BRIEF COMMUNICATION

# A condition for small bootstrap current in three-dimensional toroidal configurations

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## Abstract

For three-dimensional magnetic confinement configurations with stellarator symmetry [1] it is shown that the condition, that the maximum of the strength of the magnetic field on a magnetic surface constitutes a line orthogonal to the field lines and crosses the symmetry line, renders the strength of the bootstrap current density small compared to this quantity in quasi-axisymmetric (qa) [1, 2] and quasi-helically (qh) [3, 4] symmetric configurations.

Three-dimensional configurations offer the possibility of an island divertor concept [5]. Configurations with small bootstrap current facilitate the operation of an island divertor. Wendelstein 7-X [6] realizes the small boostrap current property by a suitable adjustment of toroidal and helical plasma curvature [7]. Here, a different condition for rendering the bootstrap current small is described which could be exploited in a future stellarator design. A standard method for assessing the long-mean-free-path limit of the bootstrap current density on a magnetic surface in a three-dimensional magnetic confinement configuration is the evaluation of the so-called bootstrap current coefficient $G_b$, e.g. conveniently summarized in [8] and more directly derived in [9]. A key ingredient of this procedure is the consideration of the field line passing through the maximum of the field strength, $B_{max}$, on that surface and evaluating the particle motion along this field line. Accordingly, the generating mechanism of the bs current density can be visualized by considering passing particle orbits barely passing the point of maximum field strength. Figure 1 shows such orbits for a qa [1] and a qh [3] configuration, and illustrates why the bs current density is opposite in qa and qh configurations. In nearly quasi-isodynamic [10, 11] (qi) configurations the bs current density nearly vanishes, and dedicated optimization renders it vanishing quite accurately [12][2]. True qi configurations exhibit the maximum of the field strength on a magnetic surface not occurring in a point but as the orthogonal to the field lines through the symmetry line, so that the condition found here is in accordance with a qi configuration. A configuration satisfying the $B_{max}$ condition without being close to a qi configuration

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[1] In cylindrical coordinates the points $r, \phi, z$ and $r, -\phi, -z$ are equivalent in the case of stellarator symmetry; the line $z = 0,\ \phi = 0$ or $\pi$ then is a symmetry line.

[2] Also, [11, 12] give references on computational tools for drift-kinetic calculations.

was obtained from a W7-X type of configuration [13] by optimization towards a $B_{max}$ which constitutes the line orthogonal to the field lines which intersects the symmetry line. The structure of $B$ obtained is shown in Fig. 2 and, indeed, shows no apparent specific structure except its $B_{max}$-line. Drift surfaces of barely passing particles are shown in Fig. 3 for this configuration. Since these drift surfaces appear to indicate approximate cancellation of the driving mechanism for the bs current, the conjecture that the bs current be small is plausible. Evaluations starting at the toroidally outer and inner position on the $B_{max}$-line give opposite results; evaluation as a function of $\theta$ shows that the average value $< G_b(\theta) >$ nearly vanishes, see Fig. 4, and supports the conjecture. Additionally, the bootstrap behavior can be investigated by computing mono-energetic drift-kinetic distribution functions[3] [7], as they are even in $v_{||}/v$ for vanishing bs current density. Results for various configurations are seen in Fig. 5 and confirm the conjecture.

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[3] These computations as well as the derivation of $G_b$ neglect collisions other than pitch-angle scattering.

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## Figure captions

Fig. 1 a, b. Cross sections of two magnetic surfaces are shown in magnetic polar coordinates $s$ and $\theta$ (with $s$ the normalized toroidal flux and $\theta$ the poloidal magnetic Boozer coordinate): s=0.25, the start surface for the guiding centre trajectories and s=0.5. Also shown are cross sections of drift surfaces of barely passing trajectories, passing in opposite toroidal directions: in a) for a qa configuration [2], in b) for a qh configuration [3]. In order to make the deviations clearly visible $\alpha$-particles in fusion-type devices ( $B = 5\,\mathrm{T}$, V=$10^3$ m$^3$ ) are used. The small initial parallel velocity is chosen just sufficiently large to guarantee many toroidal transits despite the slight imperfection of the realization of the quasi-symmetry.

Fig. 2

Contours of the field strength in the configuration optimized in order to achieve $B_{max} \approx const$ on the orthogonal through the symmetry line.

Fig. 3.

Same as for Fig. 1, but for the configuration of Fig. 2 at two starting points on the $B_{max}$-line on the outer ( $\theta = 0$ ) and the inner ( $\theta = \pi$ ) sides of the flux surface.

Fig. 4.

Results for the bootstrap current coefficient as a function of normalized flux for the starting points $\theta = 0, \pi$ and the average of $G_b(\theta)$.

Fig. 5.

Averaged mono-energetic guiding-center particle distribution functions (see. eg [7]) as functions of the pitch-angle variable $v_{||}/v$ for a qa [2] (black), a qh [3] (red), a W7-X-type [13] configuration (green dashed), the configuration of Fig. 2 (green), an $l = 2$ stellarator (number of periods 5, aspect ratio $\approx$ 11, rotational transform $\approx$ 0.5) modified in the same way (blue) and the same configuration at a ten times smaller mean free path (blue dashed).

## References

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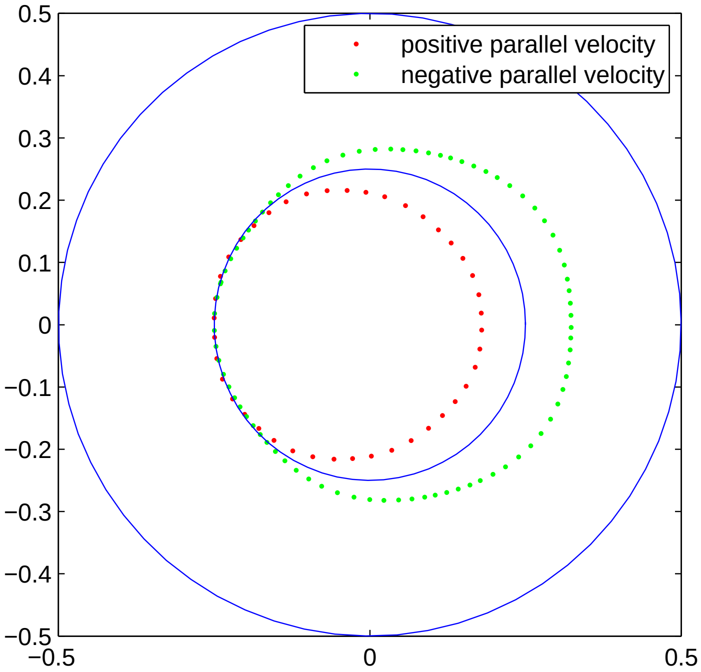

Figure 1a

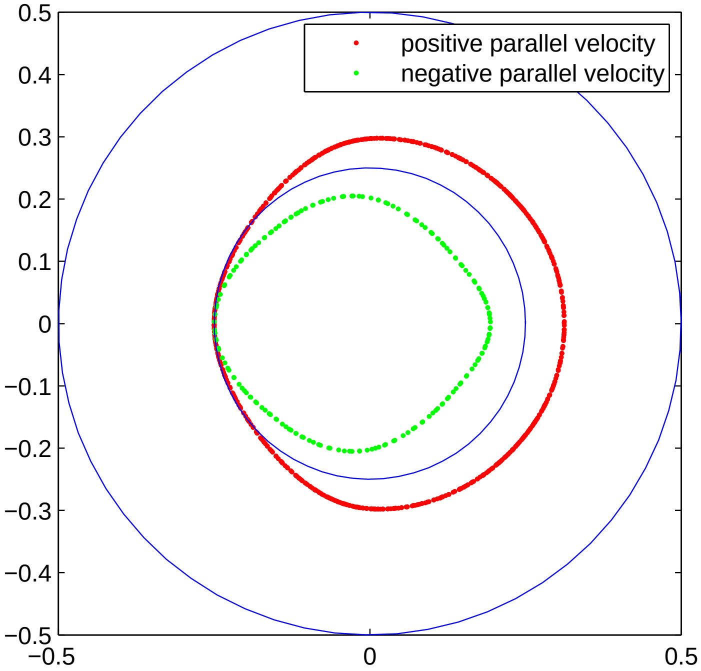

Figure 1b

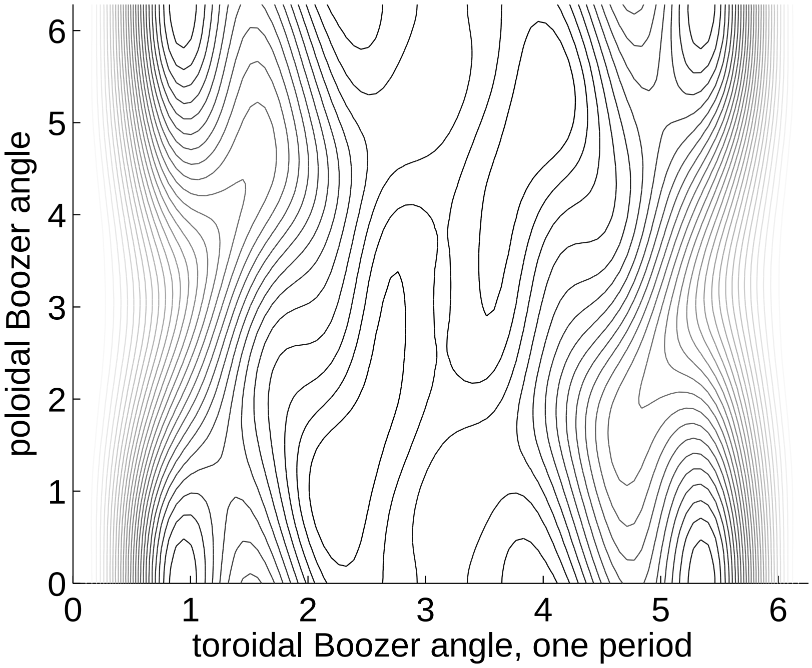

Figure 2

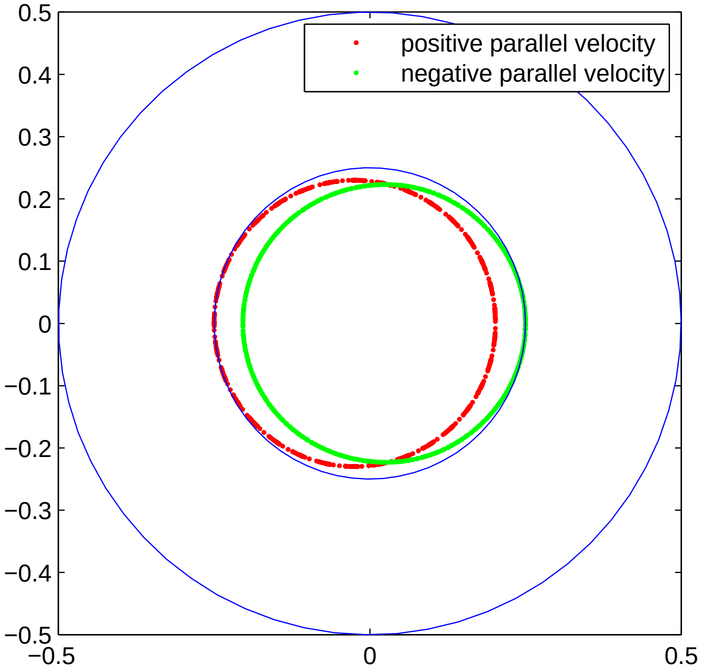

Figure 3

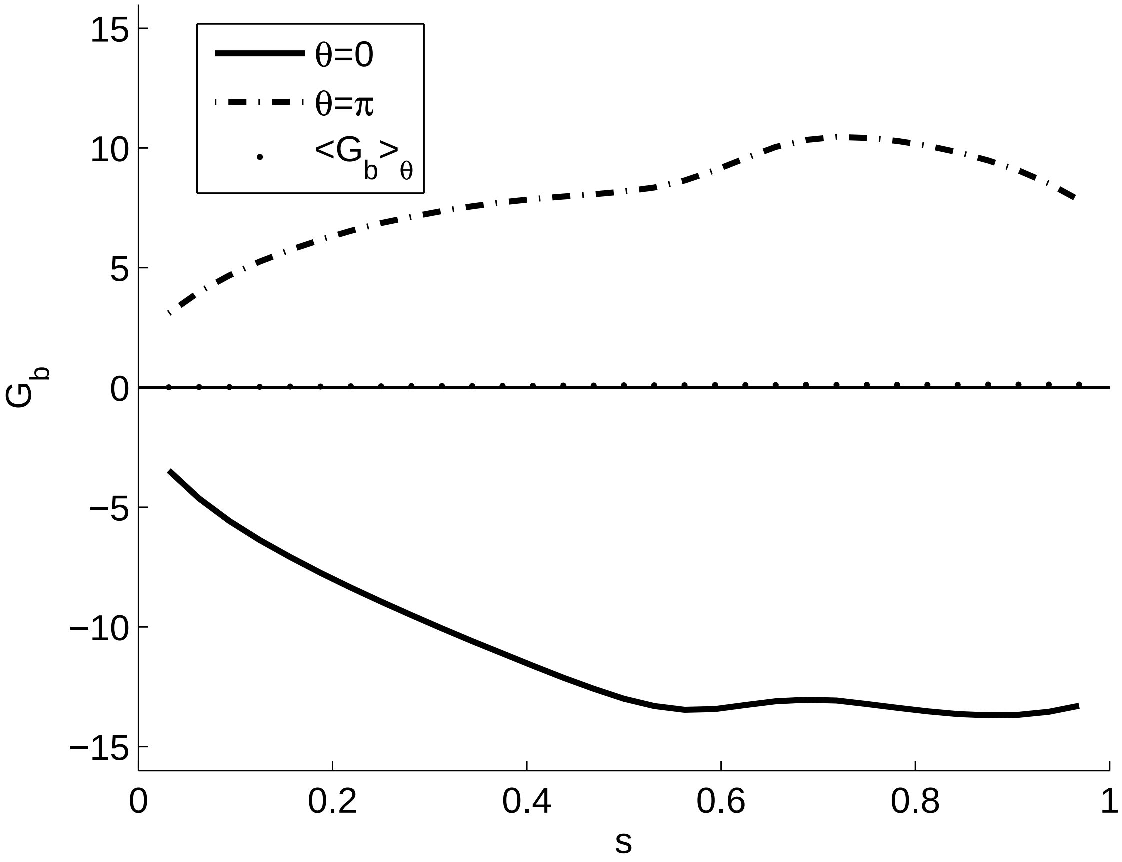

Figure 4

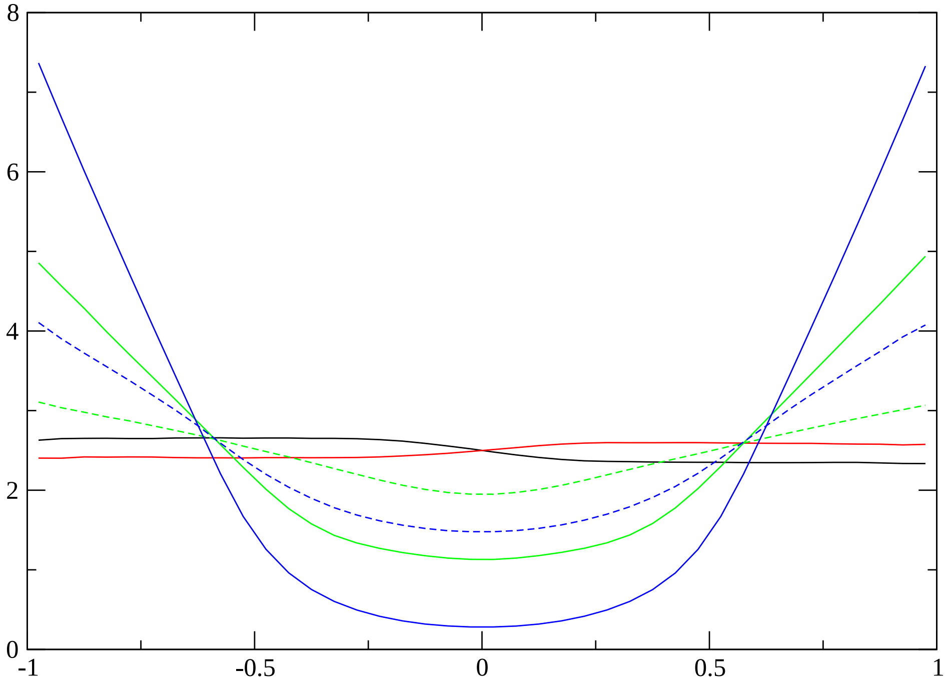

Figure 5