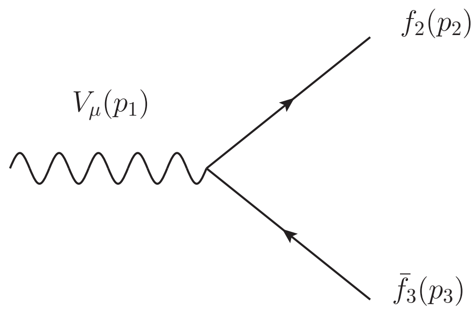


FIG. 24: Gauge boson decay into fermions.

The probability amplitude for the process shown in Fig. 24 to happen is given by

$$\sum_{spins} |\mathcal{M}|^2 = \frac{4}{3}\left[\left((p_2\cdot p_3) + \frac{2(p_1\cdot p_2)(p_1\cdot p_3)}{M_V^2}\right)(|V_V|^2 + |A_V|^2) - 4m_2m_3(|V_V|^2 - |A_V|^2)\right],$$

where M_V refers to the mass of a general spin-1 massive gauge boson V, and m_2 and m_3 refers to the mass of the fermions of momentum p_2 and p_3, respectively (see Fig. 24). The V_V and A_V refers to the vector and axial component of the Feynman rule $Vf\bar{f}$ respectively (see section of Feynman rules).

Now let us analyze the kinematics at the center of mass frame, shown by Fig. 25. In the center of mass

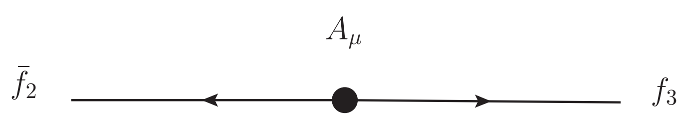


FIG. 25: Diagram of the decay of a heavy gauge boson into two fermions in the center of mass frame.

frame one can define the momenta according to the figure as

$$\begin{aligned} p_1^\mu &= (M_A, 0, 0, 0), \\ p_2^\mu &= (p^*, 0, 0, -p^*), \\ p_3^\mu &= (p^*, 0, 0, p^*). \end{aligned}$$

After applying energy-momentum conservation,

$$\begin{aligned} (p_2\cdot p_3) &= \frac{1}{2}\left(M_A^2 - (m_2^2 + m_3^2)\right), \\ (p_1\cdot p_2) &= M_A\sqrt{p^{*2} + m_2^2}, \\ (p_1\cdot p_3) &= M_A\sqrt{p^{*2} + m_3^2}, \end{aligned}$$

where

$$p^* = \frac{\sqrt{(m_2^2 - m_3^2)^2 - M_A^2\left(2(m_2^2 + m_3^2) - M_A^2\right)}}{2M_A}.$$

Using the above relations, the amplitude in the center of mass frame read as

$$\sum_{spins} |\mathcal{M}|^2 = \frac{4}{3}\left[\frac{1}{2}\left(2M_V^2 - (m_2^2+m_3^2) - \frac{(m_2^2-m_3^2)^2}{M_V^2}\right)(|V_V|^2+|A_V|^2) + 4m_2m_3(|V_V|^2-|A_V|^2)\right].$$

So, in the case of the W_R^+ boson, since $|V_V|^2 = |A_V|^2 =$, the above expression simplifies to

$$\Gamma(W^+_{R,L\mu} \to \bar{f}_2 f_3) = \frac{1}{12\pi M^2_{W^+_{R,L}}}\left(Q_R^f\, g_{R,L}\right)^2\left[2M^2_{W^+_{R,L}} - (m_2^2+m_3^2) - \frac{(m_2^2-m_3^2)^2}{M^2_{W^+_{R,L}}}\right],$$

where $Q_R^f = \frac{1}{2\sqrt{2}}$.

For the $Z^{'}$ case we can also do some simplifications since $m_2 = m_3 = m_f$, so that

$$\sum_{spins} |\mathcal{M}|^2 = \frac{4}{3}\left[(M_V^2 - m_f^2)(|V_V|^2+|A_V|^2) + 4m_f^2(|V_V|^2-|A_V|^2)\right].$$

The decay rate for any two-body decay in the center of mass frame is given by

$$\Gamma = \frac{p^*}{32\pi^2 M_Z^2}\int \sum_{spins}|\mathcal{M}|^2 d\Omega.$$

Thus,

$$\begin{aligned}\Gamma(V_\mu \to \bar{f}_2 f_3) \;&=\; \frac{1}{12\pi M_V^3}\left[\frac{1}{2}\left(2M_V^2 - (m_2^2+m_3^2) - \frac{(m_2^2-m_3^2)^2}{M_V^2}\right)(|V_V|^2+|A_V|^2) + \right.\\ &\left. + 4m_2m_3(|V_V|^2-|A_V|^2)\right]\sqrt{(m_2^2-m_3^2)^2 - M_V^2(2(m_2^2+m_3^2)-M_V^2)}.\end{aligned}$$

Again, by considering the corresponding simplifications to the cases W_R and $Z^{'}$ mentioned above, the decay widths read as

$$\begin{aligned}\Gamma(Z^{'} \to f\bar{f}) \;&=\; \frac{1}{12\pi M^2_{Z'}}\left[(M^2_{Z'}+3m_f^2)|V_{Z'}|^2 + (M^2_{Z'}-5m_f^2)|A_{Z'}|^2\right]\sqrt{M_{Z'}-4m_f^2},\\ \Gamma(W^+_{R,L\mu} \to \bar{f}_2 f_3) \;&=\; \frac{1}{12\pi M^2_{W^+_{R,L}}}\left(\frac{1}{2\sqrt{2}}g_{R,L}\right)^2\left[2M^2_{W^+_{R,L}} - (m_2^2+m_3^2) - \frac{(m_2^2-m_3^2)^2}{M^2_{W^+_{R,L}}}\right]\\ &\frac{\sqrt{(m_2^2-m_3^2)^2 - M^2_{W^+_{R,L}}\left(2(m_2^2+m_3^2) - M^2_{W^+_{R,L}}\right)}}{M_{W^+_{R,L}}}.\end{aligned}$$

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Acknowledgments

I would like to thank, in the first place, my supervisor Pavel Fileviez Perez, for his guidance through the thesis, for his infinite dedication, for his personal advices, for his motivation, for teaching me everything he thought I could understand and more. He has introduced me in the research world in the best possible way, by proposing me two different, beautiful and exciting topics which made me fall even deeper in love with physics.

My thanks also to Sebastian Ohmer, with whom I have collaborated during the second half of my thesis. I greatly appreciate his help, his patient and all the useful things I have learnt from him during our discussions.

I thank professor Jaeckel for accepting being the co-advisor of this thesis and correcting it. I am very grateful for his kindness.

I am thoroughly thankful to La Caixa (Spain) and the DAAD (Germany) for the scholarship that covered my master. Without their financial support I would not have been able to pursue my master in such an incredible university as the University of Heidelberg.

I would like to thank Manfred Lindner, who gave me the opportunity to attend the summer school TAE 2016 to increase my knowledgment in Particle Physics, which I really appreciate.

Thanks to all the MPIK members, who have always been there for helping me in any little or big issue, for answering my doubts or just for chatting or drinking a coffee together. I specially would like to thank my companions in this battlefield: Dani, Philipp, Miguel, Helena, Luisa, Tom, Moritz, Kai, Julia, Rasmus, Johannes, with whom I have shared lots of funny meals, silly discussions, thesis-breaks and some crazy parties; to the old crew, always very helpful with their experiences in physics and life; Alex Helmboldt and Pascal Humbert, for being the coolest standard model tutors one could ever have, who motivated me to pursue my master thesis in this wonderful field at the MPIK; and Juri Smirnov for all his kindness, motivation and help. Thanks to Kevin Max, who has always helped me in any matter; including the translation of the abstract of this thesis to German or the decision of which camera should I buy. And, of course, special thanks to my awesome officemate and friend Clara Miralles Vila, also adventure mate for already six years, for her useful (and not that useful) discussions, her attention, her unconditional support, her love and her smile.

I would also like to thank all my friends, Heidelbergers and non-Heidelbergers, for all the support and enjoyment of these two wonderful years, specialy to Fabrizio Rompineve, who made my days in Heidelberg bright in such a way I did not miss the spanish sun so much; and to Alycia Debnam-Carey for her inspiration.

I finally would like to thank my parents, Juanjo and Visan, who have always supported and encouraged me during my studies, bringing me infinite love; my lovely siblings, Joan and Maria (bichito), for always believing in me before anyone else did or when no one else would, and the rest of my family, who have always been there for me, even if they were one and a half thousand kilometers appart.