Causality Constraints on Massive Gravity

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The parameter space of the de Rham–Gabadadze–Tolley massive gravity ought to be constrained essentially to a line. The theory is shown to admit pp-wave backgrounds on which linear fluctuations otherwise undergo significant time advances, potentially leading to closed time-like curves. This classical phenomenon takes place well within the theory's validity regime.

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I. INTRODUCTION

A non-zero graviton mass is an interesting theoretical possibility that modifies General Relativity in the infrared and hence may ameliorate the cosmological constant problem. It is not so easy, though, to construct consistent theories of massive gravity. Such attempts were initiated long ago by Fierz and Pauli [1], who wrote down a ghost-free linearized Lagrangian for a massive graviton in flat space. However, it was not until recently that a consistent non-linear theory could be constructed [2, 3], thanks to de Rham, Gabadadze and Tolley (dRGT). The dRGT massive gravity is remarkable in that it overcame the Boulware–Deser ghost problem [4], formerly believed to plague any non-linear theory of massive-gravity with instabilities.

In this Letter, we consider 4D massive gravity theories that admit Minkowski as a solution. They constitute a family of Lagrangians that include the graviton mass m as well as two dimensionless parameters α_3 and α_4:

$$\mathcal{L} = \tfrac{1}{2}M_{\rm P}^2\sqrt{-g}\left[R + m^2\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right], \qquad (1)$$

where the three possible potential terms are

$$\begin{aligned}
\mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2],\\
\mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3],\\
\mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4],
\end{aligned} \qquad (2)$$

with the notation $[X] \equiv X^\mu{}_\mu$ for the tensor

$$\mathcal{K}^\mu{}_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\rho}f_{\rho\nu}}\,, \qquad (3)$$

and its various powers, where $f_{\mu\nu}$ is the reference metric which we will assume to be flat: $f_{\mu\nu} = \eta_{\mu\nu}$.

Eqs. (1)–(3) present the theory in the so-called unitary gauge, in which a Hamiltonian analysis has confirmed the non-existence of the ghost at the full non-linear level for generic values of the parameters [5]. One may wonder if the absence of unphysical modes in a theory guarantees its classical consistency. After all, there are various known instances where this is not true [7–9]. In the context of massive gravity, this issue was raised and critically addressed already in [10].

The purpose of this Letter is to show that the generic dRGT theory is plagued with causality violation well below the strong-coupling scale $\Lambda = \sqrt[3]{m^2 M_{\rm P}}$. More precisely, the theory admits pp-wave backgrounds that let the longitudinal modes of massive-gravity fluctuations undergo measurable time advances. This potentially leads to closed time-like curves, thereby violating causality. The remedy is to constrain the parameter space of the theory appropriately. Our main result is that the parameter α_3 is essentially set to

$$\alpha_3 = -\tfrac{1}{2}\,. \qquad (4)$$

II. PP-WAVE SOLUTIONS

Let us introduce the light-cone coordinate system $(u, v, \vec{x})$, where $u = t - x_3$, $v = t + x_3$, and $\vec{x} = (x_1, x_2)$. In these coordinates, a generic pp-wave spacetime has the following metric:

$$\mathrm{d}s^2 = -\mathrm{d}u\mathrm{d}v + F(u,\vec{x})\mathrm{d}u^2 + \mathrm{d}\vec{x}^2. \qquad (5)$$

This geometry enjoys the null Killing vector ∂_v. One can introduce a covariantly constant null vector $l_\mu = \delta_{\mu u}$ to write this metric in the Kerr–Schild form,

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + F l_\mu l_\nu. \qquad (6)$$

To see if massive gravity admits pp-waves solutions, let us first write down the equations of motion resulting from the Lagrangian (1). They are

$$G_{\mu\nu} + m^2\mathcal{X}_{\mu\nu} = 0, \qquad (7)$$

where $G_{\mu\nu}$ is the Einstein tensor and $\mathcal{X}_{\mu\nu}$ is given by

$$\begin{aligned}
\mathcal{X}_{\mu\nu} = {}& \mathcal{K}_{\mu\nu} - [\mathcal{K}]\,g_{\mu\nu}\\
&- \alpha\left(\mathcal{K}^2_{\mu\nu} - [\mathcal{K}]\,\mathcal{K}_{\mu\nu} + \tfrac{1}{2}g_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right)\\
&- \beta\left(\mathcal{K}^3_{\mu\nu} - [\mathcal{K}]\,\mathcal{K}^2_{\mu\nu} + \tfrac{1}{2}\mathcal{K}_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right)\\
&+ \tfrac{1}{6}\beta g_{\mu\nu}\left([\mathcal{K}]^3 - 3[\mathcal{K}^2][\mathcal{K}]^2 + 2[\mathcal{K}^3]\right),
\end{aligned} \qquad (8)$$

while the parameters α and β are given in terms of the original ones as $\alpha \equiv 3\alpha_3 + 1,\ \beta \equiv -3(\alpha_3 + 4\alpha_4)$.

The metric (6) yields the following Einstein tensor: $G_{\mu\nu} = -\frac{1}{2} l_\mu l_\nu \partial^2 F$, with $\partial^2 \equiv \partial_\mu \partial^\mu$. To compute $\mathcal{X}_{\mu\nu}$, note that $\mathcal{K}^\mu{}_\nu = \delta^\mu_\nu - \sqrt{\bar{g}^{\mu\rho}(\bar{g}_{\rho\nu} - F l_\rho l_\nu)} = \frac{1}{2} F l^\mu l_\nu$. Since l_μ is null, $[\mathcal{K}] = 0$ and $\mathcal{K}^2_{\mu\nu} = 0$. It is then clear from Eq. (8) that $\mathcal{X}_{\mu\nu} = \mathcal{K}_{\mu\nu} = \frac{1}{2} F l_\mu l_\nu$. Therefore, the metric (6) will be a solution of the massive gravity equation (7), provided that the function F satisfies the massive Klein–Gordon equation. The latter actually reduces to the 2D screened Poisson equation since $F = F(u, \vec{x})$ is independent of v:

$$\left(\partial^2 - m^2\right) F = \left(\partial_i \partial_i - m^2\right) F = 0. \tag{9}$$

Assuming rotational symmetry on the transverse plane, this equation has the following solution at $\vec{x} \neq 0$:

$$F = A(u) K_0(m|\vec{x}|), \tag{10}$$

where K_0 is the zeroth-order modified Bessel function of the second kind, while $A(u)$ is arbitrary in u.

Let us choose the profile of a "sandwich wave" displayed in Fig. 1:

$$A(u) = \begin{cases} a \exp\left[-\frac{\lambda^2 u^2}{(u^2-\lambda^2)^2}\right] & \text{if } u \in [-\lambda, \lambda], \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where a is a numerical constant and λ is a length scale [6]. Eq. (11) defines a smooth function $A(u) \in \mathrm{C}^\infty(\mathbb{R})$, with a compact support $[-\lambda, \lambda]$. The sandwich wave moves at the speed of light in the v-direction. Its amplitude and width are defined by a and λ respectively.

One might wonder about the singularity of the metric at $|\vec{x}| = 0$. In fact, such a geometry may be viewed as arising from the stress-energy tensor

$$T_{\mu\nu} = \pi M_{\mathrm{P}}^2 A(u) \delta^2(\vec{x})\, l_\mu l_\nu, \tag{12}$$

which saturates the null-energy condition. Then, the energy E of the source is quantified by $M_{\mathrm{P}}^2 \lambda$.

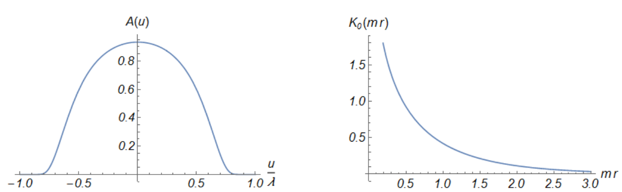

FIG. 1. Profiles of the sandwich wave: u-direction profile (left) and radial profile in the transverse plane (right).

For future convenience, we choose the amplitude a such that $\int_{-\lambda}^{+\lambda} \mathrm{d}u\, A(u) = \lambda$. This amounts to the choice $a \approx 0.93 = \mathcal{O}(1)$. We also choose the width λ to be larger than the resolution length of the effective field theory: $\lambda \gtrsim 1/\Lambda$. The latter choice is possible for a very large energy of the source: $E \gg \Lambda$. This situation is completely acceptable and does not at all invalidate the effective field theory description [17].

III. LINEAR FLUCTUATIONS

On the pp-wave geometry described in the previous section, let us consider linear massive-gravity fluctuations, $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$. Schematically, their equations of motion read [11]

$$\delta G_{\mu\nu} + m^2 \delta \mathcal{X}_{\mu\nu} = 0, \tag{13}$$

with the quantity $\delta G_{\mu\nu}$ given by

$$\begin{aligned}\delta G_{\mu\nu} = &-\tfrac{1}{2}\left(\nabla^2 h_{\mu\nu} - 2\nabla^\rho \nabla_{(\mu} h_{\nu)\rho} + \nabla_\mu \nabla_\nu h\right) \\ &+ \tfrac{1}{2}\bar{g}_{\mu\nu}\left(\nabla^2 h - \nabla\cdot\nabla\cdot h + \bar{R}^{\rho\sigma} h_{\rho\sigma}\right) - \tfrac{1}{2}\bar{R} h_{\mu\nu},\end{aligned} \tag{14}$$

where ∇_μ is the covariant derivative built from the background metric $\bar{g}_{\mu\nu}$, dot denotes a contraction of indices and $h \equiv \bar{g}^{\mu\nu} h_{\mu\nu}$. To find an expression for $\delta\mathcal{X}_{\mu\nu}$, we first need the variation of the tensor $\mathcal{K}^\mu{}_\nu$ defined in Eq. (3). An explicit computation gives

$$\delta\mathcal{K}^\mu{}_\nu = \tfrac{1}{2} h^\mu_\nu + \tfrac{1}{8} F\left(l^\mu l\cdot h_\nu - 3 l_\nu l\cdot h^\mu\right) - \tfrac{1}{16} F^2 l^\mu l_\nu l\cdot h\cdot l. \tag{15}$$

Then, varying Eqs. (8) one finds from a straightforward calculation that

$$\begin{aligned}\delta\mathcal{X}_{\mu\nu} = &\tfrac{1}{2}\left(h_{\mu\nu} - \bar{g}_{\mu\nu} h\right) - \tfrac{2\alpha-1}{4} F l_{(\mu} l\cdot h_{\nu)} + \tfrac{\alpha}{4} F l_\mu l_\nu h \\ &+ \left(\tfrac{\alpha+1}{4}\bar{g}_{\mu\nu} - \tfrac{1}{16} F l_\mu l_\nu\right) F l\cdot h\cdot l.\end{aligned} \tag{16}$$

Note that the parameter β has dropped out! In other words, the linearized fluctuations on the pp-wave background are insensitive to β. The subsequent analysis therefore holds for any value of this parameter.

One can now proceed to derive the scalar and vector constraints. We would not bore the reader with the tedious details, and just present the final results. The trace constraint reads

$$h = \left(\alpha + \tfrac{1}{2}\right)\left[F l\cdot h\cdot l + \tfrac{2}{3m^2} F_{,\mu}\left(\partial^\mu l\cdot h\cdot l - l\cdot\partial\, l\cdot h^\mu\right)\right], \tag{17}$$

whereas the divergence constraint is given by

$$\begin{aligned}\mathcal{C}_\mu = &-\tfrac{\alpha}{2} F \partial_\mu l\cdot h\cdot l + \tfrac{2\alpha-1}{4} F l\cdot\partial\, l\cdot h_\mu + \tfrac{2\alpha+3}{4} F_{,\rho}\, l_\mu l\cdot h^\rho \\ &+ \tfrac{1}{4} F l_\mu l\cdot\partial\, h - \left(\tfrac{\alpha+1}{2} F_{,\mu} + \tfrac{2\alpha+1}{16} F^2 l_\mu l\cdot\partial\right) l\cdot h\cdot l,\end{aligned} \tag{18}$$

where $\mathcal{C}_\mu \equiv \nabla\cdot h_\mu - \nabla_\mu h = \partial\cdot h_\mu - \partial_\mu h - \frac{1}{2} F_{,\mu} l\cdot h\cdot l$, and $F_{,\mu}$ is a shorthand notation for $\partial_\mu F$.

The derivation relies on the assumption that the fluctuations do not propagate through $\vec{x} = 0$, so that the background equation (9) can be used. Note that it involves not just the divergences and trace of Eq. (13), but also contractions thereof with the null vector l^μ. Of particular interest is the quantity $\nabla^\mu \delta G_{\mu\nu}$, which actually reduces to terms containing only single derivatives of the fluctuations, thanks to the identity (5.3) of Ref. [11]. Also, one needs the background Riemann tensor, which reads $\bar{R}^\rho{}_{\sigma\mu\nu} = l_\sigma l_{[\mu}\partial_{\nu]}\partial^\rho F - l^\rho l_{[\mu}\partial_{\nu]}\partial_\sigma F$.

The 5 constraints (17)–(18) render non-dynamical 5 components of the symmetric tensor $h_{\mu\nu}$, leaving one with 5 dynamical degrees of freedom, as expected. To be more explicit, we rewrite the scalar constraint as

$$h = 4F\hat{\alpha}h_{vv} + \tfrac{1}{\hat{m}^2}\, F_{,i}\left(\partial_i h_{vv} - \partial_v h_{vi}\right), \qquad (19)$$

where we have defined $\hat{\alpha} \equiv \alpha + \frac{1}{2}$, and $\frac{1}{\hat{m}^2} \equiv \frac{8}{3m^2}\hat{\alpha}$. Because $h = (h_{11} + h_{22}) - 4\left(h_{uv} + F h_{vv}\right)$, Eq. (19) determines completely the linear combination $(h_{11} + h_{22})$ in terms of other components. On the other hand, the vector constraint (18) lets one set the 4 components $h_{u\mu}$ to be non-dynamical, since their v-derivatives are completely determined. Therefore, the dynamical degrees of freedom are the two transverse modes: $(h_{11} - h_{22})$ and h_{12}, plus the three longitudinal ones: h_{vi} and h_{vv}.

To study the true dynamics, let us use commutators of covariant derivatives to rewrite Eq. (13) as

$$\left(\nabla^2 - m^2\right) h_{\mu\nu} = \Delta\mathcal{R}_{\mu\nu}, \qquad (20)$$

where the right hand side is written solely in terms of the constraints and curvatures, and is given by

$$\begin{aligned}\Delta\mathcal{R}_{\mu\nu} &= 2\nabla_{(\mu}\mathcal{C}_{\nu)} + \nabla_\mu\nabla_\nu h - 2F_{,\rho(\mu}l_{\nu)}l\cdot h^\rho + F_{,\mu\nu}l\cdot h\cdot l \\ &- \bar{g}_{\mu\nu}\nabla\cdot\mathcal{C} - m^2\bar{g}_{\mu\nu}\left[h - \tfrac{1}{4}(2\hat{\alpha}-1)Fl\cdot h\cdot l\right] + \cdots, \qquad (21)\end{aligned}$$

with ellipses standing for terms that do not contribute to the physical modes. One can substitute the right hand sides of the constraints (17)–(18) in Eq. (20) to write down the true dynamical equations. It turns out that the equations of motion for the longitudinal modes completely decouple. They have the following form:

$$\begin{aligned}\left(\partial^2 - m^2\right) h_{vi} &= Y_{ij}\partial_v^2 h_{vj} + Y_i\partial_v h_{vv}, \\ \left(\partial^2 - m^2\right) h_{vv} &= Z_i\partial_v^3 h_{vi} + Z\partial_v^2 h_{vv},\end{aligned} \qquad (22)$$

where we have defined the following operators:

$$\begin{aligned}Y_{ij} &= 2(\hat{\alpha}-1)F\delta_{ij} - \tfrac{1}{\hat{m}^2}\left(F_{,ij} + F_{,j}\partial_i\right), \\ Y_i &= 2\hat{\alpha}\left(F_{,i} + F\partial_i\right) + 2F_{,i} + \tfrac{1}{\hat{m}^2}\left(F_{,ij} + F_{,j}\partial_i\right)\partial_j, \\ Z_i &= -\tfrac{1}{\hat{m}^2}F_{,i}\,, \quad Z = 4\left(\hat{\alpha} - \tfrac{1}{2}\right)F + \tfrac{1}{\hat{m}^2}F_{,i}\partial_i\,.\end{aligned} \qquad (23)$$

The transverse derivatives of F are given, in terms of the unit transverse-position vector $\vec{n} \equiv \vec{x}/|\vec{x}|$, as

$$\begin{aligned}F_{,i} &= -mFn_i\,\tfrac{K_1(m|\vec{x}|)}{K_0(m|\vec{x}|)}, \\ F_{,ij} &= m^2F\Big[n_in_j + \tfrac{K_1(m|\vec{x}|)}{m|\vec{x}|\,K_0(m|\vec{x}|)}(2n_in_j - \delta_{ij})\Big]\,.\end{aligned} \qquad (24)$$

IV. SHAPIRO TIME DELAY / ADVANCE

One of the classic tests of General Relativity is the Shapiro time delay [13] suffered by a light ray while passing by a massive body. We would like to compute this delay (or advance) for the longitudinal modes of the massive-gravity fluctuation upon crossing the sandwich wave. To this end, we note that the general solutions of Eq. (20) and the constraints (17)–(18) can be written as superpositions of eigensolutions of the form:

$$h_{\mu\nu}(u,v,\vec{x}) = \tilde{h}_{\mu\nu}(u)\, e^{i(pv + \vec{q}\cdot\vec{x})}, \qquad (25)$$

where p and $\vec{q}$ are the momenta in the u-direction and the transverse directions respectively.

Note that $\vec{q} = \vec{q}(u)$ since the probe will experience a radial impulse in the transverse plane during the course of the sandwich wave, $u \in [-\lambda, \lambda]$. Let $\vec{q}_-$ and $\vec{q}_+$ be the incoming and outgoing transverse momenta respectively. We denote by $\vec{b}$ the impact parameter vector (in the transverse plane) at $u = -\lambda$. The unit vector along this direction is $\vec{e} \equiv \vec{b}/b$, where $b = |\vec{b}|$. We choose $\vec{q}_-$ to be aligned with $\vec{b}$, i.e., $\vec{q}_- = q_-\vec{e}$ with $q_- > 0$.

We will consider the following regime of parameters:

$$\Lambda \gtrsim \frac{1}{\lambda} \gg p \gg q_- \gg \frac{1}{b} \gg m. \qquad (26)$$

The above parametric relations may very well be accommodated because the separation between the scales Λ and m is huge $\sim \sqrt[3]{M_\mathrm{P}/m}$. The condition $q_-b \gg 1$ ensures that the probe is far away from $\vec{x} = 0$. For simplicity of analysis, we take the particle to be ultrarelativistic with $p \gg q_-$, but all momenta are much smaller than Λ. On the other hand, the sandwich wave is chosen to be very thin compared to the length scales characterizing the probe: $\lambda p \ll 1$, but thick enough to be "seen" in the effective theory: $\lambda\Lambda \gtrsim 1$. Finally, the choice of a small impact parameter, $mb \ll 1$, amplifies the effects of the sandwich wave on the probe.

While the probe particle is passing through the sandwich wave, its transverse position $\vec{x}$ will change slightly: $|\vec{x} - \vec{b}| \lesssim \lambda$, $|\vec{n} - \vec{e}| \lesssim \lambda/b$. We will neglect these small changes. The radial impulse deflects the particle but keeps $\vec{q}(u)$ aligned with $\vec{e}$: $\vec{q}(u) = q(u)\vec{e}$. Note that $q(u)$ remains positive and small compared to p. To see this, let us use the deflection formula (A.36) of Ref. [12], which is a valid approximation because the sandwich wave is thin. With $E \sim M_\mathrm{P}^2\lambda$ and $\vec{q}_+ \equiv q_+\vec{e}$, we can write $(q_-/p) - (q_+/p) \sim \lambda/b$. Given the separation of scales (26), we conclude that $q_+ > 0$ and $q_+ \approx q_-$. The same conclusion holds for $q(u)$ as it varies continuously.

Let us collectively denote the longitudinal modes as $\{\Phi_I(u)\}$ with $I = 1,2,3$, defined as

$$\Phi_1 = e_i\tilde{h}_{vi}, \qquad \Phi_2 = \varepsilon_{ij}e_i\tilde{h}_{vj}, \qquad \Phi_3 = \tilde{h}_{vv}, \qquad (27)$$

where ε_{ij} is the Levi–Civita symbol in the transverse plane. Now, plugging the expressions (25) into Eqs. (22)

and using the redefinitions (27) results in the following first-order coupled differential equations:

$$(\partial_u - ip\gamma)\,\Phi_I(u) = ipA(u)\mathcal{M}_{IJ}\Phi_J(u), \tag{28}$$

where $\gamma \equiv \frac{1}{4}(q^2+m^2)/p^2$, and the 3×3 matrix $\mathcal{M}$ contains the functions $K_0(mb)\equiv k_0$ and $K_1(mb)\equiv k_1$ in the following non-zero components:

$$\begin{aligned}
&\mathcal{M}_{11} = \tfrac{\hat\alpha-3}{6}\,k_0 + \tfrac{2\hat\alpha(1-iqb)}{3mb}\,k_1,\\
&\mathcal{M}_{13} = -\tfrac{7\hat\alpha q}{6p}\,k_0 - \tfrac{4\hat\alpha q+3i(\hat\alpha+1)m^2b-4i\hat\alpha q^2 b}{6pmb}\,k_1,\\
&\mathcal{M}_{22} = -\tfrac{\hat\alpha+1}{2}\,k_0 - \tfrac{2\hat\alpha}{3mb}\,k_1,\\
&\mathcal{M}_{31} = -\tfrac{2i\hat\alpha p}{3m}\,k_1, \quad \mathcal{M}_{33} = -\tfrac{2\hat\alpha+1}{2}\,k_0 + \tfrac{2i\hat\alpha q}{3m}\,k_1.
\end{aligned} \tag{29}$$

Let the eigenvalues of $\mathcal{M}$ be μ_I. The matrix $\mathcal{P}$ composed of the eigenvectors of $\mathcal{M}$ is u-dependent, but this dependency is as small as $q(u)/p$. Then, in terms of the modes $\Phi'_I \equiv \mathcal{P}^{-1}_{IJ}\Phi_J$, Eqs. (28) are approximately diagonal, and hence can be integrated to

$$\Phi'_I(+\lambda) \approx \Phi'_I(-\lambda)e^{ip\int_{-\lambda}^{+\lambda}\mathrm{d}u\,(\gamma+\mu_I A(u))}. \tag{30}$$

The integral in the exponent is to be understood as the shift in the v-coordinate suffered by the I-th mode upon crossing the sandwich wave [9]. To find the shift relative to massless propagation in flat space, we write the relevant terms originating from γ:

$$\Delta\gamma = \tfrac{1}{4}m^2/p^2 + \tfrac{1}{4}(q^2-q_-^2)/p^2. \tag{31}$$

The first piece comes from the non-zero graviton mass, whereas the second from the non-zero curvature.

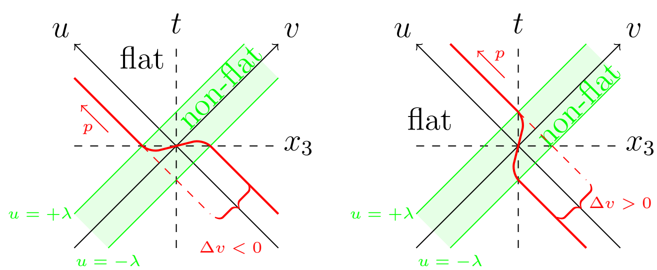

FIG. 2. Upon crossing the sandwich wave the probe undergoes a time advance (left) or a time delay (right).

The eigenvalues μ_I are independent of p and q. For small impact parameter $mb \ll 1$ they reduce to

$$\mu_1 = \tfrac{2\hat\alpha}{3m^2b^2}, \quad \mu_2 = -\tfrac{2\hat\alpha}{3m^2b^2}, \quad \mu_3 = \tfrac{2\hat\alpha+1}{2}\ln(mb), \tag{32}$$

and dominate over the $\Delta\gamma$-contributions. Then, the v-shift relative to flat-space massless propagation reads

$$\Delta v_I \equiv \int_{-\lambda}^{+\lambda}\mathrm{d}u\,(\Delta\gamma + A(u)\mu_I) \approx \lambda\mu_I. \tag{33}$$

A positive shift corresponds to a time delay, whereas a negative Δv corresponds to a time advance (see Fig. 2).

Since μ_1 and μ_2 have opposite signs, Eq. (33) says that any non-zero value of $\hat\alpha$ will lead to a time advance either for Φ'_1 or Φ'_2. Whenever $|\hat\alpha| \gtrsim m^2b^2$, the time advance is larger than the resolution time of the effective theory: $\Delta v \gtrsim 1/\Lambda$. On the other hand, if $\hat\alpha = 0$, there is no time advance on the pp-wave background [18].

V. CLOSED TIME-LIKE CURVES

The argument that time advances lead to close time-like curves (CTC) is standard. For the sake of self-containedness we just present the arrangements of Appendix G of Ref. [9]. Strictly speaking, one would need a refined version of the simplistic setup appearing in Fig. 3.

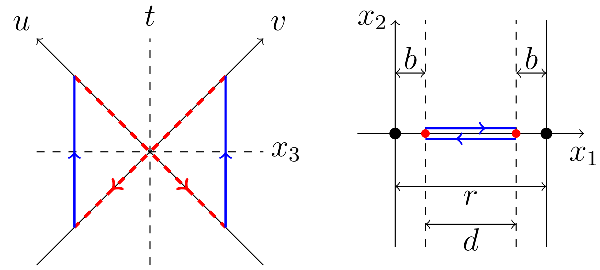

FIG. 3. Motion of a probe that follows a CTC, projected on the u-v plane (left) and on the transverse plane (right).

We imagine two sandwich waves moving in opposite directions, centered respectively at $u = 0$ and $v = 0$, separated in the transverse plane by a distance r. The probe crosses the waves one after the other, and acquires time advances $|\Delta v| = |\Delta u| \sim |\hat\alpha|\lambda/(m^2b^2)$. Note that there is a contribution to the time delay from the non-zero mass, since the probe travels a finite distance $\sim |\Delta v|$. This contribution goes as $-\Delta v(m^2/p^2)$ and is therefore negligible in front of Δv. Right after each wave passes by, a mirror is needed to control the motion in the transverse plane. The mirrors must be set in appropriate angles to counter deflections.

In between the two waves the probe travels a transverse distance $d = r - 2b$. In order to form a CTC we would need $d \sim |\hat\alpha|\lambda/(m^2b^2)$. We also require $d \gg 1/m$, so that the waves have negligible overlap at $u = v = 0$. These requirements combine into $m^2b^2 \ll |\hat\alpha| m\lambda$. In other words, the small numbers $\epsilon_1 \equiv mb$ and $\epsilon_2 \equiv \lambda/b$ should be chosen such that $\epsilon_1/\epsilon_2 \ll |\hat\alpha|$. With the present LIGO bound [14] on the graviton mass one can maintain the separation of scales (26) while making the ratio ϵ_1/ϵ_2 as small as 10^{-6}. Therefore, our argument leaves room for the following parameter region [19]:

$$|\hat\alpha| \sim |\alpha_3 + \tfrac{1}{2}| \lesssim 10^{-6}. \tag{34}$$

This is in spirit the line $\alpha_3 = -1/2$, reported in Eq. (4).

VI. REMARKS

We have shown that unless the parameter space of the dRGT massive gravity is constrained essentially to the line (4), CTCs can be formed. This is presumably an IR manifestation of the restrictions arising from requiring a sensible UV-completion [16]. In this regard it is interesting to look at the Cheung–Remmen parameter island (see Fig. 4), singled out by positivity constraints on scattering amplitudes [15]. In the parameter plane

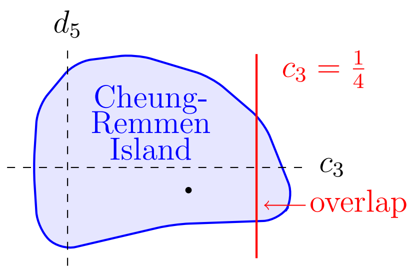

FIG. 4. A cartoon of the Cheung-Remmen parameter island. The red line $c_3 = 1/4$ corresponds to our result $\alpha_3 = -1/2$.

of $(c_3, d_5) \equiv (-\alpha_3/2, -\alpha_4/4)$, our result corresponds to the line $c_3 = 1/4$, and it rules out the minimal model represented by the black dot of Fig. 4 at $c_3 = 1/6$ and $d_5 = -1/48$. The positivity constraints are necessary but not sufficient for the existence of a UV-completion. Our analysis, on the other hand, considers only one class of backgrounds allowed by the dRGT theory. While it is inspiring to find an overlap of the two sets of results, it is likely that the parameter space will be constrained even further by considering other non-trivial backgrounds.

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[17] The situation is analogous to having a macroscopic (super-Planckian) black hole in General Relativity.
[18] This reduces the constraint (17) simply to $h = 0$. A non-zero trace may serve as a scalar constraint, but will always go along with causality violation (see also [10]).
[19] The allowed strip of region will get only thinner with improved bounds on the graviton mass.