Physics Letters B 781 (2018) 547–552

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Universal formula for the holographic speed of sound

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ARTICLE INFO

Article history:
Received 4 February 2018
Received in revised form 10 April 2018
Accepted 12 April 2018
Available online 21 April 2018
Editor: M. Cvetič

ABSTRACT

We consider planar hairy black holes in five dimensions with a real scalar field in the Breitenlohner–Freedman window and derive a universal formula for the holographic speed of sound for any mixed boundary conditions of the scalar field. As an example, we numerically construct the most general class of planar black holes coupled to a single scalar field in the consistent truncation of type IIB supergravity that preserves the $SO(3) \times SO(3)$ R-symmetry group of the gauge theory. For this particular family of solutions, we find that the speed of sound exceeds the conformal value. From a phenomenological point of view, the fact that the conformal bound can be violated by choosing the right mixed boundary conditions is relevant for the existence of neutron stars with a certain mass-size relationship for which a large value of the speed of sound codifies a stiff equation of state. In the way, we also shed light on a puzzle regarding the appearance of the scalar charges in the first law. Finally, we generalize the formula of the speed of sound to arbitrary dimensional scalar-metric theories whose parameters lie within the Breitenlohner–Freedman window.

1. Introduction

Gauge/gravity duality [1] is a very powerful conjecture, linking together two apparently disparate descriptions of physics: a gravitational field theory and a quantum field theory in a lower dimension. One of its best known results is the universal value of the ratio of the shear viscosity to entropy density [2]. Finite 't Hooft coupling corrections on the field theory side correspond to α' corrections on the gravity side, inclusion of which modifies the ratio, thus violating the conjectured viscosity bound [3,4]. The computation of the transport coefficients thus seems to have a strong dependence on the bulk Lagrangian and the computation of the holographic speed of sound [5–9] was done with this idea in mind. Indeed, a bound on the speed of sound was also expected to exist [10], as well as a dynamical bound of the bulk viscosity [11]. The fact that different gravity Lagrangians are the key to describing different speeds of sound has been extensively used in the literature (see for instance [12]), but not much emphasis was put on the boundary conditions.

The main goal of this Letter is to obtain a universal formula for the holographic speed of sound in gravity theories with scalar fields. If the scalar field is in the Breitenlohner–Freedman (**BF**) window [13,14]

$$-\frac{(D-1)^2}{4l^2} = m_{BF}^2 \leq m^2 < m_{BF}^2 + l^{-2}, \tag{1}$$

where D is the bulk spacetime dimension and l the AdS radius, the holographic speed of sound depends as much on the details of the scalar field potential as it does on the boundary conditions that the scalar field satisfies. The universal formula we obtain takes the details of this relation into account.

To obtain this general result, we face the problem of solving the bulk theory. We tackle this by means of a generalization of the designer gravity soliton line [15–17]. The key point is that scalar fields satisfying (1) admit an infinite number of possible boundary conditions, which can be traced back to the existence, in any dimension, of two normalizable modes [18]. In general, these bound-

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https://doi.org/10.1016/j.physletb.2018.04.028

ary conditions can break the conformal symmetry in the boundary [19,20]. When the scalar field mass is in the BF window (1), it provides two asymptotic integration constants (α,β) to the system. In addition, a static black hole metric provides one extra integration constant, μ, related to its mass. The solution space of the fully back-reacted metric plus the scalar field is characterized by three integration constants [21]. The solution of the non-linear system of differential equations provides the map

$$\alpha=\alpha(\varphi_h,\mathcal{A}),\qquad \beta=\beta(\varphi_h,\mathcal{A}),\qquad \mu=\mu(\varphi_h,\mathcal{A}),\tag{2}$$

where $(\varphi_h,\mathcal{A})$ are the horizon data, namely the value of the scalar field at the horizon, φ_h, and the normalized black hole area,[1] $\mathcal{A}$. The proof that dependence on $\mathcal{A}$ of the map (2) is universal and related only to the mass of the scalar field is the second essential result of this letter.

An interesting phenomenological aspect where our results have an important impact is neutron stars, which provide a unique laboratory for the study of ultra-dense nuclear matter. It is therefore of interest to construct a phenomenological model using holography; however doubts concerning the applicability of the duality to the description of neutron stars [22] due to a possible bound of the speed of sound of strongly coupled theories with gravity duals were raised in [23]. We point out that this also depends on the mixed boundary conditions, a point that was not considered in [23], and explicitly show that, indeed, the bound can be violated.

We begin with a concise review of deformations of the gauge theory along the lines of [24]. We explicitly construct the map (2) that allows us to obtain a general formula for the speed of sound. This formula is universal as is valid for all boundary conditions. Then, we present an explicit example in the consistent truncation of type IIB supergravity that breaks the isometries of the S^5 to $SO(3)\times SO(3)$ and discuss the physics of a double trace deformation. We find that the speed of sound attains a maximum value at $c_s^2\approx 0.458$. We discuss the validity of our result for any value of the coupling constant of the deformation and its relation to the renormalization group (RG) flow in the gauge theory. In the last section we provide a formula for any scalar field with a mass in the range (1).

2. The Gauge theory deformation and holography

Here we briefly review the relation between the field theory deformation and its holographic interpretation. In field theory one would like to add to the action a functional of the form [24]

$$\Delta I=\left(\frac{N}{2\pi}\right)^2\int_{\partial M} W(\mathcal{O})\sqrt{-\gamma^{(0)}}d^4x,\tag{3}$$

where $W(\mathcal{O})$ is an arbitrary function[2] of a single trace operator $\mathcal{O}$. Gravitational variables are recovered with the standard AdS/CFT identification $N^2=\frac{\pi}{2}\frac{l^3}{G}$ [1]. The bulk metric is

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=\frac{l^2}{r^2}dr^2+\frac{r^2}{l^2}\left[\gamma_{ab}^{(0)}+O(r^{-2})\right]dx^a dx^b.\tag{4}$$

The gravitational action is

$$I[g,\varphi]=\frac{1}{8\pi G}\int_M d^5x\sqrt{-g}\left[\frac{R}{2}-\frac{1}{2}(\partial\varphi)^2-V(\varphi)\right]+I_{ct}+I_{ct}^{\varphi}+\Delta I,\tag{5}$$

where I_{ct} and I_{ct}^{φ} are counterterms that render the variational principle well posed [17,25–32]. When the mass of the scalar field saturates the BF bound its fall-off is

$$\varphi=\frac{\alpha l^4}{r^2}\ln\left(\frac{r}{r_0}\right)+\frac{\beta l^4}{r^2}+O\left(\frac{\ln(r)^2}{r^3}\right),\tag{6}$$

where α and β are normalized to have engineering dimension 2. Adding the relevant scalar counterterm [17,30], is possible to identify the vacuum expectation value (**VEV**) of the single trace operator with $\langle\mathcal{O}\rangle=\beta$ as well as its source $J=\alpha-\frac{dW(\beta)}{d\beta}$ [30].

3. The black hole surface

In this section we shall outline the construction of the surface defined by the embedding (2). We consider the class of metrics in five dimensions

$$ds^2=e^A(-f dt^2+d\Sigma)+e^B\frac{dr^2}{f},\tag{7}$$

where $d\Sigma$ is a Ricci flat surface. For static metrics is possible to introduce the new variables $X=\frac{d\varphi}{dA}$ and $Y=Xf\frac{d\varphi}{df}$ and show that the Einstein equations are satisfied if $\frac{dB}{d\varphi}=-\frac{2}{X}\frac{dX}{d\varphi}+\frac{4}{3}X$ and

$$\begin{aligned}\frac{dX}{d\varphi}&=\left(3\frac{dV}{d\varphi}+4XV\right)\frac{\left(2X^2Y-6Y-3X^2\right)}{12VXY}\\ \frac{dY}{d\varphi}&=\left(3\frac{dV}{d\varphi}+2XV\right)\frac{\left(2X^2Y-6Y-3X^2\right)}{6VX^2}.\end{aligned}\tag{8}$$

It follows that $\frac{dX}{d\varphi}$ is finite at the horizon, located at $Y(\varphi_h)=0$, if and only if $X(\varphi_h)=-\frac{3}{4V}\frac{dV}{d\varphi}\Big|_{\varphi=\varphi_h}$. Furthermore, we readily see that these equations decouple and reduce to the single master equation

$$\begin{aligned}&-3\left(3\frac{dV}{d\varphi}+4XV\right)\frac{d^2X}{d\varphi^2}X+\left(-9\frac{dV}{d\varphi}+12XV\right)\left(\frac{dX}{d\varphi}\right)^2\\ &+\left[\left(X^2+3\right)8XV+\left(18X^2+1\right)\frac{dV}{d\varphi}+9X\frac{d^2V}{d\varphi^2}\right]\frac{dX}{d\varphi}=0.\end{aligned}\tag{9}$$

It is a consequence of the fall-off of the scalar (6) that, asymptotically, $e^A\varphi\sim\alpha l^2\ln(\frac{r}{r_0})$. The derivative of this relation with respect to $\ln r\sim\frac{A}{2}$ yields $2e^A\left(\varphi+\frac{d\varphi}{dA}\right)\sim\alpha l^2$. Thus,

$$l^2\alpha(\varphi_h,\mathcal{A})=\lim_{\varphi\to 0}2(X+\varphi)e^A=\alpha_h(\varphi_h)\mathcal{A}^{\frac{2}{3}}.\tag{10}$$

We see that α is generically a function of φ_h times a very precise function of the normalized black hole area. When there exists a black hole the metric fall-off can be taken as

$$\begin{aligned}e^A f&=\frac{r^2}{l^2}-\frac{\mu l^6}{r^2}+O(r^{-3}),\\ e^A&=\frac{r^2}{l^2}+O(r^{-3}),\qquad \frac{e^B}{f}=\frac{l^2}{r^2}+O(\frac{\ln(r)^2}{r^6}),\end{aligned}\tag{11}$$

where the normalization of the integration constant μ is chosen to match its mass scaling dimension, $\Delta(\mu)=4$ with its engineering

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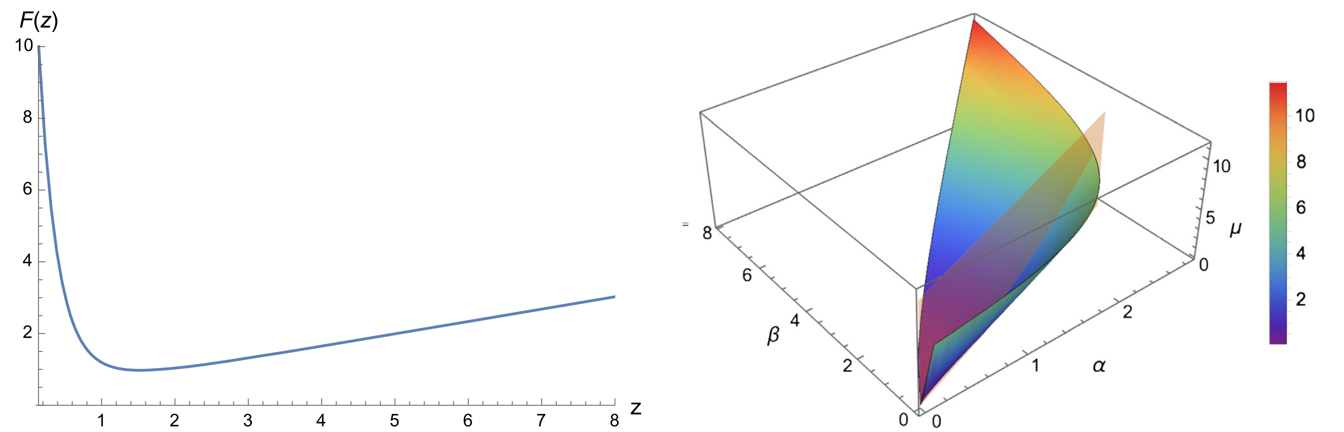
[1] We shall consider planar horizons with one compact direction. In this case $s=\frac{\mathcal{A}}{4G}$ is the entropy density.

[2] While the function W is formally arbitrary, it has to be such that there is either a soliton or a hairy black hole solution in the spacetime bulk. Otherwise the variational principle cannot attain a minimum due to potential bulk singularities.



(a) Black hole line defined by equation (17).

(b) Black hole surface $\mu = \mu(\alpha, \beta)$ and boundary condition surface $\beta = \alpha$. The color coding corresponds to the magnitude of μ.

Fig. 1. Black hole line and black hole surface for theories with potential (20).

dimension. The introduction of the scalar field yields a dual perfect fluid with energy momentum tensor [17]

$$-\frac{2}{\sqrt{-\gamma^{(0)}}}\frac{\delta I}{\delta\gamma^{ab(0)}} = \langle \mathcal{T}_{ab}\rangle = (\rho + p)\, u_a u_b + p\gamma^{(0)}_{ab}, \tag{12}$$

where

$$p = \frac{l^3}{8\pi G}\left[\frac{\mu}{2} + \frac{1}{8}\left(\alpha^2 - 4\alpha\beta + 8W(\beta)\right)\right], \tag{13}$$

$$\rho = \frac{l^3}{8\pi G}\left[\frac{3\mu}{2} - \frac{1}{8}\left(\alpha^2 - 4\alpha\beta + 8W(\beta)\right)\right], \tag{14}$$

with $u = \partial_t$. Note that when the scalar field vanishes (14) we recover the standard relation $\rho = 3p$ for a thermal gas of massless particles [33]. Using the Smarr formula of [21] we get

$$\mu\left(\varphi_h, \mathcal{A}\right) = l^{-4}\mu_h\left(\varphi_h\right)\mathcal{A}^{\frac{4}{3}}. \tag{15}$$

Inserting (2) in the first law of black hole thermodynamics, $\delta\rho = T\delta s$, with the knowledge of (15) and (10), it shows that the terms proportional to $\delta\mathcal{A}$ cancel, provided

$$\beta\left(\varphi_h, \mathcal{A}\right) = -\frac{1}{2}\alpha\ln\left|\frac{\alpha}{\alpha_0}\right| + \alpha z_h\left(\varphi_h\right), \tag{16}$$

where $z_h(\varphi_h)$ is a function of the value of the scalar at the horizon. The equations (10), (15) and (16) define the black hole surface.

4. Universal formula for the speed of sound

A useful consequence of the general considerations made so far, it is that there are two $\mathcal{A}$-independent functions

$$z \equiv \frac{\beta}{\alpha} + \frac{1}{2}\ln\left|\frac{\alpha}{\alpha_0}\right|, \qquad \frac{\mu}{\alpha^2} \equiv F(z) = \lim_{r\to\infty}\frac{(1-f)\,A^2}{4\varphi^2}. \tag{17}$$

The characterization of the speed of sound for a single scalar field theory can then be reduced to finding the black hole line $F(z)$. AdS-invariant boundary conditions correspond to $z = z^*$, where z^* is a fixed number. In this case the energy density, pressure and speed of sound are automatically $\rho = \frac{3l^3}{2\kappa}F(z^*)\alpha^2$, $p = \frac{l^3}{2\kappa}F(z^*)\alpha^2$, $c_s^2 = \left(\frac{\partial p}{\partial\rho}\right)_J = \frac{1}{3}$ where the partial derivative is taken at fixed source, $J = \alpha - \frac{dW(\beta)}{d\beta}$. It follows that there are infinite number of theories with $c_s^2 = \frac{1}{3}$. To move away from this point one should consider a generic boundary condition, of the form $z = \omega(\alpha)$, the knowledge of the black hole line (17) allows the construction of the derivative of the pressure and density at fixed source and finally from (13) and (14) a formula for the speed of sound:

$$\left(\frac{\partial p}{\partial\alpha}\right)_J = \frac{l^3}{8\pi G}\left[\alpha F + \frac{\alpha^2\dot{F}\omega'}{2} + \frac{1}{2}\alpha^2\omega'\right],$$
$$\left(\frac{\partial\rho}{\partial\alpha}\right)_J = \frac{l^3}{8\pi G}\left[3\alpha F + \frac{3}{2}\alpha^2\dot{F}\omega' - \frac{1}{2}\alpha^2\omega'\right], \tag{18}$$

$$c_s^2 = \left(\frac{\partial p}{\partial\rho}\right)_J = \frac{1}{3} + \frac{4}{3}\frac{\alpha\omega'}{6F + 3\alpha\dot{F}\omega' - \alpha\omega'}, \tag{19}$$

where $\dot{F} = \frac{dF}{dz}$, $\omega' = \frac{d\omega}{d\alpha}$. It follows that the speed of sound can be fully characterized using the black hole line defined by F and the boundary condition defined by $\omega(\alpha)$.

In holographic terms, (19) yields the dependence of the speed of sound on the VEV β associated with the source J. Note that the formula (19) is universal and can be constructed for any theory in terms of its black hole line and boundary condition. It is worth mentioning that the second term in (19) does not have a definite sign, so a priori it is not clear if the speed of sound for these theories will be bounded by the conformal value $c_s^2 = 1/3$. (19) indeed reproduces the result for a gas of massless particles for AdS invariant boundary conditions where $\omega' = 0$.

A minimal example that captures the ideas discussed here is a consistent single scalar field truncation of maximal supergravity in five dimensions:

$$V\left(\varphi\right) = -\frac{3}{2l^2}\left[3 + \cosh\left(\frac{2\sqrt{6}}{3}\varphi\right)\right], \tag{20}$$

which breaks the isometries of the S^5 to $SO(3)\times SO(3)$ [34]. Using the standard AdS/CFT dictionary, the operator dual to β is $\mathcal{O} = \frac{1}{N}\mathrm{Tr}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 - \phi_4^2 - \phi_5^2 - \phi_6^2\right)$ where the ϕ_I are the Super Yang–Mills scalars [35].

The AdS background with $\varphi = 0$ is maximally supersymmetric. Black hole solutions dressed with non-trivial scalars can be easily constructed numerically using a simple shooting method. Doing so for the theories determined from the potential (20) we find the black hole line shown in Fig. 1a. The function $F(z)$ acquires a minimum at $z_{\min} \approx 1.532$ at which $F(z_{\min}) \approx 0.975$. In Fig. 1b

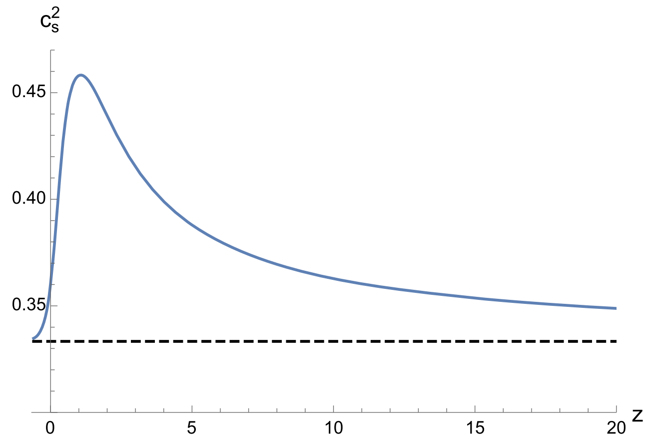


Fig. 2. Speed of sound as a function of z for the family of boundary conditions $\beta = \lambda\alpha$. The dashed line depicts the conformal value $c_s^2 = 1/3$, which is approached for large and small z.

we display the surface $\mu = \mu(\alpha, \beta)$ alongside with the boundary condition $\beta = \alpha$. Their intersection defines a uni-parametric family of black holes as described above. Note that because of the reflection symmetry $\varphi \to -\varphi$, the black hole surface satisfies $\mu(-\alpha, -\beta) = \mu(\alpha, \beta)$ (to ease visualization the region of negative α, β is not shown in Fig. 1b). Furthermore, our numerics indicate that black holes with different signs of α and β cannot exist.

For the linear boundary condition $\beta = \lambda\alpha$, we plot the speed of sound as a function of z in Fig. 2 — for this deformation the energy density is obtained by replacing $W = \frac{\beta^2}{2\lambda}$ and $\alpha = \frac{\beta}{\lambda}$ in (14). This yields $\rho = \frac{\alpha^2 l^3}{\kappa}\left(\frac{3}{2}F(z) - \frac{1}{8}\right)$. This is indeed an everywhere positive convex function for all the deformations we are considering. In the plots we have set $z = \frac{1}{2}\ln|\alpha| + 1$. So, when $\alpha \gg 1$, then z is very large and the deformed theory explores very high energies, where the conformal value for the speed of sound is recovered as one would expect. When $\alpha \sim 0$ then $z < 0$ and the deformation vanishes. The conformal value for the speed of sound is again recovered in this limit. In the intermediate regime the speed of sound is larger than the conformal value, attaining a maximum at $c_s^2 \approx 0.458$; a very interacting state of matter which might be relevant for the description of neutron stars, where the speed of sound is believed to be bounded only by the speed of light [22]. We note that the conjectured bound ($c_s^2 \leq 1/3$) for the speed of sound in theories with an holographic dual was only supposed to hold when there is no chemical potential [10]. This bound was indeed shown to be violated when a chemical potential is introduced in [12]. In this sense, this is the first example of the violation of the conjectured bound with an everywhere positive convex energy functional.

5. The renormalization group

The field theory variables α and β have an ambiguity when written in terms of the gravity variable φ, see (6). This is parametrized by the constant r_0 necessary to make the argument of the logarithm dimensionless. This is interpreted on the field theory side as a standard one-loop renormalization [24]. We take the bare coupling constant λ_Λ and the bare field β_Λ to be defined at the cutoff scale Λ and the renormalized coupling constant λ_ν and renormalized field β_ν defined at the mass scale ν. The independence of the bulk field from the renormalization procedure yields

$$\varphi l^{-4} = \frac{\beta_\nu\left(\lambda_\nu^{-1}\ln(r\nu) + 1\right)}{r^2} + O(\frac{\ln(r)^2}{r^3})$$

$$= \frac{\beta_\Lambda\left(\lambda_\Lambda^{-1}\ln(r\Lambda) + 1\right)}{r^2} + O(\frac{\ln(r)^2}{r^3}), \tag{21}$$

where the latter equality implies $\beta_\nu\lambda_\nu^{-1} = \beta_\Lambda\lambda_\Lambda^{-1}$ and $\lambda_\nu = \lambda_\Lambda + \ln(\Lambda/\nu)$ which has the form of a one loop renormalization of a dimensionless coupling constant [24]. In the plot of the speed of sound of the previous section we have set $z = \frac{1}{2}\ln|\alpha| + 1$, which means setting the renormalized coupling constant to one and measuring α in terms of the scale given by ν^2:

$$z = \frac{1}{2}\ln\left|\frac{\alpha}{\nu^2}\right| + \lambda_\nu. \tag{22}$$

One can obtain the relation between ν^2, λ_ν and the energy density ρ_ν with the equation $\rho_\nu = \frac{\nu^2 l^3}{\kappa}\left(\frac{3}{2}F(\lambda_\nu) - \frac{1}{8}\right)$. The energy density and the value of the dressed coupling constant are indeed necessary to compare the theoretical prediction with the experiment. A change in the values that we picked would imply a simple shift in the graph of Fig. 2.

6. Other scaling and spacetime dimensions

Scalar fields with mass in the BF window (1) fall-off as

$$\varphi \sim \frac{\alpha}{r^{\Delta_-}} + \frac{\beta}{r^{\Delta_+}} + O(r^{-\Delta_- -1}), \tag{23}$$

where $2 < \Delta_- < \Delta_+ = D - 1 - \Delta_-$ and $\frac{\Delta_+}{\Delta_-}$ is not an integer [19]. The energy density is known for these theories [20]. Using the Smarr formula of [21] and the fact that $\rho - (D-2)p$ vanishes for AdS invariant boundary conditions yields the pressure. In these cases the black hole surface takes the form

$$\alpha = \alpha_h(\varphi_h)\,\mathcal{A}^{\frac{\Delta_-}{D-2}}, \qquad \mu(\varphi_h, \mathcal{A}) = \mu_h(\varphi_h)\,\mathcal{A}^{\frac{D-1}{D-2}},$$
$$\beta = z(\varphi_h)\,\alpha^{\frac{\Delta_+}{\Delta_-}}, \tag{24}$$

and using the black hole line $\mu = F(z)\alpha^{\frac{D-1}{\Delta_-}}$ and the boundary condition $z = \omega(\alpha)$, the universal formula for the speed of sound is

$$c_s^2 = \frac{1}{D-2} + 2\frac{D-1}{D-2} \times \frac{\omega'\alpha(D-1-2\Delta_-)\Delta_-^2}{(D-2)(D-1)\left((D-1)F + \omega'\alpha\Delta_-\dot{F}\right) - 2\Delta_-^2(D-1-2\Delta\_)\omega'\alpha}. \tag{25}$$

7. Conclusions

We have investigated the role of mixed boundary conditions of the scalar field in the physics of hairy black holes withing gauge/gravity dualities. As we are going to detail below, our analysis is relevant in a broad sense and can be concretely used for phenomenological models in condensed matter, field theory, astrophysics, and cosmology.

We have crucially exploited that the physics of the system is not only a function of the bulk Lagrangian, but also of the boundary conditions of the scalar fields in the BF window (1). We have obtained a universal formula of the speed of sound in gravity theories with scalar fields where the boundary condition appears explicitly through ω in (25). Since similar techniques can be applied in cosmology, our result could be useful for obtaining the speed of sound of primordial fluctuations in supergravity models of inflation, e.g. [37]. Another important phenomenological consequence of our result is that the conformal bound on the speed of sound for strongly coupled field theories with gravity duals can be violated by choosing suitable boundary conditions. Therefore, one can model neutron stars withing gauge/gravity dualities and the bound on neutron star mass of [23] when the speed of sound is less than $1/\sqrt{3}$ is no longer relevant. As expected, the maximum value of the speed of sound is invariant under the RG flow of the deformation. Our analysis was possible due to the obtention of the map (24), which bring in a new general understanding of the non-linear equation (9). It is worth noting that the Smarr relation for planar black holes is necessary to construct our universal formula for the speed of sound. When the black hole has a different topology, in general there is no scaling relation and therefore, is not possible to find the IR-UV map that follows from it. Further analysis is required in these non-planar cases.

This deeper understanding of holography and gravity also sheds light in a long debate about the scalar charge in gravitational theories. By explicitly obtaining the map (2), we have shown that there is only one independent integration constant that characterizes the system, which can be also interpreted as a proof of a no-hair theorem (in the 'parlance' of general relativity, the hair is secondary). In [36] it was proposed that a the first law can have extra contributions due to the existence of more asymptotic integration constants. Our analysis proves that, indeed, there is no need for a modification of the first law by including the "scalar charges" and so the puzzle raised by [36] for asymptotically AdS hairy black holes is solved.

In summary, we have shown that an infinite number of inequivalent thermodynamic systems can be modeled using the same bulk Lagrangian by changing the boundary terms associated with deformations on the dual field theory. This provides an interesting mechanism that can be used to fit phenomenological data while keeping the gravity Lagrangian fixed. Of course, the set of deformations one is allowed to introduce are required to fulfill some physical criteria of consistency, such as positivity of the energy or reality of the speed of sound. The double trace deformation discussed in this paper is a simple example that provides a convex energy functional, so we expect this background solution to be stable under metric perturbations. A more ambitious and natural goal would be to fit the equation of state given by lattice QCD (for references see the recent review [38]) using our proposed mechanism. We expect to address this interesting point in future work.

Acknowledgements

We thank Andrei Starinets for valuable discussions. Research of AA is supported in part by FONDECYT Grants 1141073 and 1170279 and Newton-Picarte Grants DPI20140053 and DPI20140115. The work of TA is supported by the European Research Council under the European Union's Seventh Framework Programme (ERC Grant agreement 307955). The work of DA is supported by the FONDECYT Grant 1161418 and Newton-Picarte Grant DPI20140115. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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