Displacement noise-free gravitational-wave detection

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Abstract

The major design of current GW detector is Fabry-Perot Michelson interferometer, which uses laser interferometry. In order to reach high sensitivity, it needs to decrease all displacement noises of mirrors. Recently, new idea that combines many signals and cancels all displacement noises, was suggested. However, this has serious problems: (i) sensitive frequency-band is too high to apply to a ground-based GW interferometer. (ii) GW response function becomes f^2 in low frequency. We naively extended the design with Fabry-Perot cavity and tried resolving these problems. We found that, unfortunately, the results were negative and no improvement in the sensitivity compared with the design without FP cavity, obtained.

1 Introduction

The first generation of kilometer-scale, ground-based laser interferometer gravitational-wave (GW) detectors, located in the United States (LIGO), Europe (VIRGO and GEO 600), and Japan (TAMA 300), has begun its search for gravitational wave radiation and has yielded scientific results. The development of interferometers of the next-generation, such as Advanced-LIGO (in U.S.) and LCGT (in Japan), is underway. These current major design of GW detectors is called Fabry-Perot (FP) Michelson interferometer, in which laser is split into two directions at the beam splitter, is reflected by FP cavities, and then, is combined again at the beam splitter[1]. The output light from the photodetector contains the phase shift due to the change of an arm length in each direction, including GW signals, the displacement noise of mirrors and other noise. So far, much effort to improve sensitivity has been devoted to reducing the displacement noise, caused by seismic oscillation, thermal Brownian motion, radiation pressure of light, and etc.

Recently, the new idea, that cancels the displacement noise by combining the signals, is suggested[2]. This design of an interferometer is called displacement noise-free interferometer (DFI). DFI is similar to time-delay interferometry (TDI), which cancels all laser noise by combining the signals and will be used in Laser Interferometer Space Antenna (LISA), but there is a difference in the way of combining the signals. Extending the previous idea, it was showed that not only displacement noise but also laser noise could be canceled at the same time[3].

DFI is possible because the responses of the GW and the displacement noise is different on the frequency band higher than the frequency corresponding to the size of a detector. The displacement of a mirror acts at the time of the emission and reflection of light. On the other hand, the GW acts during the light trip between the mirrors. It should be noted that, of course, we cannot distinguish them in lower frequency than the size of a detector because GW can be regarded as the tidal force to act on the mirrors.

There is serious problems to apply DFI to a ground-based interferometer. As mentioned above, GWs

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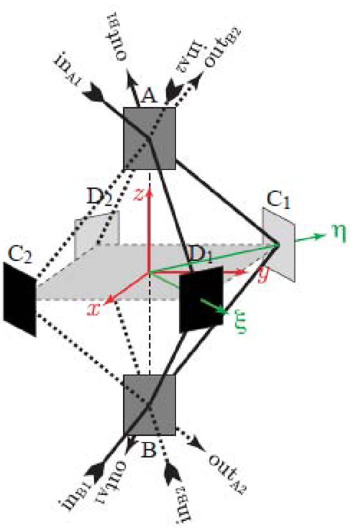

Figure 1: Design of DFI without FP cavity[4].

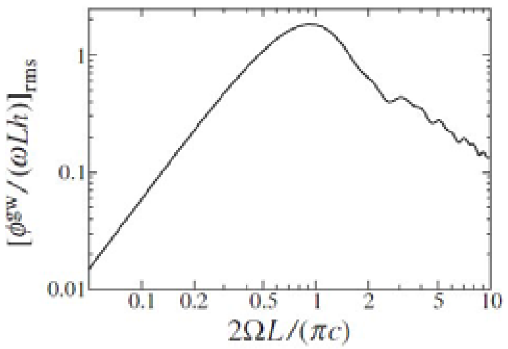

Figure 2: rms response function of DFI without FP cavity[4].

and the displacement of mirrors cannot be distinguished in low frequency. This worsens the response function for GWs in low frequency, which has the frequency dependence f^2. In addition, the peak of response function located around $100kHz$, corresponding to the arm length of the detector, for example, $3km$. Thus, the sensitivity worsens in our interested observational frequency, in which GWs from astrophysical objects can exist. To resolve this problem, we tried constructing DFI design with FP cavity. First, we will review the previous design of DFI without FP cavity briefly. Next, we will describe the design with FP cavity.

2 DFI without FP cavity

First, in order to take a brief look at the principle of DFI, let us consider N-detector system. Suppose that there are N test masses, located in D-dimensions and lights are emitted and receipted between each test mass. In this case, $N(N-1)$ signals can be acquired. On the other hand, the number of displacement noise is $N \times D$ and the number of laser noise is N. Thus, all noise can be canceled if the number of the signal is larger than that of the noise. In other words, this is expressed as the condition, $N \geq D+2$.

In practice, however, it is difficult to realize a composite mirror, which emits or reflects light in more than two directions. So, more practical design without composite mirrors, are proposed[4]. This consists of two Machzender interferometers, in which lasers are injected from the top and bottom, as showed in Fig2. In the figure, beam splitters are located at the top and bottom, and the mirrors are at four vertices in the middle. The phase shift of light from the start $(ct_0, \mathbf{x}_0)$ to the end $(ct, \mathbf{x})$ is expressed to the linear order of h as,

$$\phi^{gw} = \frac{\omega L}{2c} \int_0^1 d\zeta h_{ij}^{TT}(t_0 + L\zeta, \mathbf{x}_0 + \mathbf{N}\ell\zeta) N_i N_j \tag{1}$$

where c is the speed of light, ω is the angular frequency of light, $|\mathbf{x}-\mathbf{x}_0| = c|t-t_0| = L$ is a nonperturbed distance between two test masses and $\mathbf{N}$ is defined as $\mathbf{N} \equiv (\mathbf{x}-\mathbf{x}_0)/L$. GW is written as h_{ij}^{TT} in Transvers-Traceless gauge.

Let us define the phase shift of signal for a Machzender interferometer,

$$\delta\phi_{A1} \equiv \phi_{(A\to C1\to B)} - \phi_{(A\to D1\to B)}, \qquad \delta\phi_{A2} \equiv \phi_{(A\to D2\to B)} - \phi_{(A\to C2\to B)}, \tag{2}$$

$$\delta\phi_{B1} \equiv \phi_{(B\to C1\to A)} - \phi_{(B\to D1\to A)}, \qquad \delta\phi_{B2} \equiv \phi_{(B\to D2\to A)} - \phi_{(B\to C2\to A)}. \tag{3}$$

Combining these signals to cancel the all displacements of mirrors and beam splitters, gives following the signal,

$$\phi_{DFI} \equiv [\delta\phi_{A1} - \delta\phi_{B1}] - [\delta\phi_{A2} - \delta\phi_{B2}]. \tag{4}$$

Fourier-transforming ϕ_{DFI} and taking average for GWs from all directions on the celestial sphere, we can obtain the root-mean-squared GW response function of DFI, as showed Fig2. The behavior of the response function is different in above and below the frequency corresponding to the size of the detector, L. Above the characteristic frequency, the response becomes f^{-1} because the contribution of GW in a phase shift during the light trip, is integrated and canceled partially. Below the frequency, the response becomes f^2 because we cannot distinguish the effects of GW and the displacement of a mirror, and DFI combination also cancels GW signals. Due to this fact, the sensitivity worsens in our interested observational frequency even though the only noise is white shot noise.

3 DFI with FP cavity

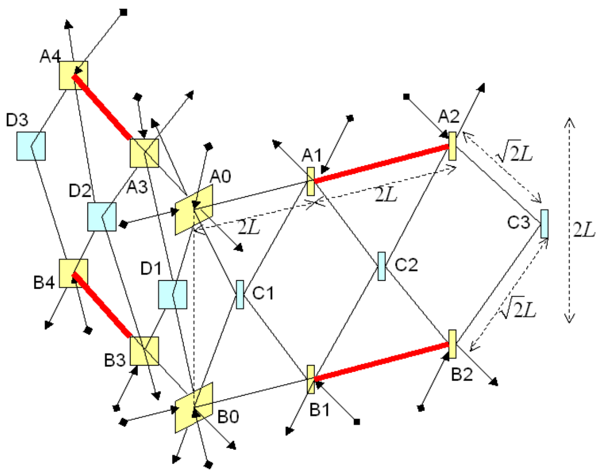

Figure 3: Design of DFI with FP cavity.

In order to figure out the problem of DFI in the previous section, we introduced the FP cavity into DFI and thought out the design like Fig3. The yellow objects are mirrors and beam splitters and the light blue are completely reflecting mirrors. The parts $A1-A2$, $B1-B2$, $A3-A4$, $B3-B4$ are FP cavities.

In Fourier domain, the phase shift due to GW and the displacement of mirrors from $(t-L_{12}, \mathbf{X}_1+\mathbf{x}_1)$ to $(t, \mathbf{X}_2+\mathbf{x}_2)$, is given as[3],

$$\begin{aligned} \tilde{\delta\phi}_{12}(\Omega) &= \omega/c \times [\, \mathbf{n}_{12} \cdot \{\tilde{\mathbf{x}}_2 - e^{-i\Omega L_{12}} \tilde{\mathbf{x}}_1\} \\ &+ \mathbf{n}_{12} \otimes \mathbf{n}_{12} \;:\; \frac{1}{2} \{ \sum_p \mathbf{e}^p \tilde{h}_p \frac{e^{i\Omega(-\mathbf{e}_z \cdot \mathbf{X}_2)} - e^{-i\Omega(L_{12}+\mathbf{e}_z \cdot \mathbf{X}_1)}}{i\Omega(1-\mathbf{e}_z \cdot \mathbf{n}_{12})} \} \,] \end{aligned} \tag{5}$$

where L_{12} is the arm length between test mass 1 and 2, $\mathbf{X}_i$ and $\mathbf{x}_i$, $i = 1, 2$ are the nonperturbed and perturbed positions of test masses respectively. $\mathbf{n}_{12}$ is defined as $\mathbf{n}_{12} \equiv (\mathbf{X}_2 - \mathbf{X}_1)/L_{12}$. Noted that $\mathbf{x}_i$ depends on the time of light emission and reception. GW is propagating in the direction of $\mathbf{e}_z$. The polarization tensors of GW are difined as

$$\mathbf{e}_+ \equiv \mathbf{e}_x \otimes \mathbf{e}_x - \mathbf{e}_y \otimes \mathbf{e}_y, \qquad \mathbf{e}_\times \equiv \mathbf{e}_x \otimes \mathbf{e}_y + \mathbf{e}_y \otimes \mathbf{e}_x. \tag{6}$$

The response of the FP cavity is well known. Using the above expression and taking the sum of the signals for each travels in FP cavity, we can derive the phase shift due to GW and the displacements of mirrors and obtain the simple form,

$$\tilde{\Phi}_{12} = \alpha \tilde{\delta\phi}_{12} \tag{7}$$

where

$$\alpha \equiv \frac{e^{-i\Omega L_{12}}}{1 - R^2 e^{-2i\Omega L_{12}}} \tag{8}$$

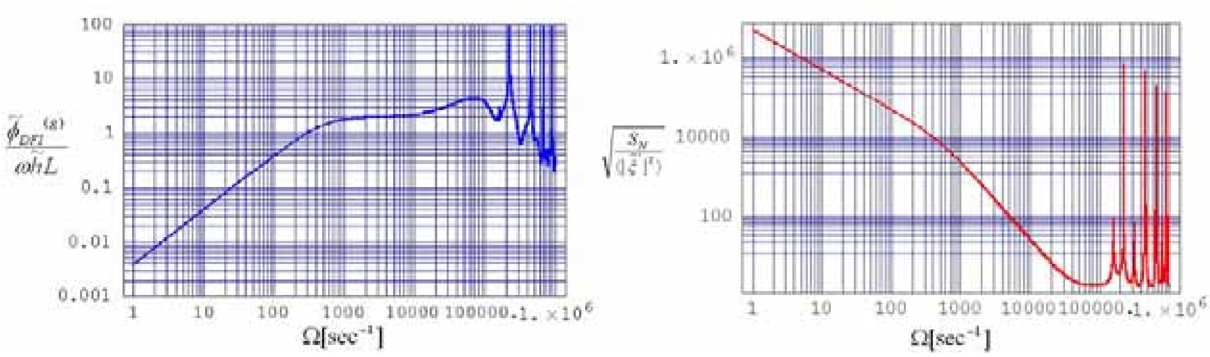

Figure 4: GW responce function (left) and shot noise spectrum (right) of DFI with FP cavity. Here we selected the parameters $R = 0.99$ and $L = 3km$. This correspons to $\Omega_{FP} \approx 6.3 \times 10^5 sec^{-1}$ and $\Omega_L \approx 500 sec^{-1}$. The values in the vertical axis are normalized by some factor.

R is the reflectivity of the mirrors of FP cavity.

Now we are ready to construct DFI signal. Using arbitrary coefficients, we can write down it like,

$$\tilde{\delta\phi}_{DFI}(\Omega) \equiv \tilde{\delta\phi}_{FPM1}(\Omega) - \tilde{\delta\phi}_{FPM2}(\Omega) + \sum_{i=0}^{4} \gamma_i \, \tilde{\delta\phi}_{MZi}(\Omega). \tag{9}$$

where $\tilde{\delta\phi}_{FPM1}$ and $\tilde{\delta\phi}_{FPM2}$ are the signals of Fabry-Perot Michelson interferometer at the top and bottom in Fig3 respectively, and $\tilde{\delta\phi}_{MZi}$, $i = 0, \cdots, 4$ are the signals of Machzender interferometer concerning A_i and B_i respectively. γ_i, $i = 0, \cdots, 4$ are arbitrary coefficients, determined by the condition that all displacement noise are canceled, and become

$$\begin{aligned} \gamma_0 &= \frac{\sqrt{2}}{1 - e^{-2\sqrt{2}i\Omega L}}, \quad \gamma_1 = \frac{\{2\alpha \, cos(2\Omega L) \, e^{-2i\Omega L} + 1\}e^{-2i\Omega L}}{\sqrt{2}(1 - e^{-2\sqrt{2}i\Omega L})}, \quad \gamma_2 = \frac{-\sqrt{2}\alpha e^{-4i\Omega L}}{1 - e^{-2\sqrt{2}i\Omega L}} \\ \gamma_3 &= -\gamma_1, \quad \gamma_4 = -\gamma_2 \end{aligned} \tag{10}$$

Thus, the signal $\tilde{\delta\phi}_{DFI}$ has no displacement noise and is limited only by shot noise. The results are shown in Fig4. GW response function is shifted toward low frequency by virtue of FP cavity, and flat region appears between the characteristic frequency determined by the size of the detector and that determined by FP cavity, in the spectrum. These are $\Omega_L = c/L$ and $\Omega_{FP} = T^2 c/(4L)$ respectively, where T is the transmissivity of the mirrors of FP cavity. In this case, however, the spectrum of shot noise after the cancellation has nontrivial dependence on frequency, though the spectrum of an original shot noise has flat frequency dependence. So, in total, shot noise sets off the advantage of GW response and the sensitivity shows the same frequency dependence as that of DFI without FP cavity! Unfortunately, we had negative results.

In this work, we found that the naive attempt to lower the peak frequency in the sensitivity with FP cavity, doesn't work well. In order to apply the DFI to a ground-based GW interferometer, it needs other ideas to shift the peak frequency of the sensitivity.

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