Can we measure individual black-hole spins from gravitational-wave observations?

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Measurements of black-hole spins in gravitational wave (GW) observations with ground-based detectors are expected to be hampered by partial degeneracies between the two spins, and between the spins and the binary's mass ratio during the inspiral. If the inspiral and merger-ringdown parts of the GW signal happen to both be in the sensitive frequency band of a GW detector, can we hope to measure both spins and break this degeneracy? Are two-spin models really necessary or are single-spin models sufficient? Using Bayesian parameter estimation we will investigate these questions for a range of configurations over the parameter space for an effective-one-body reduced order model with spins aligned with the orbital angular momentum.

Keywords: Gravitational waves; black hole binaries; reduced order modeling; effective-one-body; MG14 Proceedings.

1. Introduction and Methods

The motivation for this study is twofold: (i) we want to investigate the *degeneracies* inherent in GWs emitted from BH binary coalescences and see how well individual spins can be measured under ideal circumstances using complete inspiral-merger-ringdown signals, (ii) we would like to find out whether two-spin models are really necessary for GW data analysis, or whether single-spin models will be sufficient.

To describe the waveform we use a *reduced order model* (ROM)[1,2] of the spin aligned *effective-one-body* model calibrated to numerical relativity simulations, "SEOBNRv2".[3] SEOBNRv2 solves a complicated system of ODEs and is too slow to be used directly for parameter estimation studies which routinely require millions of likelihood evaluations. The ROM provides speedups of several thousands and is essential to make such studies feasible.

We carry out a Bayesian parameter estimation study with the ensemble sampler `emcee`[4] with the following simplifying assumptions: We sample the 4-dimensional space of intrinsic binary parameters (component masses and aligned spins) while keeping extrinsic parameters, such as sky location, orientation, distance fixed. We use a likelihood that is maximized over recovered signal-to-noise ratio (SNR)[5] and time and phase of coalescence. We have checked that our qualitative results remain unchanged by comparing with a full multi-detector likelihood and the nested sampling code in `lalinference`.[6]

2. Degeneracies between Spin Parameters in Inspiral and Merger-ringdown

Complete GW signals from BH binary coalescences consist of *inspiral, merger and ringdown* parts. In the inspiral regime the waveform is well described by post-Newtonian techniques (PN).[7]. The study of PN waveforms in the Fisher matrix approximation[8,9] has shown that there exist strong correlations between binary parameters. The chirp mass $M_c = M_{\text{tot}}\eta^{3/5}$ is the best measured parameter, followed by the symmetric mass-ratio $\eta = m_1 m_2/M_{\text{tot}}^2$, and spins where the total mass is $M_{\text{tot}} = m_1 + m_2$.

The dimensionless aligned component spins χ_i enter the PN phase in a special combination, the "reduced spin"[8,10]

$$\chi = \chi_s + \delta\,\chi_a - \frac{76\eta}{113}\chi_s, \tag{1}$$

where $\chi_s = (\chi_1 + \chi_2)/2$, $\chi_a = (\chi_1 - \chi_2)/2$ and $\delta = (m_1 - m_2)/M_{\text{tot}}$. One can mimic the effect of mass-ratio on the waveform by changing the reduced spin.[11] χ can be measured reasonably well in the inspiral and therefore it will be difficult to infer the individual spins with good accuracy. In particular, for unequal mass systems, the spin on the larger object will primarily determine χ and will be measurable to much higher accuracy than the spin on the smaller body.

During the merger and ringdown the *final mass* and *final spin* are the quantities that can be most accurately measured[13,14], and these involve a different combination of spins than χ [12]. This leads us to expect that the accuracy of spin measurements can be improved by using complete waveforms over the PN measurement accuracy.

3. Can we Break the Spin Degeneracy?

We discuss the results of parameter estimation simulations for a range of configurations at mass-ratio $q = 4$, equal aligned spins $\chi_1 = \chi_2 = -0.9, 0, 0.5, 0.9$ and total mass chosen such that inspiral and merger ringdown both contribute significant power. We use the aLIGO design power spectral density for the detector noise with a lower cutoff of 10 Hz. We consider an optimistically high SNR of 30 for advanced ground based detectors.

Fig. 1 shows 2-dimensional posteriors for the symmetric mass-ratio η and reduced spin χ and for the aligned component spins χ_1 and χ_2. The $\eta - \chi$ posteriors are highly correlated and their slope depends strongly on the injected spins. For highly aligned spins this posterior becomes almost parallel to the η axis and thus allows very precise measurement of the reduced spin χ, while the measurement is worst for high anti-aligned spins.

The $\chi_1 - \chi_2$ posteriors also show significant correlation. While the spin on the larger BH χ_2 can be measured with decent accuracy (better for high aligned spins, worse for small or anti-aligned spins), the spin on the smaller BH χ_1 cannot be determined at all, except for highly aligned systems. In these cases, the Kerr limit

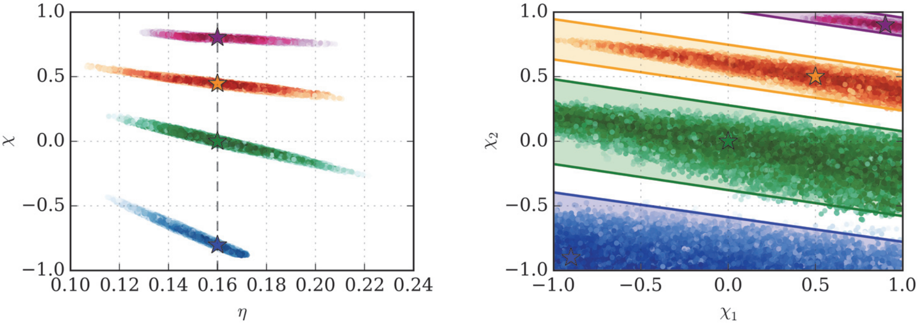

Fig. 1. The $\eta - \chi$ (left) and $\chi_1 - \chi_2$ (right) posteriors for injections at mass-ratio $q = 4$, total mass $M_{\rm tot} = 50_\odot$ and spins $\chi_1 = \chi_2 = -0.9, 0, 0.5, 0.9$ at SNR 30. Bands covering the support of the χ posterior obtained from a single-spin model have been overlaid on the $\chi_1 - \chi_2$ plot.

$|\chi_i| \leq 1$ constrains the individual spins, especially the spin of the small BH; but this is due to a boundary effect, *not* a break in the degeneracy.

In addition to the posteriors obtained from the two-spin model we have superimposed bands computed from the reduced spin posterior and mass-ratio posterior of a single spin model (using equal spins and Eq. (1)). We see that inference with a single-spin model leads to very similar conclusions on the support of the individual spin posteriors. Thus the question arises: when are two spin models really warranted and what additional information can they provide in practice? The two-spin model allows one to compute the maximum a posteriori probability for χ_1, a useful point estimate of the true value of χ_1, while a single-spin model only yields a distribution for χ which runs rather parallel to the χ_1 axis for unequal mass systems. The shape of credible regions (containing a certain amount of posterior probability) in the two component spins may also be of interest for high total mass, where the spins are less correlated.

The mass-ratio–spin degeneracy is not absolute, and at sufficient SNR we expect to be able to measure the second spin. How high must the SNR be? We consider a non-spinning system to avoid the χ_2 extreme-Kerr boundary. At high SNRs systematic errors in the ROM and the underlying EOB model become important. While the numbers may change a bit, we still expect to get a qualitatively correct picture from our analysis. Fig. 2 (right panel) shows a series of 95% credible regions for $q = 4$ and $M_{\rm tot} = 100M_\odot$ and we see that we need to go to SNRs of order 100 to learn something about χ_1. While this may appear an unrealistically high SNR, the situation is still much improved compared to low mass systems (Fig. 2 left panel) where SNRs of order 1000 would be required to restrict χ_1.[9].

The present study does not include effects of higher spherical harmonic modes of the GW signal because spinning inspiral-merger-ringdown waveforms including these effects are not yet available. Higher modes are expected to become important around total masses of $100M_\odot$ and for high mass-ratios[14,15] and their effect will be

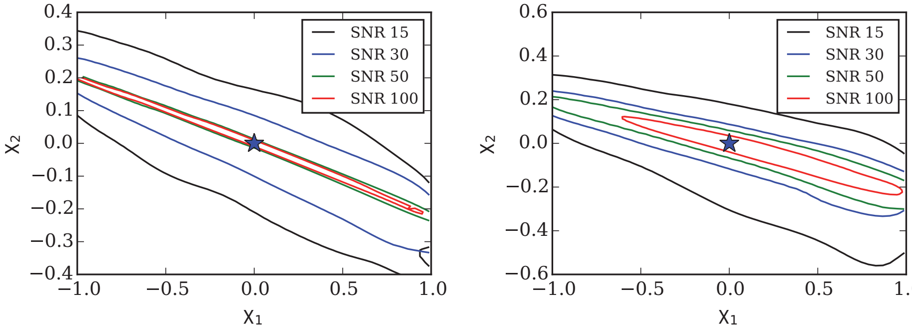

Fig. 2. 95% joint credible regions of the component spins χ_1, χ_2 for $q = 4$, $\chi_1 = \chi_2 = 0$, $M_{\rm tot} = 12M_\odot$ (left panel) $M_{\rm tot} = 100M_\odot$ (right panel) system are shown as a function of the SNR. The blue star denotes the injected spins.

investigated in future work. The waveform used here also does not include effects of precession of the orbital plane and of the BH spins. At total masses where the merger ringdown contributes significantly to the overall power the number of precession cycles is very small and therefore we do not expect precession change the qualitative picture. A more extensive study[16] discusses aligned-spin systems near equal-mass and at highly unequal mass.

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